Construction of the approximate stability equations of motion for an elastic cylindrical body under axial compressive load

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Abstract. A method of constructing nonlinear motion stability equations for isotropic elastic bodies is developed for cylindrical bodies of standard material of the 2nd order subjected to the action of “dead” axial compressive forces. The perturbation of the displacement vector is given by its approximate decomposition with respect to the base of tensor functions of the 1st, 2nd and 3d valences. The time-dependent tensor coefficients of the decomposition satisfy the system of ordinary differential equations. Based on the obtained equations one can investigate the balance stability of cylindrical bodies under various fixing conditions.

Key words: elastic body, stress tensor, the perturbation of the displacement vector, stability equations of motion, standard material of the 2nd order, compresive load.

INTRODUCTION

Mathematical models of mechanics of a deformable elastic body is a scientific basis for the development of engineering methods for design and technologies of manufacturing modern structures and devices. The most common model of the mechanical system is the classical mechanics model for an elastic (thermoelastic) body [7, 12, 14]. A number of works are devoted to constructing the mathematical models, development and generalization of the methods for solving spatial boundary value problems in the theory of elasticity and thermoelasticity for bodies of finite size e.g. [9, 15].

Cylindrical bodies often serve as bases of structures used in modern machine building, instrument ingeneering, civil ingeneering [1, 13]. In operational process effects undesirable in sense of strength and reliability of machinery, equipment, and buildings occur. Therefore problems of dynamical stability of structures, especially in nonlinear formulation, are among the most important in elasticity theory [10, 11].

As a rule, to construct the mathematical models of equations of motion (equilibrium) stability of elastic bodies one needs to know the basic (unperturbed) solution. It is usually found out by methods of quasi-static linear elasticity theory [4, 5, 6, 16].

We use the mathematical model [17] for solving spatial problems of nonlinear dynamic elasticity theory applying the method of expansion in tensor functions developed in [8]. In [2] and [3], based on the proposed mathematical model, we have constructed the equations for investigation of motion (balance) stability of elastic systems and, in particular, the linearized equations of motion stability for a body of Murnagan’s material. Next, we have derived (in a certain approximation) the system of ordinary differential equations of motion stability for a straight circular cylinder of Murnagan’s material and standard material of the 2nd order subjected to the action of the complex surface load.

In the present work on the basis of the technique, proposed in these works, we obtain the system of equations for investigation of motion stability of an elastic cylinder of standard material of the 2nd order subjected to the action of axial compressive forces.

MATHEMATICAL MODEL AND RESEARCH TECHNIQUE

In the mathematical model introduced in [17] we consider two configurations of an isotropic elastic body K. The reference γ_0 configuration is natural (undeformed), there are no stresses and deformations in the body. The actual γ_τ configuration is basic (unperturbed), it arises as a result of the action of body and surface forces on the body K starting from the moment $\tau = \tau_0$.

The vector $\vec{u}$ and the tensor P denote the perturbations of the displacement vector and the Piola-Kirchhoff stress tensor, respectively, in γ_τ -configuration. The vector $\vec{u}$ is given by its decomposition with respect to the base of tensor functions $\left\{\Phi^{(i-1)}(\vec{R}_0)\right\}$, where $\vec{R}_0 = \vec{r}_0 - \vec{r}_{30}$, $\vec{r}_0$ and $\vec{r}_{30}$ stand for the radius vector of an arbitrary point of the body K and the radius vector of some fixed point (e.g., the center of mass of the body K) in the actual configuration:

$$\vec{u} = \sum_{i=1}^{N} \Phi^{(i-1)}(\vec{R}_0) \overset{i-1}{\cdot} u^{(i)}(\tau) \ , \qquad (1)$$

where: the coefficients $u^{(i)}(\tau)$ are time-dependent tensor functions.

In [3] we formulate the problem of motion stability of a homogeneous elastic body K with the density ρ:

$$\int_{X_0} [\rho_0 \frac{d^2\vec{u}}{d\tau^2} \otimes \hat{\Phi}^{(i-1)} + \vec{\ni}_0^{\,k} \cdot \hat{P} \otimes \frac{\partial \hat{\Phi}^{(i-1)}}{\partial \xi^k}] dV_0 =$$
$$= \int_{X_0} \rho_0 \ \vec{f} \otimes \hat{\Phi}^{(i-1)} dV_0 + \int_{\partial X_0} \vec{n}_0 \cdot \hat{P} \otimes \hat{\Phi}^{(i-1)}] d\Sigma_0,$$
$$i = \overline{1, N}, \qquad (2)$$

with the boundary conditions:

$$\vec{n}_0 \cdot \hat{P}\Big|_{\partial X_0} = (\vec{q}_* \frac{d\Sigma_*}{d\Sigma_0} - \vec{q} \frac{d\Sigma}{d\Sigma_0})\Big|_{\partial X_0}. \qquad (3)$$

Here $\otimes$ denotes the operation of tensor product, $\{\xi^i\}$ are the Lagrange coordinates, $\{\vec{\ni}_i^{\,0}\}, \{\vec{\ni}_0^{\,j}\}$ are the vector basis in the reference configuration and the biorthogonal vector basis, respectively (for the Cartesian coordinate system they coincide), X_0, ∂X_0 are the domain of the body K and its boundary surface in the reference configuration, $\vec{f}$ is the perturbation of the vector of mass forces, $\vec{n}_0$ is the vector of outer normal to the surface ∂X_0, $\vec{q}$ is the vector of surface forces per unit area of the γ_τ-configuration, $\vec{q}_*$ is its value in the γ_τ^*-configuration, $d\Sigma_0, d\Sigma, d\Sigma_*$ are the surface area elements, respectively, in the γ_0, γ_τ, and γ_τ^*-configurations (γ_τ^* is the actual configuration, which corresponds to a perturbation of the initial conditions in the γ_τ–configuration).

The dynamic boundary conditions (3) are taken into account in the surface integral in the right-hand side of the relation (2). Note that, if the mass and the surface loads are "dead", then $\vec{q}_* d\Sigma_* = \vec{q} d\Sigma$, $\vec{f}_* = \vec{f}$ and, therefore, the right-hand side of (2) is equal to zero.

While investigating the stability of a straight circular cylinder of standard material of the 2nd order with the radius r and the height h, we remain three summands in the decomposition of the perturbation of the displacement vector (1) and take $\{\vec{R}_0^{\,N}\}$, where $\vec{R}_0^{\,k}$ is k-multiple tensor product of the vector $\vec{R}_0$ by itself, as the base of the decomposition, namely, we suppose that

$$\vec{u} = u^{(1)} + \vec{R}_0 \cdot u^{(2)} + \vec{R}_0 \otimes \vec{R}_0 \cdot\cdot u^{(3)}. \qquad (4)$$

We assume that the axis of the cylinder coincides with the axis $O\xi^3 (-\frac{h}{2} \le \xi^3 \le \frac{h}{2})$.

The Piola-Kirchhoff stress tensor for standard material of the 2nd order up to the second-order terms with respect to the gradient $\vec{\nabla}_0 \otimes \vec{u}_0$ has the following form [17]:

$$P_0(\vec{\nabla}_0 \otimes \vec{r}) = (I + \vec{\nabla}_0 \otimes \vec{u}_0) \cdot T(\vec{u}_0) +$$
$$+ \frac{1}{2} \lambda \vec{\nabla}_0 \otimes \vec{u}_0 \cdot\cdot \vec{\nabla}_0 \otimes \vec{u}_0^{\,T} I + \mu \vec{\nabla}_0 \otimes \vec{u}_0^{\,T} \cdot \vec{\nabla}_0 \otimes \vec{u}_0, \qquad (5)$$

where:

$$T(\vec{u}_0) = \lambda \vec{\nabla}_0 \cdot \vec{u}_0 I + 2\mu \varepsilon(\vec{u}_0),$$
$$\hat{\varepsilon}(\vec{u}_0) = \frac{1}{2}(\vec{\nabla}_0 \otimes \vec{u}_0 + \vec{\nabla}_0 \otimes \vec{u}_0^{\,T}), \qquad (6)$$

are the Cauchy stress tensor and the strain tensor in the linear elasticity theory, $\vec{\nabla}_0 = \vec{\ni}_0^{\,i} \frac{\partial}{\partial \xi^i}$ is the Hamilton nabla-operator in the reference configuration, $\hat{I}$ is the unit tensor, λ, μ are the Lame parameters, the index "T" denotes the transposition operation. Then the perturbation of the Piola-Kirchhoff stress tensor takes the form:

$$P = \vec{\nabla}_0 \otimes \vec{u}_0 \cdot T(\vec{u}_0) + \vec{\nabla}_0 \otimes \vec{u} \cdot T(\vec{u}_0) +$$
$$+ \lambda \vec{\nabla}_0 \otimes \vec{u}_0 \cdot\cdot \vec{\nabla}_0 \otimes \vec{u}^{\,T} I + \mu(\vec{\nabla}_0 \otimes \vec{u}_0^{\,T} \cdot \vec{\nabla}_0 \otimes \vec{u} +$$
$$+ \vec{\nabla}_0 \otimes \vec{u}^{\,T} \cdot \vec{\nabla}_0 \otimes \vec{u}_0) + (I + \vec{\nabla}_0 \otimes \vec{u}) \cdot T(\vec{u}) + \qquad (7)$$
$$+ \frac{1}{2} \lambda \vec{\nabla}_0 \otimes \vec{u} \cdot\cdot \vec{\nabla}_0 \otimes \vec{u}_0^{\,T} I + \mu \vec{\nabla}_0 \otimes \vec{u}^{\,T} \cdot \vec{\nabla}_0 \otimes \vec{u}.$$

Substituting (4) and (7) into (2) and taking some transforms we obtain the system of three tensor ordinary differential equations of the motion stability for a given cylinder of standard material of the 2nd order. Neglecting nonlinear summands we obtain the approximate linearized system of the motion stability for such cylindrical body [2]:

$$\sum_{\beta=1}^{3} [\rho_0 \hat{M}^{(i+\beta)} \overset{\beta}{\cdot} \frac{d^2 \hat{u}^{(\beta)}}{d\tau^2} + \hat{J}^{(i+\beta)} \overset{\beta}{\cdot} \hat{u}^{(\beta)}] = \hat{F}^{(i)}, i = \overline{1,3}, \qquad (8)$$

where:

$$M^{(i+\beta)} = \int_{X_0} \vec{\ni}_0^{\,s} \otimes \Phi^{(i-1)} \otimes \vec{\ni}_s^{\,0} \otimes \Phi^{(\beta-1)} dV_0,$$
$$J^{(i+\beta)} = \delta^{kt} \int_{X_0} A_{ts}^{qj} \ \vec{\ni}_0^{\,s} \otimes \frac{\partial \Phi^{(i-1)}}{\partial \xi^k} \otimes \vec{\ni}_q^{\,0} \otimes \frac{\partial \Phi^{(\beta-1)}}{\partial \xi^j} dV_0, \qquad (9)$$
$$F^{(i)} = \int_{X_0} \rho_0 \vec{f} \otimes \Phi^{(i-1)} dV_0 + \int_{\partial X_0} \vec{n}_0 \cdot P \otimes \Phi^{(i-1)} d\Sigma_0,$$
$$A_{ts}^{qj} = \lambda \delta^{qj} \delta_{st} + \lambda a_{ts} \delta^{qj} + \lambda a^{jq} \delta_{st} + \mu(\delta_t^i \delta_s^q +$$
$$+ \delta_t^q \delta_s^j) + 2\mu \varepsilon_{t\cdot}^{\cdot j} \delta_s^q + t_{\cdot s}^{q\cdot} \delta_t^j + \mu a_{t\cdot}^{\cdot q} \delta_s^j + \mu a_{\cdot s}^{j\cdot} \delta_t^q,$$
$$\vec{\nabla}_0 \otimes \vec{u}_0 = a^{ij} \ \vec{\ni}_i^{\,0} \otimes \vec{\ni}_j^{\,0} = a_{ij} \ \vec{\ni}_0^{\,i} \otimes \vec{\ni}_0^{\,j} = a_{\cdot j}^{i\cdot} \ \vec{\ni}_i^{\,0} \otimes \vec{\ni}_0^{\,j} =$$
$$= a_{i\cdot}^{\cdot j} \ \vec{\ni}_0^{\,i} \otimes \vec{\ni}_j^{\,0},$$

$$T(\vec{u}_0) = t^{ij}\, \vec{\ni}_i^{\,0} \otimes \vec{\ni}_j^{\,0} = t_{ij}\, \vec{\ni}_0^{\,i} \otimes \vec{\ni}_0^{\,j} = t_{.j}^{i.}\, \vec{\ni}_i^{\,0} \otimes \vec{\ni}_0^{\,j} = t_{i.}^{.j}\, \vec{\ni}_0^{\,i} \otimes \vec{\ni}_j^{\,0},$$

$$\varepsilon(\vec{u}_0) = \varepsilon^{ij}\, \vec{\ni}_i^{\,0} \otimes \vec{\ni}_j^{\,0} = \varepsilon_{ij}\, \vec{\ni}_0^{\,i} \otimes \vec{\ni}_0^{\,j} = \varepsilon_{.j}^{i.}\, \vec{\ni}_i^{\,0} \otimes \vec{\ni}_0^{\,j} = \varepsilon_{i.}^{.j}\, \vec{\ni}_0^{\,i} \otimes \vec{\ni}_j^{\,0}, \quad (10)$$

$\delta^{ij} = \delta_{ij} = \delta_j^i$ are the Kronecker symbols.

APPROXIMATE LINEARIZED EQUATIONS OF THE MOTION STABILITY FOR A CYLINDRICAL BODY OF STANDARD MATERIAL OF THE 2ND ORDER UNDER AXIAL COMPRESSIVE LOADS

Let a given cylindrical body be subjected to the action of axial compressive forces which in the actual configuration are specified by the stress vector:

$$\vec{q} = \begin{cases} -N_0 \overrightarrow{n_0}, & (\xi^1, \xi^2, \xi^3) \in \Sigma_1 \cup \Sigma_2, \\ 0, & (\xi^1, \xi^2, \xi^3) \in \Sigma_3 \end{cases}, \quad (11)$$

where: Σ_1, Σ_2 are the upper and the lower bases of the cylinder, and Σ_3 is its lateral surface, N_0 is the density of axial compressive forces uniformly distributed along Σ_1 and Σ_2. The mass load is assumed to be “dead”. As a base solution we take the solution of a corresponding problem formulated within the framework of the stationary linear elasticity theory [12]

$$\vec{u}_0 = \nu Q(\xi^1 \vec{\ni}_1^{\,0} + \xi^2 \vec{\ni}_2^{\,0}) - Q\xi^3 \vec{\ni}_3^{\,0}, \quad (12)$$

where: $Q = \dfrac{N_0}{E}, \nu = \dfrac{\lambda}{2(\lambda+\mu)}$ is the Poisson coefficient, and $E = 2\mu(1+\nu)$ is the Young’s elastic modulus.

We obtain from (12):

$$\vec{\nabla}_0 \otimes \vec{u}_0 = \nu Q\, \vec{\ni}_\alpha^{\,0} \otimes \vec{\ni}_0^{\,\alpha} - Q\, \vec{\ni}_3^{\,0} \otimes \vec{\ni}_3^{\,0},$$

$$\varepsilon(\vec{u}_0) = \nu Q\, \vec{\ni}_\alpha^{\,0} \otimes \vec{\ni}_0^{\,\alpha} - Q\, \vec{\ni}_3^{\,0} \otimes \vec{\ni}_3^{\,0},$$

$$\vec{\nabla}_0 \cdot \vec{u}_0 = (2\nu - 1)Q, \quad T(\vec{u}_0) = -EQ\, \vec{\ni}_3^{\,0} \otimes \vec{\ni}_3^{\,0},$$

whence, we have

$$a^{ij} = \varepsilon^{ij} = \begin{cases} \nu Q, & i = j = \overline{1,2}, \\ -Q, & i = j = 3, \\ 0, & i \neq j, \end{cases} \quad (13)$$

$$t^{ij} = \begin{cases} -EQ, & i = j = 3 \\ 0, & otherwise \end{cases}.$$

Calculating coefficients (9) with regard for (13), substituting them into linearized system (8) and performing appropriate convolutions, we obtain the approximate linearized system of the motion stability for the problem in coordinate form:

$$8\frac{d^2u_m}{d\tau^2} + \tau^2\left(\frac{d^2u_{11m}}{d\tau^2} + \frac{d^2u_{22m}}{d\tau^2}\right) + \frac{h^2}{3}\frac{d^2u_{33m}}{d\tau^2} = \frac{8F_m}{\rho_0 V} \quad (m = 1,2,3),$$

$$3\rho_0 r^2 \frac{d^2u_{11}}{d\tau^2} + 12(1+2\nu Q)((\lambda+2\mu)u_{11} + \lambda u_{22}) + 12\lambda(1+Q(\nu-1))u_{33} = \frac{12F_{11}}{V},$$

$$\rho_0 h^2 \frac{d^2u_{33}}{d\tau^2} + 12(1+Q(\nu-1))(u_{11}+u_{22}) + 12((\lambda+2\mu)(1-2Q) - EQ)u_{33} = \frac{12F_{33}}{V},$$

$$3\rho_0 r^2 \frac{d^2u_{12}}{d\tau^2} + 12\mu(1+2\nu Q)(u_{12}+u_{21}) = \frac{12F_{21}}{V},$$

$$3\rho_0 r^2 \frac{d^2u_{21}}{d\tau^2} + 12\mu(1+2\nu Q)(u_{12}+u_{21}) = \frac{12F_{12}}{V},$$

$$3\rho_0 r^2 \frac{d^2u_{13}}{d\tau^2} + 12\mu((1-2Q)u_{13} + (1+Q(\nu-1))u_{31}) = \frac{12F_{31}}{V},$$

$$\rho_0 h^2 \frac{d^2u_{31}}{d\tau^2} + 12\mu(1+Q(\nu-1))u_{13} + (1-2Q)u_{31}) = \frac{12F_{13}}{V},$$

$$3\rho_0 r^2 \frac{d^2u_{23}}{d\tau^2} + 12\mu((1-2Q)u_{23} + (1+Q(\nu-1))u_{32}) = \frac{12F_{32}}{V},$$

$$\rho_0 h^2 \frac{d^2u_{32}}{d\tau^2} + 12\mu((1+Q(\nu-1))u_{23} + (1-2Q)u_{32}) = \frac{12F_{23}}{V},$$

$$\rho_0[12\frac{d^2u_1}{d\tau^2}+r^2(3\frac{d^2u_{111}}{d\tau^2}+\frac{d^2u_{221}}{d\tau^2})+\frac{h^2}{2}\frac{d^2u_{331}}{d\tau^2}]++24[(1+2\nu Q)((\lambda+2\mu)u_{111}+\lambda u_{122})+$$

$$+\lambda(1+Q(\nu-1))u_{133}]=\frac{48F_{111}}{Vr^2},$$

$$\rho_0[12\frac{d^2u_2}{d\tau^2}+r^2(3\frac{d^2u_{112}}{d\tau^2}+\frac{d^2u_{222}}{d\tau^2})+\frac{h^2}{2}\frac{d^2u_{332}}{d\tau^2}]+24\mu(1+2\nu Q)(u_{112}+u_{211})=\frac{48F_{211}}{Vr^2},$$

$$\rho_0[12\frac{d^2u_3}{d\tau^2}+r^2(3\frac{d^2u_{113}}{d\tau^2}+\frac{d^2u_{223}}{d\tau^2})+\frac{h^2}{2}\frac{d^2u_{333}}{d\tau^2}]+24\mu[(1-2Q)u_{113}+(1+Q(\nu-1))u_{311}]=\frac{48F_{311}}{Vr^2},$$

$$\rho_0[12\frac{d^2u_1}{d\tau^2}+r^2(3\frac{d^2u_{111}}{d\tau^2}+\frac{d^2u_{221}}{d\tau^2})+\frac{h^2}{2}\frac{d^2u_{331}}{d\tau^2}]+24\mu(1+2\nu Q)(u_{122}+u_{221})=\frac{48F_{122}}{Vr^2},$$

$$\rho_0[12\frac{d^2u_2}{d\tau^2}+r^2(\frac{d^2u_{112}}{d\tau^2}+3\frac{d^2u_{222}}{d\tau^2})+\frac{h^2}{2}\frac{d^2u_{332}}{d\tau^2}]+24[(1+2\nu Q)(\lambda u_{211}+(\lambda+2\mu)u_{222})+$$

$$+\lambda(1+Q(\nu-1))u_{233}]=\frac{48F_{222}}{Vr^2},$$

$$\rho_0[12\frac{d^2u_3}{d\tau^2}+r^2(\frac{d^2u_{113}}{d\tau^2}+\frac{d^2u_{223}}{d\tau^2})+\frac{h^2}{2}\frac{d^2u_{333}}{d\tau^2}]+24\mu[(1-2Q)u_{223}+(1+Q(\nu-1))u_{322}]=\frac{48F_{322}}{Vr^2},$$

$$\rho_0[4\frac{d^2u_1}{d\tau^2}+\frac{r^2}{2}(\frac{d^2u_{111}}{d\tau^2}+\frac{d^2u_{221}}{d\tau^2})+\frac{3h^2}{10}\frac{d^2u_{331}}{d\tau^2}]+8\mu[(1+Q(\nu-1))u_{133}+(1-2Q)u_{331}]=\frac{48F_{133}}{Vh^2},$$

$$\rho_0[4\frac{d^2u_2}{d\tau^2}+\frac{r^2}{2}(\frac{d^2u_{112}}{d\tau^2}+\frac{d^2u_{222}}{d\tau^2})+\frac{3h^2}{10}\frac{d^2u_{332}}{d\tau^2}]+8\mu[(1+Q(\nu-1))u_{233}+(1-2Q)u_{332}]=\frac{48F_{233}}{Vh^2},$$

$$\rho_0[4\frac{d^2u_3}{d\tau^2}+\frac{r^2}{2}(\frac{d^2u_{113}}{d\tau^2}+\frac{d^2u_{223}}{d\tau^2})+\frac{3h^2}{10}\frac{d^2u_{333}}{d\tau^2}]+$$

$$+8[\lambda(1+Q(\nu-1))(u_{311}+u_{322})+((\lambda+2\nu)(1-2Q)-EQ)u_{333}]=\frac{48F_{333}}{Vh^2},$$

$$\rho_0r^2\frac{d^2u_{211}}{d\tau^2}+6[\lambda(1+Q(\nu-1))u_{233}+(1+2\nu Q)((\lambda+3\mu)u_{211}+\lambda u_{222}+\mu u_{112})]=\frac{24F_{112}}{Vr^2},$$

$$\rho_0r^2\frac{d^2u_{122}}{d\tau^2}+6[(1+2\nu Q)(\lambda u_{111}+(\lambda+3\mu)u_{122}+\mu u_{221})+\lambda(1+Q(\nu-1))u_{133}]=\frac{24F_{212}}{Vr^2},$$

$$\rho_0r^2\frac{d^2u_{123}}{d\tau^2}+6\mu[(1+Q(\nu-1))(u_{231}+u_{132})+2(1-2Q)u_{123}]=\frac{24F_{312}}{Vr^2},$$

$$\rho_0r^2h^2\frac{d^2u_{311}}{d\tau^2}+4h^2[(1+2\nu Q)(\lambda+2\mu)u_{311}+\lambda u_{322})+\lambda(1+Q(\nu-1))u_{333}]+$$

$$+12\mu r^2[(1+Q(\nu-1))u_{113}+(1-2Q)u_{311}]=\frac{48F_{113}}{V},$$

$$\rho_0r^2h^2\frac{d^2u_{231}}{d\tau^2}+4\mu h^2(1+2\nu Q)(u_{312}+u_{231})+12\mu r^2[(1+Q(\nu-1))u_{123}+(1-2Q)u_{231}]=\frac{48F_{123}}{V},$$

$$\rho_0r^2h^2\frac{d^2u_{132}}{d\tau^2}+4\mu h^2(1+2\nu Q)(u_{312}+u_{231})+12\mu r^2[(1+Q(\nu-1))u_{123}+(1-2Q)u_{231}]=\frac{48F_{213}}{V},$$

$$\rho_0r^2h^2\frac{d^2u_{322}}{d\tau^2}+4h^2[(1+2\nu Q)(\lambda u_{311}+(\lambda+2\mu)u_{322})+\lambda(1+Q(\nu-1))u_{333}]+$$

$$+12\mu r^2[(1+Q(\nu-1))u_{223}+(1-2Q)u_{322}]=\frac{48F_{223}}{V},$$

$$\rho_0 r^2 h^2 \frac{d^2 u_{133}}{d\tau^2} + 4\mu h^2[(1+Q(\nu-1))u_{331} + (1-2Q)u_{133}] + 12r^2[2(\lambda+2\mu)+E)u_{33} +$$

$$+\lambda(1+Q(\nu-1))(u_{111}+u_{122}) + ((\lambda+2\mu)(1-2Q)-EQ)u_{133}] = \frac{48F_{313}}{V},$$

$$\rho_0 r^2 h^2 \frac{d^2 u_{233}}{d\tau^2} + 4\mu h^2[(1+Q(\nu-1))u_{332} + (1-2Q)u_{233}] + 12r^2 \tag{14}$$

$$[\lambda(1+Q(\nu-1))(u_{211}+u_{222}) + ((\lambda+2\mu)(1-2Q)-EQ)u_{233}] = \frac{48F_{323}}{V}.$$

Since:

$$\vec{q}\,\frac{d\Sigma}{d\Sigma_0}\Big|_{\Sigma_1\cup\Sigma_2} = -N_0\vec{n}_0\frac{d\Sigma}{d\Sigma_0}\Big|_{\Sigma_1\cup\Sigma_2} \text{ and}$$

$$\vec{q}_*\frac{d\Sigma_*}{d\Sigma_0}\Big|_{\Sigma_1\cup\Sigma_2} = -N_0\vec{n}_0\frac{d\Sigma_*}{d\Sigma_0}\Big|_{\Sigma_1\cup\Sigma_2},$$

the boundary conditions (3) for the problem are of the form:

$$\vec{n}_0 \cdot P\Big|_{\Sigma_1\cup\Sigma_2\cup\Sigma_3} = 0. \tag{15}$$

Hence, if the mass and the surface loads are “dead”, then the right-hand sides of the equations (14) are equal to zero.

CONCLUSIONS

On the basis of constructed in [Ban-Kol-2003] equations for the investigation of motion (balance) stability of elastic systems we have obtained the system of ordinary differential equations of the motion stability for a straight circular elastic cylinder of standard material of the 2nd order subjected to the action of axial compressive forces. The obtained equations enables us to investigate the balance stability of the cylinder under various fixing conditions.

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