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ROBUSTNESS STUDY OF THE ROAD PROFILE ESTIMATION TECHNIQUE UNDER UNCERTAINTY

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This paper studies profile estimation of a road. The prediction has been achieved using the Independent Component Analysis Method (ICA). The vehicle dynamic responses were calculated for different road profiles which were defined using an ISO norm. The robustness of this method was proven by implementing the stochastic Monte Carlo (MC) technique in the presence of inevitable uncertainty parameters simultaneously associated with the vehicle mass, spring stiffness and damping for different vehicle speeds and wind values. Convergence was assessed when comparing real profiles to simulated ones. The obtained results prove the efficiency of the ICA in estimating the profile variabilities under uncertainties.

Keywords: Independent Component Analysis (ICA) method, Monte Carlo (MC) method, road profiles, estimation

1. Introduction

The road profile is a significant factor that affects the vehicle performance especially riding comfort and road handling. Hence, detection of such data is crucial for accurate knowledge of the vehicle behavior on one hand and for active vehicle control systems design, on the other (Nodeh *et al*., 2021; Doumiati *et al*., 2017). The collection of road profile variability was the topic of a good number of studies. Some of these used instrumented vehicle relying on direct measurements of road irregularities such as profilographs (American Society of Testing and Materials, 2008), profilometers (Healey *et al*., 1997), laser and cameras (Xue *et al*., 2020). Nevertheless, these instruments have high operation costs so their frequent use in detecting the road profile is impractical. Adding to that, in snowy environments, laser sensors cannot be used (Nodeh *et al*., 2021). Several researchers tried to overcome these drawbacks, suggesting the use of road estimators based on accelerometers which are easier to process. For instance, González *et al.* (2008) used accelerometers to estimate road roughness at a constant vehicle speed but this estimation was not confirmed when the speed changed. Similarly, Hong *et al.* (2002) proposed an estimation method at a constant speed based on the Fourier Transform of the road. Other studies focused on estimation of the road profile based on vehicle dynamic responses. Among these studies, we can cite that of Imine *et al.* (2005) in which the authors presented a model-based sliding mode observer of a full-car model, where the speed was considered constant. In Heyns *et al.* (2012), a road roughness monitoring system was suggested using a Bayesian estimator under variable speed, but it required a priori information of the road. Fauriat *et al.* (2016) proposed an algorithm based on the cross-entropy method that used Monte Carlo simulations in order to get the optimal road profile estimation. This numerical technique, however, needs a long computation time. In a real context, the suspension system model is multi-dimensional so its parameters are uncertain due to variation of the sprung mass, tire stiffness and damping. In this context, Chaabane *et al.* (2019) proposed a comparison between the augmented Kalman filter estimation technique and the Independent Component Analysis (ICA) method. The authors showed that the Kalman filtering displayed greater sensitivity to both sprung mass and vehicle speed variations. However, it was remarked that the Kalman filter displayed certain losses in the frequency spectrum mainly at high speeds (Chaabane *et al.*, 2019) compared to the ICA. So, for an accurate road profile estimation, the uncertain parameters of the vehicle model have to be taken into account. To this end, this paper proposes the use of the ICA technique to reconstruct the road variability. Ben Hassen *et al.* (2019b) has already referred to this method and estimated the road profile using dynamic responses of the full vehicle model. As shown in the indicated study and many other research works, the proposed method was fast, easy and simple to apply in order to estimate the road profile. It was also used for different vehicle models (quarter car, pitch model, roll model) and exhibited a high accuracy in the estimation process. The road profiles were adopted according to ISO 8608 standard. Ben Hassen *et al.* (2019a) proved that the ICA technique can estimate road irregularities even with the use of a non--linear vehicle model (non-linear suspension spring and damper). A great deal of research dealt with uncertainties. Among the used methods in these works, we can cite the Monte Carlo MC method which presents a simple process to carry out the uncertainty propagation (Papadrakakis and Kotsopulos, 1999). Several researchers referred to this method to study robustness. As an illustration, we can mention that of Fonseca *et al.* (2007) where the authors used a novel probabilistic method to optimize the robust design of a beam truss based on the MC method. The obtained result was checked with a regular MC simulation. Che and Wang (2014) proposed to use the MC optimization technique to study the robustness of a new product design. The idea was to adjust the precision value of random design variables in MC experiments and then to validate the method. A case study was brought forward.

In this paper, relying on the previous studies we proposed to estimate several road profiles using the ICA method. A quarter vehicle model was adopted for the purpose. The novelty lies in the use of an MC method-based optimisation technique to study the ICA robustness under uncertainty. In a real case, the sprung mass, tire stiffness and damping are variable parameters. Using the MC technique, a different set of random values of these parameters are computed; then, the estimation process is applied under these uncertainties. Also the influence of the vehicle speed and noise variation (wind) are taken into account. Comparing the obtained results with the real road profiles, the ICA efficiency was proven.

The remainder of this paper is organized as follows: in first Section, the studied quarter car model was introduced together with the road profile construction. In the second Section, the MC theory was detailed. In the third Section, the ICA technique was applied and the simulation results were described; a good accuracy between the original signals and the estimated ones was achieved. Finally, Section 5 was devoted to study of the proposed method robustness varying the sprung mass, stiffness and damping and using the MC optimisation technique under several vehicle and wind speeds. Section 6 drew the main conclusions.

2. Studied model

In this Section, a combination of the ICA algorithm and the Monte Carlo method was achieved. The ICA uses dynamic responses of the vehicle model as input signals to estimate the road profile. The efficiency of this estimation was studied using the Monte Carlo method which takes into account the uncertainty of such parameters as the sprung mass, stiffness and damping. The following flowchart describes different steps we followed to study the ICA estimation robustness, taking into account parameter uncertainties.

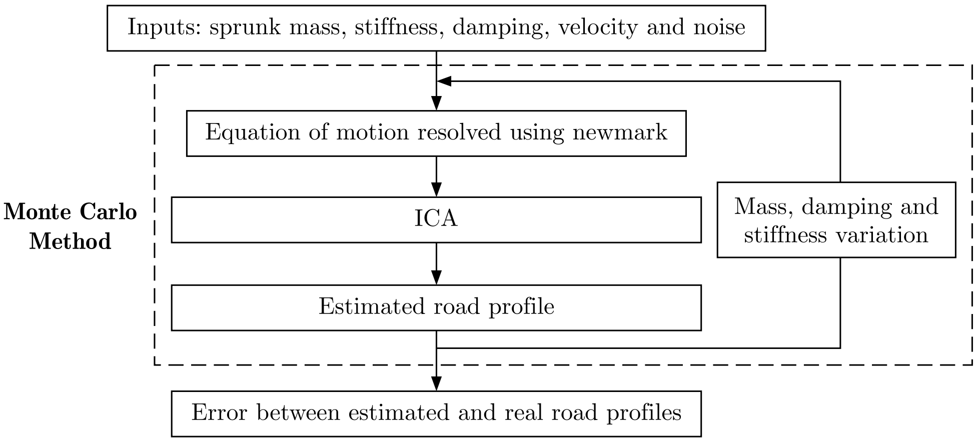


Fig. 1. ICA and MC coupling method

2.1. Vehicle model

In this study, the vehicle is modelled using the quarter car model as it is the most commonly used model to describe vehicle performances (Chaabane *et al.*, 2019). The vehicle is modelled by two masses m_1 and m_2 linked by a suspension with the stiffness k_1 and the damping c_1. The contact between the road (defined by its profile $r(t)$) and the vehicle wheel was modelled by a linear stiffness k_2. As shown in Fig. 2, the model has two degrees of freedom: X_1 and X_2 depicting the m_1 and m_2 displacements, respectively.

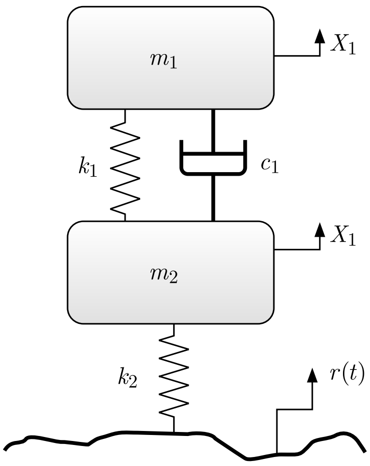


Fig. 2. Quarter car model

The equations of motion associated to the system under study are written as follows

$$
\begin{aligned}
&m_1\dot{X}_1 + k_1(X_1 - X_2) + c_1(\dot{X}_1 - \dot{X}_2) = 0 \\
&m_2\dot{X}_2 + k_1(X_1 - X_2) + c_1(\dot{X}_1 - \dot{X}_2) + k_1[X_2 - r(t)] = 0
\end{aligned}
\tag{2.1}
$$

The suspension parameters used in this paper are given in Table 1. These values were provided by Fauriat *et al.* (2016).

2.2. Road disturbance modelling: random road profile

In this study, the external excitation due to the road disturbance is expressed using the random road profile, which has been constructed according to ISO 8608 standard given in Table 2.

Table 1. Suspension system parameters

Parameter	Value	Unit
Sprung mass	$m_1 = 372$	kg
Unsprung mass	$m_2 = 59$	kg
Suspension stiffness	$k_1 = 36540$	N/m
Tire stiffness	$k_2 = 242000$	N/m
Suspension damping	$c_1 = 3300$	Ns/m

Table 2. Road profile classification

Road class	Degree of roughness $G_d(n_0)$ $[10^{-6}\mathrm{m}^3]$		
	Lower limit	Geometric mean	Upper limit
Road A	–	16	32
Road B	32	64	128
Road C	128	256	512
Road D	512	1024	2048
Road E	2048	4096	8192

The Integral White Noise Model method has been applied to establish the road roughness. It considers the road roughness as the result of a filtered white noise by

$$\dot{r}(t) = 2\pi n_0 \sqrt{G_d(n_0) V W_1(t)} \tag{2.2}$$

where $n_0 = 0,1$ cycle/m is the reference spatial frequency, $G_d(n_0)$ is the displacement PSD (given in Table 2), V is the vehicle velocity (equal to 15 m/s in our case study) and $W_1(t)$ is the Gaussian white noise with a variance equal to 1.

This road disturbance affects dynamic responses of the vehicle. These responses are measured numerically using the Newmark algorithm and then they are inserted in the ICA algorithm as observed signals denoted by $\{\mathbf{V}_{observed}\}$. This vector is composed of the mixing matrix $[\mathbf{M}_{mixing}]$ and the source signals $\{\mathbf{S}_{original}\}$ which are the road profiles as follows

$$\{\mathbf{V}_{observed}\} = [\mathbf{M}_{mixing}]\{\mathbf{S}_{original}\} \tag{2.3}$$

Generation of the estimated road profiles $\{\mathbf{S}_{estimated}\}$ by ICA requires some assumptions (Chaabane *et al.*, 2019): firstly, the elements of $\{\mathbf{S}_{estimated}\}$ should be statistically independent and secondly, the number of generated signals is equal to the observed ones. Then, $[\mathbf{M}_{unmixing}]$ which is defined as the inverse of $[\mathbf{M}_{mixing}]$ is computed and the estimated vector can be written as

$$\{\mathbf{S}_{estimated}\} = [\mathbf{M}_{unmixing}]\{\mathbf{V}_{observed}\} \tag{2.4}$$

We mentioned that some pre-treatments: centering and whitening (Chaabane *et al.*, 2019) are applied to the vector$\{\mathbf{V}_{observed}\}$. Thus, $[\mathbf{M}_{unmixing}]$ is determined and finally, $\{\mathbf{S}_{estimated}\}$ will be equal to

$$\{\mathbf{S}_{estimated}\} = [\mathbf{M}_{unmixing}]^H\{\mathbf{V}_{observed}\} \tag{2.5}$$

where $(\cdot)^H$ presents a conjugate-transpose operator.

The ICA method swaps the estimated sources and random profiles in order to get the exact road profile. Then, the MC algorithm is applied to study the robustness of the ICA under uncertainties which are the driver mass, the suspension stiffness and the damping variation. To be more realistic, the influence of the vehicle speed and the noise are also taken into account.

3. Monte Carlo theory

It is well known that the MC theory is a reliable method with successive resolutions used for deterministic systems including uncertain parameters which are expressed by random variables. Random samples are established for each uncertain parameter considering their correlations and their probability distributions. A set of parameters and deterministic calculation are estimated for each based on analytical or numerical model. The MC method is known for its ability to be applied to all system sizes and complexity degrees. A large number of iterations is needed for accurate results. The standard MC approach is described by the following equation

$$\mathbf{Y} = \mathbf{M}(\mathbf{X}) \tag{3.1}$$

where $\mathbf{M}$ describes the model under consideration, the vector $\mathbf{X} = [X_1, X_2, \ldots, X_n]^{\mathrm{T}}$ represents the uncertain input parameters while the vector $\mathbf{Y}$ contains random values of the estimated outputs. The method essentially consists of 5 steps as described in Fig. 3.

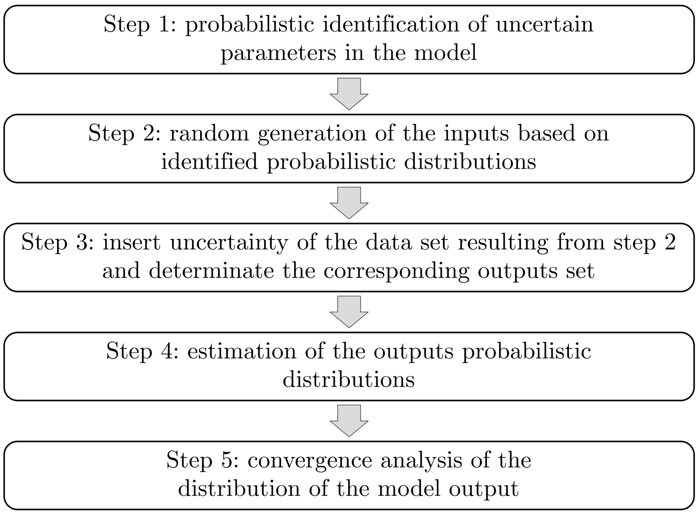


Fig. 3. MC algorithm steps

4. ICA results and discussion

4.1. Dynamic responses

Three road profile types are considered for this study: road A, road C and road E, where A is chosen as a very good quality road, C has a medium roughness and finally E is the most disturbed road. The dynamic responses corresponding to each profile type, which are the inputs for the ICA algorithm, are given in Fig. 4. Only the sprung mass acceleration is introduced as observed signals.

As presented in Fig. 4, the sprung mass acceleration is the only known input for the ICA. Starting from each response, the ICA would estimate the corresponding road profile.

4.2. Estimation of the road profile

Based on the dynamic responses of the quarter car model, the ICA method accurately estimates different road profiles as presented in Fig. 5. This figure displays a comparison between the real and estimated profiles for different road types: Figs. 5a, 5b and 5c correspond to roads A, C and E, respectively.

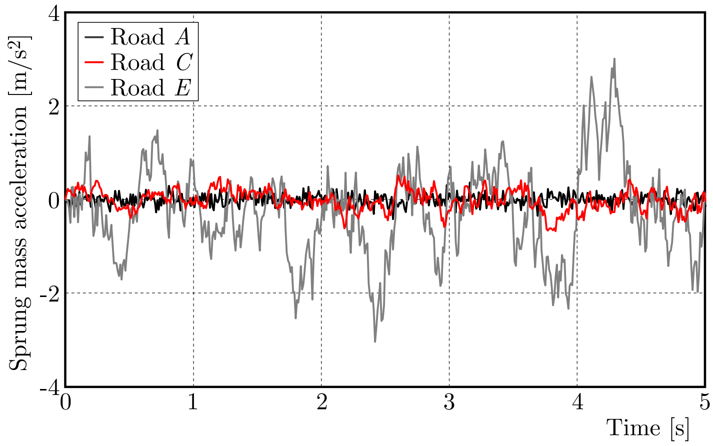


Fig. 4. Observed signals corresponding to several road profiles

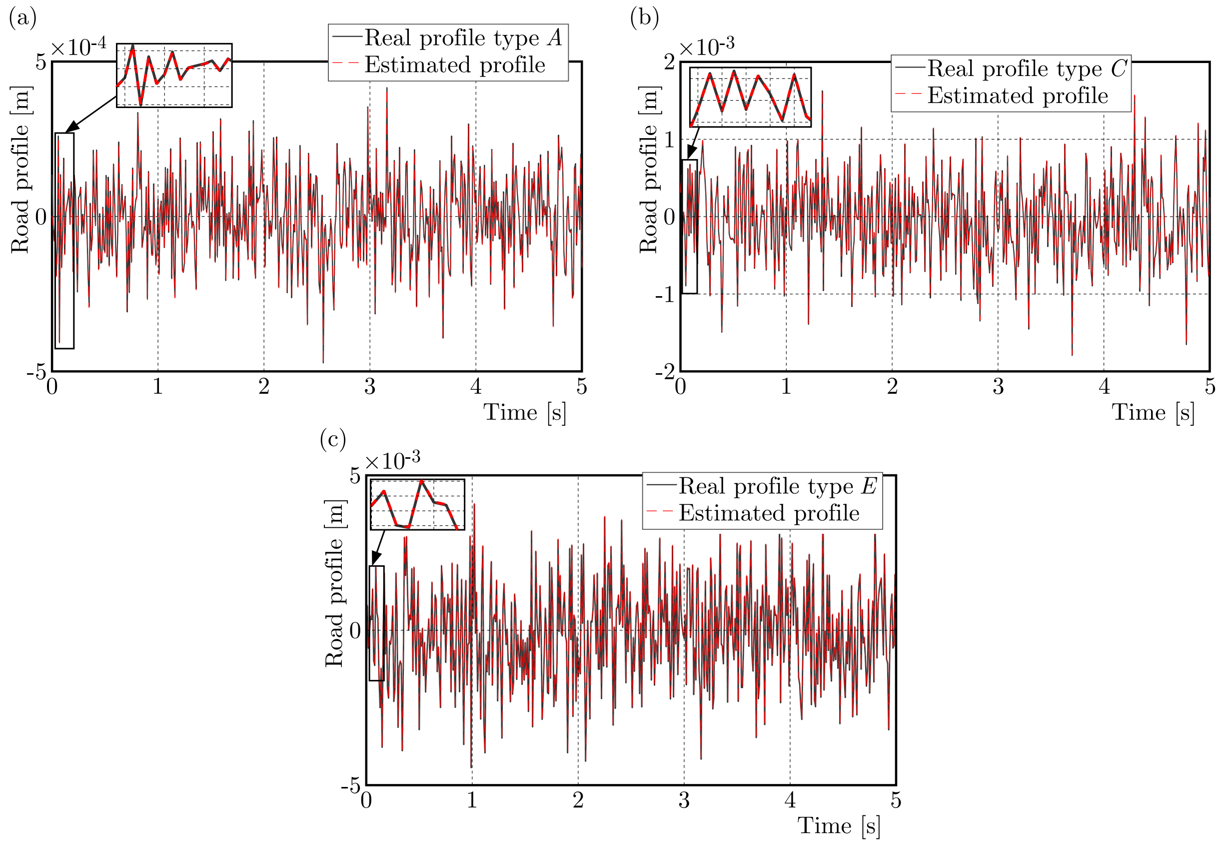


Fig. 5. Comparison between the real and estimated profiles: (a) road A, (b) road C, (c) road E

A good accuracy between the estimated profile and the real one in different studied cases can be easily noticed. This result was already proven in a previous work of Ben Hassen *et al.* (2019b). Indeed, the ICA is considered as one of the best used techniques in estimating dynamic responses. Since the sprung mass, stiffness and damping coefficient are variable parameters, the idea now is to study sensibility of the ICA method when these different parameters are uncertain. The Monte Carlo algorithm is particularly applied to study this problem, as previously mentioned. In this work, 500 samplings of 3 input variables have been calculated and then the problem is solved for each sample of input variables. Moreover, the influence of both of vehicle speed and wind is studied.

5. Robustness study

5.1. Uncertainties parameters impact

The objective here is to study the performance prediction of the ICA method which will operate in the presence of three simultaneously associated uncertainty parameters: the sprung mass, stiffness and damping. Thus, variations of 20%, 40% and 60% have been performed for each parameter. The achieved results were discussed in the following Section. Tables 4, 5 and 6 show the relative error between the real signal and the extracted one from the MC simulation for, respectively, the sprung mass, sprung stiffness and damping coefficient uncertain parameters. Three variations, 20%, 40% and 60%, were studied for each parameter. It is worth noting that there was convergence in the results. In fact, for the three uncertainty cases and the three variation values, the error did not exceed 5%. This might be interpreted as success of the combination between the MC and ICA methods in reproducing the real profile. In order to further explain, the obtained results are given in Tables 4 to 6. Table 3 has been elaborated to recapitulate the collected errors (maximal, minimal and average) between real and estimated profiles defined for each variation parameter.

Table 3. Maximum, average and minimum errors between real and estimated road profile for mass, stiffness and damping uncertainties roads A, C and E

	20%			40%			60%		
	m	k	c	m	k	c	m	k	c
Road A									
Min	0.70	0.80	1.02	1.01	1.20	2.23	1.13	2.00	2.85
Average	0.94	1.20	1.64	1.54	1.73	2.72	3.02	3.52	4.78
Max	1.25	1.74	2.25	2.04	2.26	3.50	2.31	2.99	4.02
Road C									
Min	0.90	1.10	1.70	1.27	1.64	3.23	1.50	2.41	3.92
Average	1.18	1.46	2.17	1.73	2.09	3.73	2.23	3.04	4.19
Max	1.75	2.01	2.70	2.49	3.01	4.49	3.02	3.52	4.78
Road E									
Min	1.17	1.48	2.42	1.55	2.00	4.15	1.90	3.01	4.99
Average	1.56	2.14	2.80	2.19	2.83	4.47	2.83	3.93	5.16
Max	2.27	3.01	3.31	3.50	3.90	5.09	4.05	4.52	5.71

It is easily noticeable that for the three parameter variations: mass, stiffness and damping, the errors increased with an increase of the road profile roughness. For example, for the mass sprung with 60% variation, the maximum error goes from 2.31% to 5.71%. For the stiffness parameter with 60% variation, the maximum error climbs from 2.9% to 4.52%. It can be therefore concluded that the estimation with the combination of the MC and ICA methods allows accurate reproduction of the road profile, especially for small variations. Even for great variations, the obtained results are acceptable. By comparing the maximum, minimum and average errors with respect to the variation values for each parameter, we can conclude that the error values also increased with the increase of variation. In fact, for the mass variation, the maximum error goes from 1.25% to 2.31% when the mass varies from 20% to 60%. The average error also passes from 0.94% to 1.73%. The same conclusions can be drawn for the stiffness and damping variations.

5.2. Vehicle speed impact

In order to show the impact of vehicle speed on the robustness study, three case studies have been elaborated. The first considered vehicle speed was 20 km/h; in the second case, the vehicle

Table 4. Impact of sprung mass variation on road profiles A, C and E

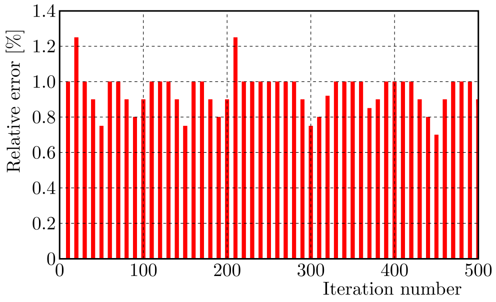
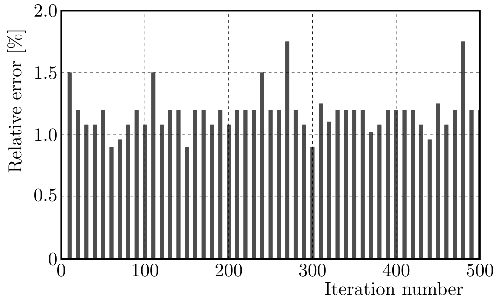
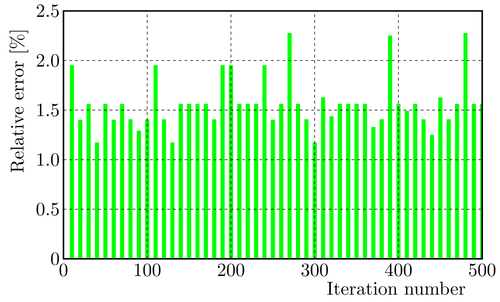
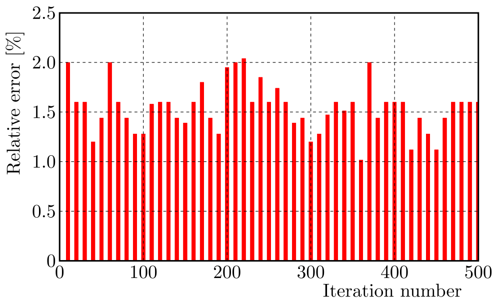
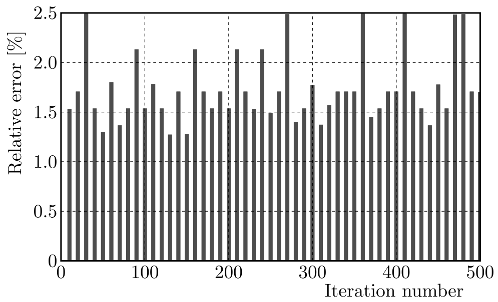
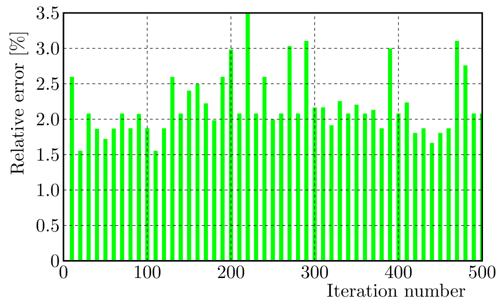
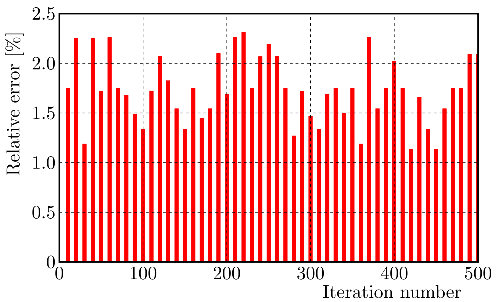
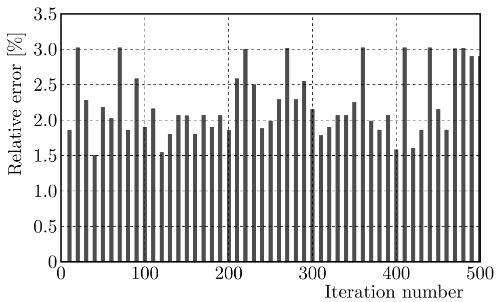
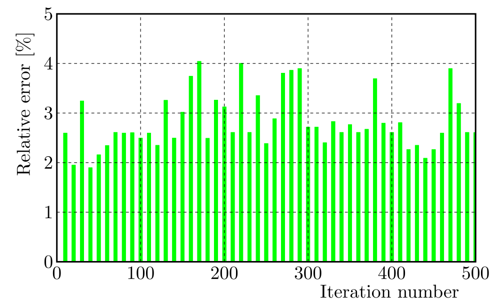
Chosen variation	Road type		
	Road A	Road C	Road E
20% m			
40% m			
60% m			

Table 5. Impact of stiffness variation on road profiles A, C and E

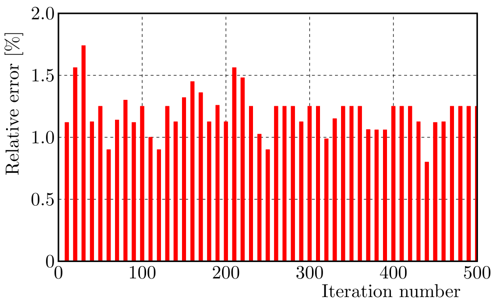
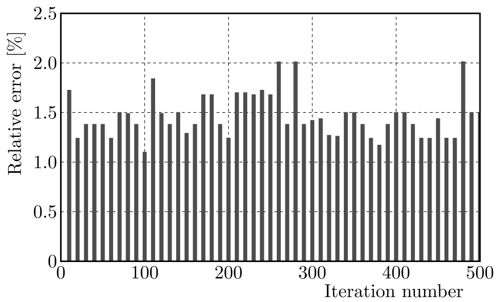
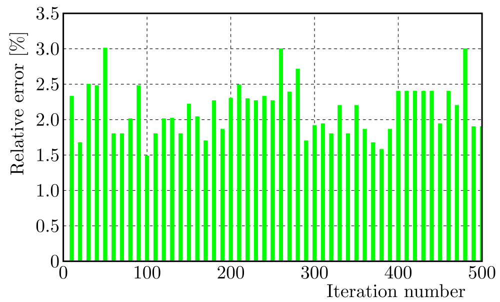
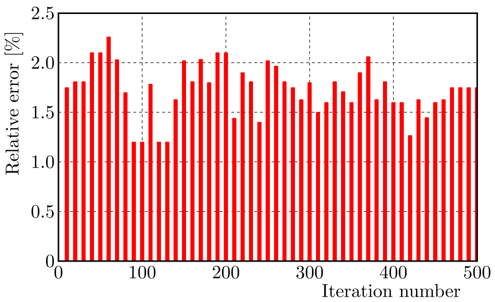
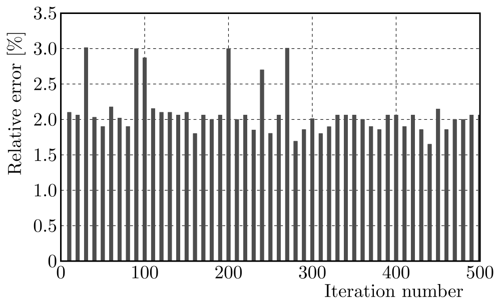
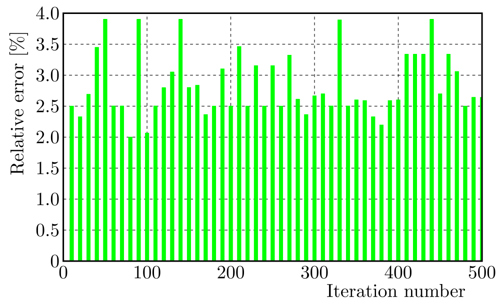
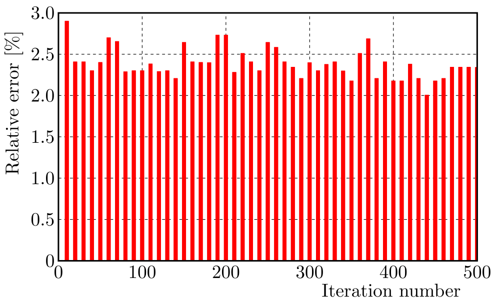
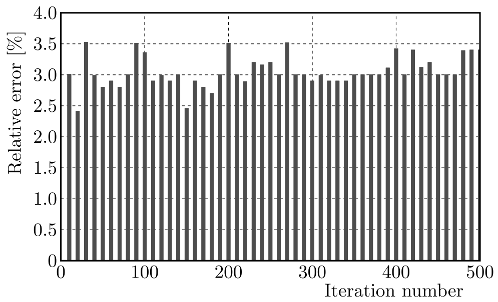
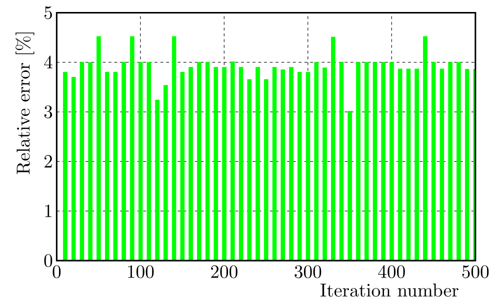
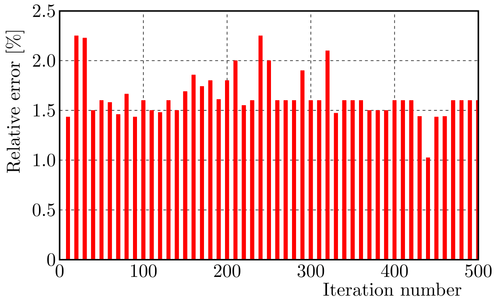
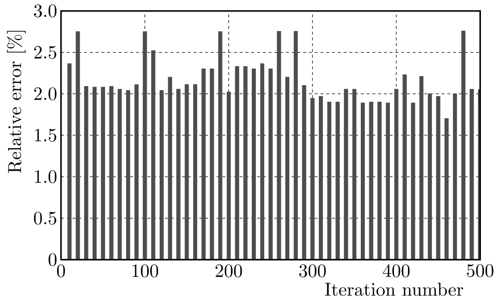
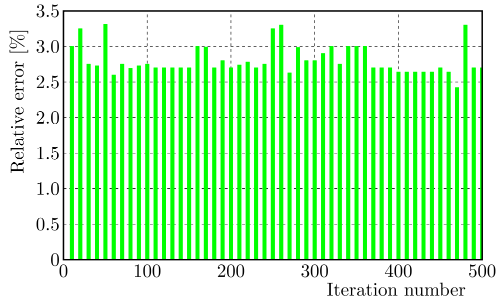
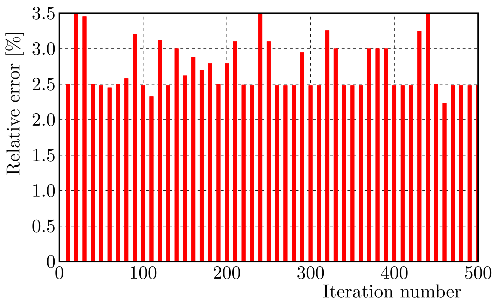
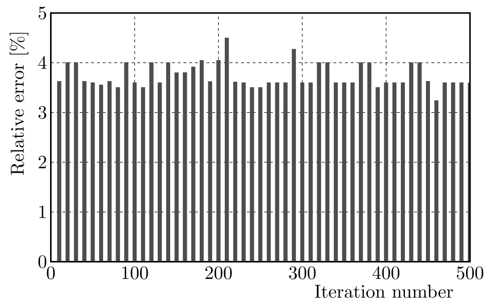
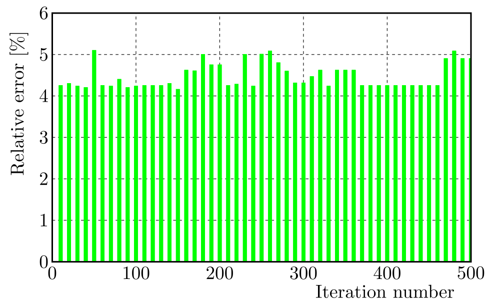
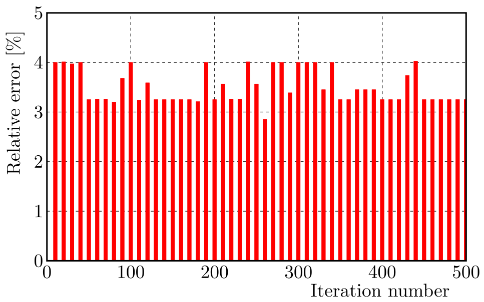
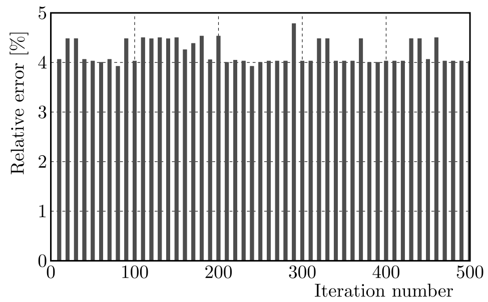
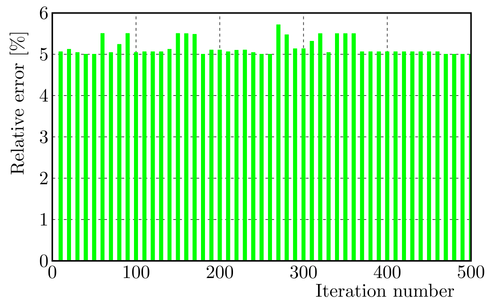
Chosen variation	Road type		
	Road A	Road C	Road E
20% k			
40% k			
60% k			

Table 6. Impact of damping variation on road profiles A, C and E

Chosen variation	Road type		
	Road A	Road C	Road E
20% C			
40% C			
60% C			

speed was 50 km/h, in the third one, we took into account a vehicle speed equal to 100 km/h. The three vehicle speeds were studied together with a 60% variation of the three other uncertain parameters. The obtained results showed that when the vehicle speed increased, the error in the estimation process increased as well, especially when driving on E road type, disturbed with a 60% damping variation. In fact, the maximum error is obtained especially for road profile E, with a vehicle speed equal to 100 km/h having damping variation equal to 60%. Despite their increase, we can say that the error values are acceptable, and that the MC method reproduces accurately the road profile for all studied cases.

Table 7. Speed impact on road E

	Road profile								
	A			*C*			*E*		
	Vehicle speed]km/h]								
	20	50	100	20	50	100	20	50	100
Min	1.5	2.8	3.5	2.8	3.8	3.8	3.8	5.0	5.0
Average	2.5	3.4	4.2	3.5	4.3	4.4	4.4	5.2	5.6
Max	3.5	4.0	5.0	4.4	5.2	5.0	5.0	5.5	6.2

5.3. Noise variation impact

In order to show the wind impact (defined as noise variation) on the vehicle, three cases have been investigated. In this work, we adopted the Signal to Noise Ratio (SNR) values suggested by Ben Hassen *et al.* (2019a), from 0.9 dB to 3.5 dB in the second case, and to 9.5 dB in the third case. The estimation quality decreased from the minimum noise to the maximum one, especially for roughness profile E. It is also worth reminding that in this study, these error values are assumed to be acceptable and the road profile is well estimated.

Table 8. Noise impacts on road E

	Road profile								
	A			*C*			*E*		
	Wind variation [db]								
	0.9	3.5	9.5	0.9	3.5	9.5	0.9	3.5	9.5
Min	3.8	3.92	3.95	4.0	4.90	5.0	6.0	6.2	7.1
Average	3.9	4.46	4.47	4.9	5.45	5.5	6.5	6.6	7.7
Max	4.0	5.00	5.00	5.8	6.60	6.0	7.0	7.0	8.3

6. Conclusion

In this study, the MC method has been used based on the Newmark technique in order to solve the equation of motion of the quarter vehicle model and then to estimate the road profile when vehicle mass, damping coefficient and sprung stiffness are uncertain. The obtained results depend on the iteration number. An increase of the iteration number leads to a better refined solution. The probabilistic behavior of the suggested uncertain parameters affects the ICA method efficiency in estimating the road disturbance profile. In the second step, the impact of variation of both vehicle speed and wind are investigated. The achieved results show that the uncertainty levels in the input data have no impact on the ICA method for several vehicle speeds and wind values when estimating different road profiles. This confirms the efficiency of the ICA in reconstructing the road disturbance. This result is very important since it allows estimation of the road profile in real time, on one hand, and it can be inserted in a control law, afterward, to ameliorate the vehicle performance.

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