Multiobjective optimization of reflector's shape

Krzysztof Wandachowicz
Politechnika Poznańska
60-965 Poznań, ul. Piotrowo 3a, e-mail: Krzysztof.Wandachowicz@put.poznan.pl

The article presents results of research on the calculation of the shape of a mirror reflector, which ensures the highest possible average illuminance, and uniformity of illuminance. A multiobjective genetic algorithm was used to carry out optimization calculations.

KEYWORDS: luminaires, optimization

1. Introduction

Constructing an objective with a higher number of objective functions may turn out to be necessary, especially when an objective with a single target function, supplemented with relevant constraints, fails to describe the posed problem in a proper way. Taking into account a higher count of criteria, which are usually contradictory to each other and are mutually incalculable, causes a situation where they cannot be brought down to a single, scalar criterion. Instead of a single objective function (1), a vector of functions is obtained and a search for the optimal compromise among them is under way [1].

$$F(x) = [F_1(x), F_2(x), ..., F_m(x)] \tag{1}$$

Increasing the number of objective functions increases the objective's difficulty. The designer should properly define the problem by introducing relevant expressions that can be described in a natural manner. Multiobjective optimization consists in minimizing or maximizing the vector of objective functions, which is subject to constraints (2).

$$\min_{x \in R^n}/\max F(x), \text{ subject to}$$
$$G_i(x) = 0, i = 1, ..., k_e; G_i(x) \le 0, i = k_e + 1, ..., k; lb \le x \le ub \tag{2}$$

In multiobjective optimization a vector of decision variables should be searched for, for which all components of the vector of objective function achieve extreme values. The objective function is a vector and if any of its components competes with another one, then finding an explicit solution will be impossible. Usually, each of the components of the vector of objective function achieves its extreme value with another vector of decision variables.

For the purpose of comparing solutions in multiobjective optimization, a concept of non-inferior solutions in Pareto meaning is introduced. A non-inferior solution is achieved when the improvement of value of one objective function causes deterioration of another value. The area Ω is assumed in the space of x variables, which satisfy all introduced constraints (3).

$$\Omega = \{x \in R^n\}, \text{ subject to}$$
$$G_i(x) = 0, i = 1,...,k_e; G_i(x) \le 0, i = k_e + 1,...,k; lb \le x \le ub \qquad (3)$$

The area Ω is matched by the area Λ in the objective's function space, while the vector of objective function maps the variables space to the objective's function space (4).

$$\Lambda = \{y \in R^m : y = F(x), x \in \Omega\} \qquad (4)$$

Point x_1 constitutes a non-inferior solution (dominates over another point x_2) if both following conditions are satisfied [1]:

1. Point x_1 ensures a solution that is not worse than the solution of point x_2 for all objective functions.
2. Point x_1 ensures a solution that better than the solution of point x_2 for at least one objective function.

Figure 1 presents several non-inferior solutions located on a curve between points A and D.

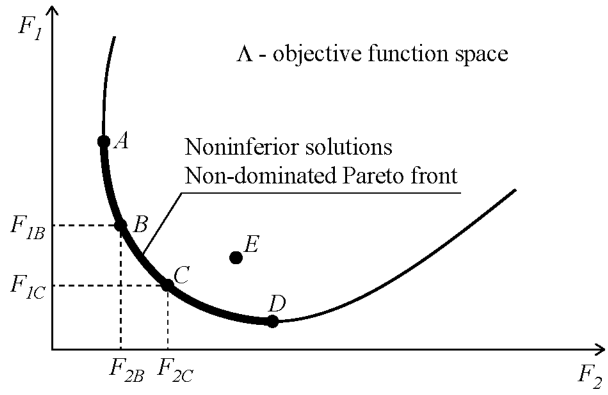

Fig. 1. Illustration of non-dominated Pareto front

Points B and C are non-dominated pints, because the improvement of the first objective function F_1 requires the deterioration of the second function F_2 (and the other way around). Point E does not belong to non-dominated solutions, because it fails to satisfy the first of the above conditions for the remaining points. Points A, B, C and D dominate over point E. The non-inferior solutions create the so-called Pareto set. Since they usually form a certain curve, the phrase "non-dominated

Pareto front" is also used [1]. Because a genetic algorithm creates a population of solutions in every subsequent iteration, then using it in multi-criteria optimization for identifying a Pareto set of optimal solutions is very beneficial.

2. Elite selection with the use of niching strategy

The application of elite selection, consisting in the guarantee of survival of non-dominated individuals, may lead to premature convergence. As early as after just several iteration, all individuals may be non-dominated and although they all form the elite, they are not necessarily in the vicinity of the real Pareto front. It is suggested to use a method based on NSGA-II, where, apart from the protection of elite individuals, clear diversification of individuals that make up the Pareto front is maintained [1].

In this method, individuals that fill in the population's unclaimed places (following the introduction of individuals from the best front) are selected taking into consideration the niching strategy, consisting in selecting points located in the least crowded areas of the last front. The procedure has no impact on the algorithm in initial iterations. That is because at that time there are numerous non-dominated and diverse fronts, which are still far from the optimal one. At that time, it is of little importance which individuals are selected to fill up the remaining unclaimed places in the new population. On the other hand, in final iterations most of individuals belong to the best, non-dominated front. Individuals guaranteeing higher diversity are selected for the new population, which consequently leads to a non-dominated front, where points that create distributed solutions are more uniform.

The evaluation of crowding is done on the basis of crowding distance d_j. It is calculated as the average length of the side of a cuboid within which a given solution is contained and no other solution from the same, non-dominated front of solutions. If the distance is large, the solution is not crowded. Individuals with the highest value of crowding distance d_j are elected for the remaining unclaimed places in the new population. Figure 2 presents a geometric interpretation of the entire expression. The value of crowding distance d_j is equal to half of the circumference of a rectangle (or a cube or cuboid for a higher count of objective functions).

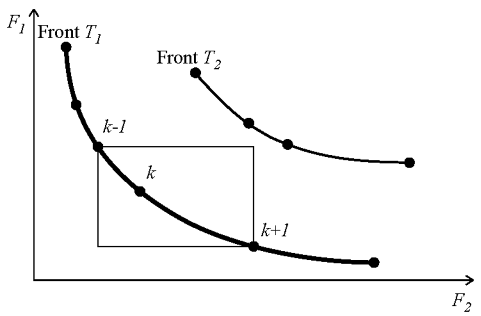

Fig. 2. Illustration of calculation of crowding distance for the kth point

Usually, most problems that require solving with a multiobjective evolution algorithm are subject to certain constraints (2). In such cases, the space of solutions is divided into two areas: the acceptable area and the unacceptable area, and all Pareto-optimal solutions must be acceptable. The method employed in this paper consists in using tournament selection, wherein a better solution is selected from two solutions from a given population. Three cases can be considered: (I) both solutions are acceptable, (II) only one solution is acceptable, (III) both solutions are unacceptable.

The definition of domination is replaced with the following phrase: point x_1 "dominates with constraints" over point x_2 if any of the following conditions is satisfied [1]:

1. Point x_1 constitutes an acceptable solution, while point x_2 is considered unacceptable.
2. Both points constitute unacceptable solutions, but point x_1 violates constraints to a lesser degree.
3. Both points constitute acceptable solutions and point x_1 dominates over point x_2 (in the sense of definitions in chapter 1). If both solutions belong to the same front, the solution located in the least crowded area of the front is selected.

Using the above definition, subsequent Pareto fronts "non-dominated with constraints" are established in a given population (Fig. 2). A tournament selection is carried out among two randomly selected points that make up these fronts, with the aim of selecting a better solution, as provided for in the above criteria. After the first series of selections, pairs are chosen at random again and a better solution is selected again. Points selected in two series create a mating pool. The best individuals have a high chance to appear there in several instances, and the worst one - in none. The mating pool is of the same size as the population.

3. Model of reflector

The reflector profile is described with Hermite interpolation where shape preservation is achieved with Fritsch-Carlson method [2, 3]. Points P_1, P_2, P_3, P_4 and P_5 (Fig. 3) are interpolation nodes through which a curve that describes the reflector's profile passes. The optimization algorithm whose subsequent steps are to lead to find the objective function minimum changes the values of decision variable Ci (relevant coordinates of points $P_2 \div P_5$).Point P_1 does not change its position, whereas the remaining four points change their locations both in X_2 ($x_{2,i}$) axis and in X_1 ($x_{1,i}$) axis. In this case, the operation of the optimization algorithm may result in the change of height and width of the reflector. The identification of location of interpolation nodes in X_1 axis is done with a single variable dx_1 that is responsible for changing the value of coordinate $x_{1,5}$ of point P_5.The coordinates of the remaining points are calculated taking into account identical distance Δx_1 in axis X_1 between the subsequent points. In this case, the optimization algorithm operates on five variables ($x_{2,2}$, $x_{2,3}$, $x_{2,4}$, $x_{2,5}$, dx_1).

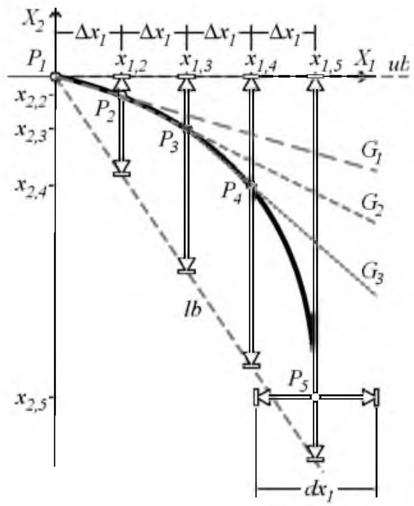

Fig. 3. Reflector profile, presentation of conditions limiting the location of interpolation nodes

The reflector features a top opening and a bottom opening shaped like a square. The shape of the four walls is generated by a profile curve described with Hermite interpolation. In the top opening, a model of the so-called LED module is placed, whose parameters match the technical specification of Fortimo LED DLM 2000 module. The module's illuminating element is a circle-shaped surface, 6 cm in diameter, covered in luminophore. The light beam distribution of this surface is almost Lambertian [4].

4. Optimization of reflector's shape

The genetic algorithm uses three main operations at each stage of creating a new generation from the current population [5]:

- selection (selection of parents from individuals in a given population),
- crossover (pairing parents to create individuals for the subsequent generation),
- mutation (introduction of random changes to selected parents).

The following solutions have been used for this paper [6]:

- floating point representation - for approximating the algorithm to the objective's space, two points located close to each other in the representation space will also be close to each other in the objective's space,
- scaling of objective function by appointing ranks equalizes the score of less adapted individuals, while maintaining a high diversity within the population,
- elite selection carries over to the next generation the entire, non-dominated and best Pareto front; the remaining free spaces in the population are for individuals created by their parents through crossing and mutation,

– heuristic crossover and mutations with Gaussian distribution (magnitude of mutation decreases with each new generation),
– the number of individuals in population is 25, the number of generations is 100.

A proprietary method with algorithm for ray tracing, implemented in Radiance system [7], was used for calculating photometric parameters (luminous intensity, illuminance). This method was described in several publications [8], which also presented results of calculations of luminaires with optical elements of various photometric properties.

The optimization objective consisted in finding a reflector shape that would allow to achieve the highest possible mean illuminance value E_{av}, while maintaining proper uniformity ratio of illuminance in the illuminated surface (ratio of minimal illuminance E_{min} to mean illuminance E_{av}). The reflector illuminates a square-shaped surface, whose side is 3 m long. The luminaire model is located 3 m over the middle of this surface.

The optimization was carried out for two objective functions (5).

$$\begin{aligned} F_1(x) &= -E_{av} \\ F_2(x) &= -100(E_{min}/E_{av}) \end{aligned} \tag{5}$$

As the optimization algorithm searches for the function's minimal value, and the posed objective aims to maximize, a minus was inserted in the objective function's formula.

The following constraints were applied to the objective's function space:

1. Constraints applied to domain $lb \le x_{2,i} \le ub$:
   – their purpose is to make the curve describing the reflector profile convex upwards, the values of points $x_{2,i}$ determining the coordinates in axis X_2 may alter within boundaries specified by straight lines ub and lb (Fig. 3).
2. Linear inequality constraints $G_i(x) \le 0$:
   – the curve describing the reflector's profile should remain monotone across the entire interval; creating a reflector's profile whose curve is not monotone may lead to longer calculations by extending the searching area with area where the optimal solution is not found,
   – point P_3 should be located below straight line G_1 (the straight line passes points P_1 and P_2), point P_4 below straight line G_2, and point P_5 below straight line G_3 (Fig. 3).

5. Calculation results

Figure 4 presents results of calculations carried out pursuant to the above described niching strategy.

A single, specific solution is not obtained and after completion of the optimization process, extra-optimization procedures, allowing the selection of a determined result from a series of points, should be used. Optimization with penalty function used to be applied in previous tests [4]. A criterion of penalizing individuals for whom the uniformity ratio of illuminance is below 70% was assumed at that time. In the case of the posed objective it seems obvious that increasing the uniformity ratio (function F_2) will result in decreasing the illuminance (function F_1). Assuming the above criterion, a solution located as close as possible to point $F_2(x) = -70$ should be sought.

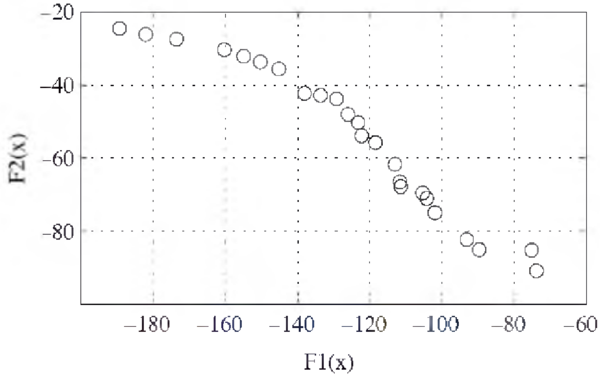

Fig. 4. Pareto front obtained with the use of niching strategy

Figure 5 presents results of calculations carried out with niching strategy. The algorithm was supplemented with a selection function, serving to limit the searched range of values of function $F_2(x)$ to the range from -60 to -80 (outside this range, values $F_2(x)$ are equated to zero).Limiting the values of objective function leads to increasing the diversity of obtained solutions in the expected area. This strategy allows to obtain results that are not worse, without changing the number of individuals in the population.

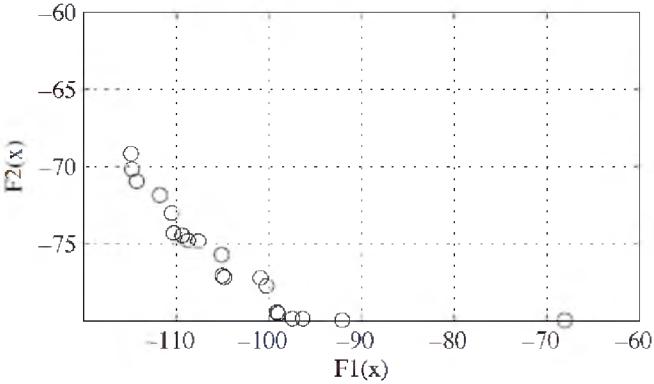

Fig. 5. The Pareto front obtained with the use of niching strategy and constraining the searched range of function values $F_2(x)$

Figure 6 presents results of calculations carried out without niching strategy. When comparing these results with results obtained with the application of niching strategy, it can be seen that a slightly worse distribution of points in Pareto front was achieved, mainly due to fewer points present in the last, best front. The relatively similar uniformity ratio of distribution of points in Pareto fronts proves that niching strategy has a comparatively weak impact on this parameter. This happens because the objective itself leads to obtaining a high diversity of results. For a reflector with smooth surface, even a slight change of shape may have a significant impact on the change of luminous intensity distribution.

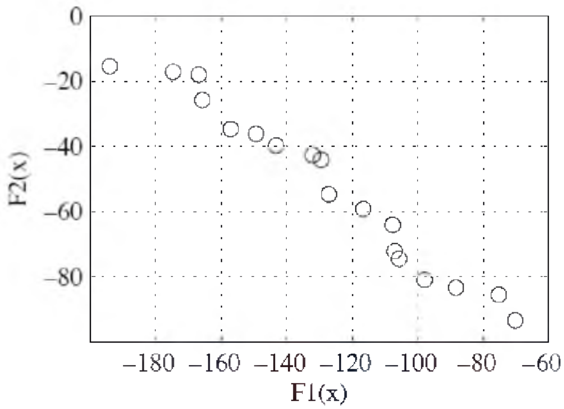

Fig. 6. Pareto front obtained without niching strategy

Table 1 presents results of calculations obtained for four various algorithms. The following were provided (negatives changed into positives): values of objective function, number of individuals creating the last, non-dominated Pareto front N_P, CPU time and the relation of the number of iterations to calculation time.

Table 1. List of calculation results

Lp	Algorithm name	$F_1(x)$	$F_2(x)$	N_P	CPU [s]	Number of iterations to calculation time
1	Niching strategy	104	71	25	3430	0.73
2	Niching strategy with constraint of range of function F_2	115	70	25	3519	0.71
3	Without niching strategy	107	72	20	3549	0.70
4	Penalty function	108	71	-	7340	0.61

The best result was obtained for the algorithm with niching strategy and constraint of range function F_2. At the same time, this algorithm ensures the highest diversity of results in the expected range of solutions. The parameter that gives the value of the number of iterations to the calculation time shows a slight deterioration in the efficiency of the multiobjective optimization algorithm in relation to singleobjective optimization with penalty function [4].

Figure 7 presents calculated illuminance distribution for the square surface with side length 3 meters and luminous intensity distribution for reflector obtained for algorithm with niching strategy and constraint of range function F_2.

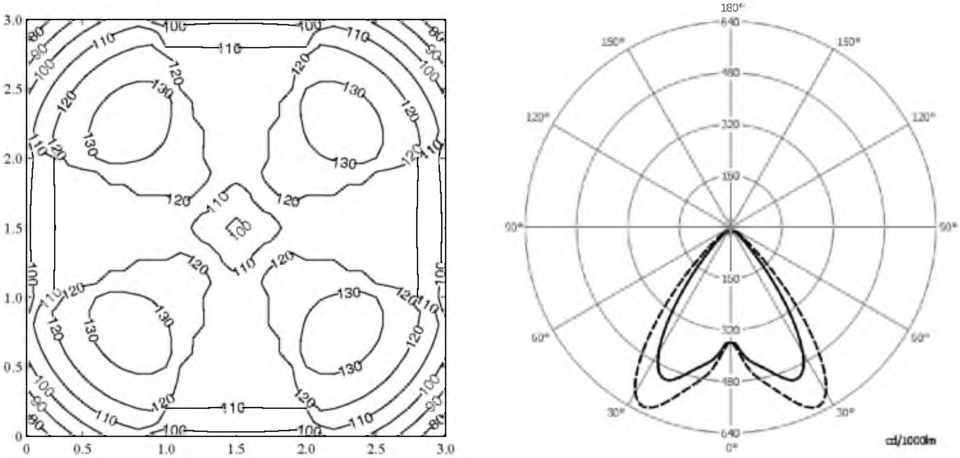

Fig. 7. The calculated illuminance distribution isoluxes for the square surface with side length 3 meters; luminous intensity curve, solid line – C0-C180 surface, broken line – C45-C225 surface

6. Conclusions

The completed tests show that the proposed method of optimization of the reflector's shape may be successfully used with the application of multiobjective optimization. Although the algorithm's efficiency decreased in relation to calculations using penalty function, ultimately, a better result was obtained in less time. The multiobjective optimization requires the use of extra-optimization criteria for the selection of the final solution. At the same time, a set of results is obtained, which can be analyzed in terms of cost effectiveness of execution.

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