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The measuring instrument accuracy class as a measure of the Type B uncertainty

Abstract

The evaluation of Type B uncertainty is done by means of scientific analysis based on all available information about possible variability of the quantity measured. In order to facilitate the obtaining of information helpful in the evaluation of Type B uncertainty, a new instrument metrological quality assessment method has been developed, which relies exclusively on the random model. The assessment result, in the form of an instrument accuracy class, can be used for estimation of the Type B uncertainty as presented in the accompanying examples.

Keywords: metrological quality of instrument, accuracy class, Type B standard uncertainty.

1. Introduction

Methods used presently for measurement uncertainty assessment rely on the Guide [1] and its Supplements [2, 3]. The Monte Carlo method extends the scope of uncertainty assessment to include nonlinear measurement equations [4-7]. Analyses carried out show, however, that in specific cases this method leads to an overestimation of the initial quantity measurement uncertainty range [6]. Also the quality of distribution fitting to the preliminary measurement results is not without significance.

The method described in the Guide [1], relying on the uncertainty propagation law, still provides a basic measurement uncertainty assessment method. It involves the calculation of Type A and B uncertainties and then, based on the measurement equation, the combined standard uncertainty and further expanded uncertainty associated with the level of probability. Calculation of Type A uncertainty does not pose any difficulty, because this is the standard deviation of the mean value of the results of measurements conducted in the conditions of repeatability [8]. In contrast, the evaluation of the value of Type B uncertainty depends significantly on the expertise and willingness of the person conducting the test. According to the Guide [1], this is done by means of scientific analysis based on all available information about possible variability of the quantity measured using the comprehensive knowledge of the measuring instrument, previous measurements, calibration and certification data, etc. The Guide provides several examples of proceeding, which obviously do not exhaust all situation that might be encountered by a researcher.

Estimating the Type B uncertainty is especially difficult, if the instrument on which measurements are conducted has an obsolete verification certificate, or does not have any at all. Of course, it would be best to apply to the relevant metrological institution for issuing such a certificate, but often this is neither, nor cheap, nor expeditious. In that case, a final project student or a doctoral student carrying out measurements reduces the Type B uncertainty assessment to calculations based solely on the instrument's resolution. Assuming the distribution of probability of possible values of the measured quantity within the range defined by the scale interval, it is easy to calculate the standard deviation value. Such an example is actually given in the Guide [1] (p. 4.3.7).

Such a simplification leads, however, to an underestimation of the uncertainty value, as it does not include the analysis of the nonlinearity, the hysteresis of an instrument and the repeatability and reproducibility of the results of measurements made using that instrument. In the 90s of the last century, a method was proposed for assessing a measuring instrument's property called its accuracy class, which considered the above-mentioned phenomena [9-12]. The measure of systematic influences was the largest difference between the ordinates of the ideal and the real characteristics, while the measure of random influences was the limiting random error. Currently, in view of the uncertainty approach based on the Guide [1], it should be regarded as obsolete. Therefore, the new works were carried out [13].

The presented report provides an attempt to develop a new method for assessing the metrological quality of an instrument based exclusively on the random model. The assessment results might be used for evaluation of Type B uncertainty following the recommendations of the Guide [1]. It should also be emphasized that the notion of accuracy class has the value of universality and can also be used for the assessment of measuring transducers at the instrument design stage. It might be regarded then as a measure of the quality of measured value conversion into electric voltage and used as important guidance for designer and constructor.

2. The idea behind the determination of the measuring instrument accuracy class

According to the VIM dictionary [8], the accuracy class is a characteristic of a measuring instrument, which tells us that the results of measurements made using that measuring instrument are burdened with uncertainties contained within specific limits. The accuracy class can be expressed with a number and in this form is used for a long time [14]. The concept is also found in other standardization publications such as [15], but is always described as a concept related to the fulfillment of certain requirements to the accuracy of the instrument. This broad definition encompasses the characteristic of an instrument, which is calculated in a procedure described later on in this paper.

Let the static characteristic of a measuring instrument or transducer be given by the relationship $y=f(x)$, where x is the input quantity, while y is the measurand. If $y\in[0,Y_m]$, then the measuring nominal range of the instrument is equal to the range of a nominal indication interval, being Y_m.

To calculate the accuracy class, the following sequence of operations needs to be performed.

2.1. Identification of the mean real characteristic

It is necessary to have a set of n standard values of the measurand x_{wi}, (i=1,...,n), which will be ordered into an increasing series, so that $x_{wi+1}>x_{wi}$. By measuring a value corresponding to the standard value, we will find the successive values of $y(x_{wi})$ from i=1 to i=n, to obtain the relationship $y_g(x_w)$. Then, proceeding from i=n−1 to i=1, we obtain the relationship $y_d(x_w)$. Very often, $y_g(x_w)\neq y_d(x_w)$, which is due to the occurrence of the hysteresis phenomenon. This operation should be repeated not less than k = 3 times. As a result, the mean real characteristic is obtained as a line connecting the points corresponding to the mean values of $y_g(x_{wi})$ and $y_d(x_{wi})$ (see Fig. 1).

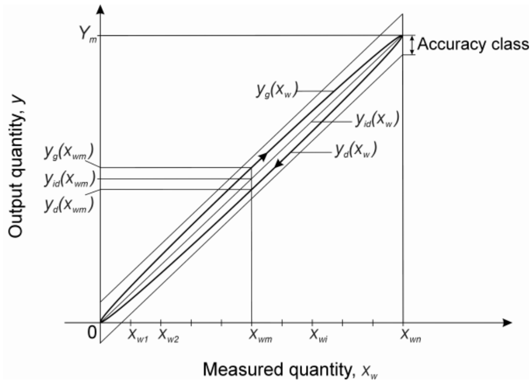

Fig. 1. The real and the ideal characteristics [10]

2.2. Identification of the ideal characteristic $y_{id}(x)$

The form of the ideal characteristic, $y_{id}(x)$, will depend on the person conducting the analysis. In the simplest case, when a ready instrument is tested, which gives a reading in the units of the measurand, the ideal characteristic is the line $y=x$. If only part of an instrument is assessed, such a transducer at the design stage, then the output quantity is usually electric voltage. In that case, the ideal characteristic can be identified by one of methods described later on in the text.

a) Approximation by the least squares method, in the analysis of a complete set of points $y_g(x_{wi})$ and $y_d(x_{wi})$, in the form of a linear regression function:

$$y = ax + b$$

or

$$y = ax \,. \tag{1}$$

depending on whether there is the need for identification of the zero error, or not it.

b) Finding the straight line passing through the extreme points resulting from the measuring range Y_m, $y=Y_m/X_m \cdot x$, where X_m is the standard value corresponding to Y_m.

c) Finding the relationship resulting from the theoretical analysis of the mathematical model of the tool, $y=f_t(x)$.

The ideal characteristic is a measurement model. It is obvious that the decision of its form has an effect on the result of quality assessment of an instrument or a measuring transducer under examination.

2.3. Making out the corrections table

Having the real and the ideal characteristics at our disposal, we can determine the differences between them, called corrections, for individual standard values. If there is a hysteresis, then the calculation should be made separately for the upper and the lower parts of the real characteristic. Then, we will obtain:

$$p_{g(ij)} = \left| y_{id}(x_{wi}) - y_{gj}(x_{wi}) \right|, \; i = 1,2,\ldots,n, j = 1,2,\ldots,k \tag{2}$$

and

$$p_{d(ij)} = \left| y_{id}(x_{wi}) - y_{dj}(x_{wi}) \right|, \; i = n\text{-}1, n\text{-}2,\ldots,1, j = 1,2,\ldots,k. \tag{3}$$

The corrections p are a measure of the nonlinearity and the measuring hysteresis.

2.4. Randomized corrections and the calculation of its expanded uncertainty

The corrections p are realizations of a new random variable P. The standard deviation of all values of variable P can be calculated as the Type A uncertainty, $u_A(p)$. Based on scientific judgment using all of the relevant information available, which may include: previous measurement data, experience with, or general knowledge of, the behavior and property of relevant materials and instruments, manufacturer's specifications, data provided in calibration and other reports, and uncertainties assigned to reference data taken from handbooks the Type B uncertainty, $u_B(p)$, can be calculated. Next, by determining the Type A and the Type B uncertainties we are able to further determine the combined standard uncertainty $u_C(p)$ and multiplying it by coverage factor of k_p=2, the expanded uncertainty U_p.

2.5. Determination of the measurement uncertainty at the point of the largest correction value

At the point of the static characteristic, (x_{wm}, $y(x_{wm})$), corresponding to the largest value p_m of the correction, a series of measurements of the standard value, x_{wm}, is made under repeatability and reproducibility conditions. While doing this, we have to make sure to distinguish between the upper and the lower branches of the real characteristic, $y_g(x_w)$ and $y_d(x_w)$. The outcome of this process is a set q of values $y_r(x_{wm})$ (r=1,...,q), where no subsets associated with individual series are distinguished. Thus, all random influences are treated jointly.

The standard deviation of this q series of results is calculated as the Type A uncertainty, u_A. Next the Type B uncertainty, u_B, can be determined in the same way as in the case of corrections p. After calculating the combined standard uncertainty, the expanded uncertainty U_m is obtained, taking k_p=2.

2.6. Calculation of the accuracy class

Having the expanded uncertainties U_p and U_m and the measuring range Y_m the following relationship can be written:

$$\kappa' = \frac{\sqrt{U_p^2 + U_m^2}}{Y_m} 100\% \,. \tag{4}$$

The accuracy class κ is a number obtained from rounding the κ' up to one significant digit in the case where the accuracy class is smaller than unity, and to an integer otherwise.

Thus, the accuracy class defines the relative interval in % of the range of a nominal indication interval, Y_m, in which the real value of the measurand lies with the probability as determined in the calculation of the uncertainty.

3. Accuracy class and the Type B uncertainty

If a measurement has been made with an instrument of the accuracy class κ and the range of a nominal indication interval, Y_m, and the obtained result has a value of $\hat{y}$ (the estimation of the real value), then the real value y_p is contained in the range $\left(\frac{\hat{y} - \kappa Y_m}{100}, \frac{\hat{y} + \kappa Y_m}{100} \right)$. The value $\kappa Y_m/100$ is half of the uncertainty range and can be used for finding the Type B uncertainty of the measuring instrument under examination. Assuming the rectangular distribution of measurand values within this range, the following is obtained: $u_B = \kappa Y_m / (100\sqrt{3})$.

4. Examples of the metrological quality assessment of an instrument using accuracy class

4.1. Displacement sensor

The instrument examined was a Z4M-W100RA type sensor manufactured by OMRON. It is a laser gauge operating on the dispersed reflected radiation principle. Featuring an analog output, the sensor is designed, among other things, for measurements of displacements, velocities or vibrations. It consists of a laser radiation source and a photodetector. The radiation source is a laser diode. According to the specification supplied by the manufacturer, the sensor's measuring range is ±40 mm relative to the focus, while the resolution at selected response speed is 150 µm (0.7 ms). The power supply voltage is within the range from 12 to 24 VDC. Used for the measurement of displacement, the sensor was subjected to detailed metrological assessment aimed at determining its accuracy class within a measuring range from 0.00 mm to 40.00 mm.

Therefore, the identification of the mean real characteristic was made under hysteresis conditions. The measurand standard were class II standard plates of an execution accuracy of 0.005 mm. The range of a nominal indication interval was Y_m=40.00 mm. A triple pass of the static characteristic was made using the standards. As a result, the mean real characteristics $y_{gj}(x_w)$ (Table 1) and $y_{dj}(x_w)$ (Table 2) were obtained. A slight hysteresis was found to occur.

As an ideal characteristic, according to (1), a straight line of the following form was taken:

$$y_{id}(x_w) = x_w .$$

Using (2) and (3), a table of corrections was made out (Table 3).

Tab. 1. Measurement results used for determining the characteristic $y_g(x_w)$

x_{wi}	$y_{gj}(x_{wi})$			$y_g(x_{wi})$
	j=1	j=2	j=3	
mm	mm	mm	mm	mm
0.00	0.00	0.00	0.00	0.000
10.00	10.06	10.06	10.06	10.060
20.00	20.00	20.02	20.02	20.013
30.00	29.98	29.98	29.98	29.980
40.00	39.89	39.89	39.89	39.890

Tab. 2. Measurement results used for determining the characteristic $y_d(x_w)$

x_{wi}	$y_{dj}(x_{wi})$			$y_d(x_{wi})$
	j=1	j=2	j=3	
mm	mm	mm	mm	mm
30.00	29.98	29.96	29.96	29.967
20.00	20.02	20.00	20.00	20.007
10.00	10.03	10.06	10.03	10.040
0.00	0.02	0.05	0.05	0.040

Tab. 3. Table of corrections

x_{wi}	p		
mm	mm	mm	mm
0.00	0.00	0.00	0.00
10.00	0.06	0.06	0.06
20.00	0.00	0.02	0.02
30.00	0.02	0.02	0.02
40.00	0.11	0.11	0.11
30.00	0.02	0.04	0.04
20.00	0.02	0.00	0.00
10.00	0.03	0.06	0.03
0.00	0.02	0.05	0.05

The Type A uncertainty was calculated as the standard deviations of all the values p of the variable P; it amounted to $u_A(p)$=6.4·10^{-3} mm for 702 degrees of freedom.

Next two Type B uncertainties were calculated. Taking into account the plate execution accuracy of ±0.005 mm and assuming a rectangular distribution, the Type B uncertainty is:

$$u_{B1}(p) = \frac{0.01}{\sqrt{12}} = 2.9 \cdot 10^{-3}, \text{ mm.}$$

The calculation of the Type B uncertainty considered also the fact that the signal from the sensor is sent to an amplifying and compensating circuit and then, in a 1 V/cm form, it is transmitted onto a computer to a National Instruments Lab-PC-1200/AI card equipped with a 12 bit-resolution AC converter. Corresponding to voltage values from the range from 0 V to 10 V are displacement values from the range from 0 mm to 100 mm. Using these data, the other Type B uncertainty could be calculated; it amounts to:

$$u_{B2}(p) = \frac{100}{2^{12}\sqrt{12}} = 0.7 \cdot 10^{-2}, \text{ mm.}$$

Having these uncertainties and taking the coverage factor equal to k_p=2, the expanded uncertainty was calculated to obtain U_p=0.02 mm.

Then, based on the results summarized in the Table 3, it was found that the greatest correction value occurred at point (40.00, 39.89) of the static characteristic, and that was p_m=0.11 mm. Under repeatability and reproducibility conditions, a series of q=30 measurements of the value x_{wm}=40.00 mm was carried out.

Next, the Type A uncertainty was calculated; it was u_A=1.1·10^{-5} mm for 870 degrees of freedom.

The Type B uncertainties, u_{B1} and u_{B2}, are the same as the Type B uncertainties $u_{B1}(p)$ and $u_{B2}(p)$.

Having the Type A uncertainty and the Type B uncertainties, the combined standard uncertainty, and then the expanded uncertainty were calculated, taking an coverage factor of k_p=2. A value of U_m=0.02 mm was obtained.

Using (4), the κ' index value was calculated, obtaining 0.07%. Hence, the accuracy class κ of the displacement sensor is 0.1. It means that with a 0.95 probability the largest difference between the measured value and the unknown real value not exceed 0.1% of the nominal indication interval Y_m=40.00 mm, i.e. 0.04 mm. The Type B uncertainty of the measuring instrument is 0.02 mm.

4.2. Temperature sensors

The concept of accuracy class can be used to compare the metrological characteristics of several instruments. To illustrate the application two temperature sensors were researched.

Sensor 1 has operating range from -40°C to 124°C according to the manufacturer information. It was surface mountable device which integrates sensor plus signal processing elements and provides a fully calibrated digital output. A temperature was measured by a band-gap method with the CMOSens® technology. The sensor was seamlessly coupled to a 14 bit analog to digital converter and a serial interface circuit. It was individually calibrated in a precision humidity chamber by manufacturer. The 2-wire serial interface and internal voltage regulation is used.

Sensor 2 has operating range from –20°C to 85°C according to the manufacturer information. The sensor was a rugged, self-sufficient system that records the result in a protected memory section with 512 bytes of SRAM. The recording was done at a user defined rate. Every device was individually calibrated in a NIST-traceable chamber. It was configured and communicates with a host-computing device through the serial 1-wire protocol. Every sensor is factory-lasered with a guaranteed unique 64-bit registration number that allows for absolute traceability.

The sensors were subjected to metrological assessment aimed at determining their accuracy class within a measuring range from 15.138°C to 45.132°C. Thus the nominal indication interval Y_m was 29.994°C. For each sensor the study was carried out on the test bench equipped with a high precision bath (Fluke 7340) and thermometer readout with a measurement probe (Fluke 1590). The tested temperature sensors were placed in an oil bath. The temperature was set in the bath and the setting temperature was read using thermometer readout with resolution equal to 0.001°C. The real temperature value indicated by the tested sensor was recorded by the software installed on computer.

The identification of the mean real characteristic was made for the sensors under hysteresis conditions. The measurements were repeated k=3 times. As an ideal characteristic a straight line (1) was assumed. The results obtained for the two sensors are summarized in Table 4.

Tab. 4. The results of calculations of the sensors accuracy class

Sensor	$u_A(p)$	$u_{B1}(p)=u_{B1}$	$u_{B2}(p)=u_{B2}$	U_p	x_{wm}
	°C	°C	°C	°C	°C
1	$4.0\cdot10^{-3}$	$2.9\cdot10^{-4}$	$2.9\cdot10^{-3}$	0.010	45.107
2	$3.3\cdot10^{-3}$	$2.9\cdot10^{-4}$	$5.9\cdot10^{-2}$	0.118	45.107

Sensor	p_m	u_A	U_m	Y_m	κ
	°C	°C	°C	°C	-
1	0.109	$9.3\cdot10^{-3}$	0.019	29.994	0.1
2	0.110	$7.3\cdot10^{-3}$	0.119	29.994	0.6

For both of them the Type A uncertainty, $u_A(p)$, was received for 1980 degrees of freedom. The Type A uncertainty, u_A, was received for 20 degrees of freedom. Calculating the Type B standard uncertainty the rectangular distribution of measurand values was assumed. The Type B uncertainty, $u_{B1}(p)$, was calculated for resolution of standard instrument equal to 0.001°C. The Type B uncertainty, $u_{B2}(p)$, was calculated taking into consideration additional informations. In the case of the sensor 1, a 14 bit analog to digital converter and range of –40°C to 124°C were taken into account. In the case of the sensor 2, a 512 bytes of SRAM used for record the result and range of –20°C to 85°C were taken into account. The Type B uncertainties u_{B1} and u_{B2} are the same as the Type B uncertainties $u_{B1}(p)$ and $u_{B2}(p)$.

The accuracy class of the temperature sensor 1 is 0.1. It means that with a 0.95 probability the largest difference between the measured value and the unknown real value not exceed 0.1% of the nominal indication interval Y_m=29.994°C, i.e. 0.030°C. The Type B uncertainty of the sensor is 0.017°C.

The accuracy class of the temperature sensor 2 is 0.6. It means that with a 0.95 probability the largest difference between the measured value and the unknown real value not exceed 0.6% of the nominal indication interval Y_m=29.994°C, i.e. 0.180°C. The Type B uncertainty of the sensor is 0.104°C.

It follows that, for measuring the temperature within the desired range sensor 1 is better despite the greater operating range.

5. Conclusions

The accuracy class described enables the determination of the metrological quality of a measuring instrument using a single synthetic indicator only.

The evaluation of an instrument's accuracy class makes it possible to consider both systematic influences that manifest themselves in nonlinearity and a hysteresis, and random influences whose effect is the limited repeatability and reproducibility.

The knowledge of the accuracy class of an instrument enables one to precisely determine the Type B uncertainty that is shown by the results of measurements made using that instrument.

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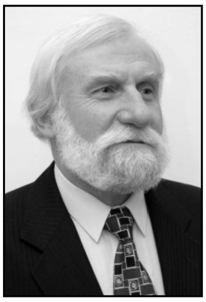

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