INFORMATION
SYSTEMS IN
MANAGEMENT

Information Systems in Management (2014) Vol. 3 (2) 113−121

POWER LAW AND SELF-SIMILARITY IN THE DISTRIBUTION OF NATIONAL INCOME

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Economic systems as complex systems are the source of many unconventional researches. The actual phenomena in complex systems have the non-scalability features and non-scalability does not exist without the power laws. This paper presents the occurrence of power series of rights and self-similarity in the distribution of national income in 2010, in terms of countries, the European Union and the Polish subregions. This means that one can study economic systems using tools of complex systems theory. This applies especially to the study of the mechanisms governing the system at the local level that generate global action.

Keywords: power-law distribution, non-scalability, self-similarity, a complex system

1. Introduction

The general theory of complex systems provides a perfect foundation for the study of rights underlying operations of such systems. Economic systems as complex systems are home to universal laws, among which the most important are exponential power laws.

The system comprises a circuit whose operation is highly complex, and its complexity does not result from the mathematical difficulties that occur when attempting to describe or modeling. Traditional methods of drawing conclusions on the basis of information, such as trend analysis, have limited the usefulness of complex systems. It is a consequence of phenomena characterizing the operation of such a system. The most important features of complex systems include [4]:

- unusual sensitivity to initial conditions,
- multiplicity of unpredictable interactions that occur between the components,
- constant evaluation.

Complex systems are home to a number of characteristic phenomena such as non-scalability dependence on power law, self-organization. In this approach, an economic system is characterized by the interaction of distributed agents, the lack of a central control center, multi-level organization, continual adaptation and the lack of overall balance. There are also complex systems that are completely non-deterministic, predicting their future behavior based on data from the past is virtually impossible. In complex systems, there is also a constant self-organization [6].

Complex system is the Internet, which is often used to study various phenomena. Also, business interactions, financial accounting are examples of issues in which there are investigated the characteristics of complex systems.

Complex systems and the actual phenomena included in them are characteristic of non-scalability, and non-scalability does not exist without power laws. Interdisciplinary researches on the prevalence of power laws confirm the operation of laws of Pareto, Zipf, Gibrat in economic systems [5, 9, 10].

2. Power law

Today, technology development made it possible to collect and study the actual data on an advanced level. Exponential distributions and the related concept of non-scalability has become the subject of many studies, not only in technical sciences, but also in the natural and social sciences. The most commonly used examples of the occurrence of power laws is the dynamics of the financial markets, the distribution of wealth in society, citation rate in scientific articles, the number of entries on the website. Exponential laws in the context of complex systems are analyzed in the study of the GDP of countries in financial profits of companies of different industries, sizes of cities.

Distribution of wealth in society was studied in the late nineteenth century by Pareto, who analyzed incomes statistics in Italy, and he said that the distribution of the power series is described in the power law. This is a consequence non-scalable distribution of wealth. His discovery is invoked today in the course of GDP in international trade, as well as, in an analysis of social inequality. Today we know that the complex system is inextricably linked to the occurrence of power distributions.

Exponential probability distributions can be expressed as a formula [3, 4]:

$$P(x) = cx^{-\alpha}, \tag{1}$$

where x is a random variable discrete or continuous, α is a constant parameter of the distribution known as the exponent or scaling parameter, c is normalization constant.
In case of continuous random variable probability density is:

$$P(x) = \frac{\alpha - 1}{x_{\min}} (\frac{x}{x_{\min}})^{-\alpha}, \tag{2}$$

where constant C was determined from the normalization condition

$$\int_{x_{\min}}^{\infty} P(x)dx = 1 \tag{3}$$

assuming that x_{min} is the smallest value of the variable x, while $\alpha > 1$.
When the random variable is discrete, power-law probability distribution is:

$$P(x) = \frac{x^{-\alpha}}{\xi(\alpha, x_{\min})}, \tag{4}$$

where

$$\xi(\alpha, x_{\min}) = \sum_{n=0}^{\infty} (n + x_{\min})^{-\alpha} \tag{5}$$

for $\alpha > 1$, is a generalized zeta function, which satisfies the normalization condition.

$$\sum_{x=x_{\min}}^{\infty} P(x) = c\xi(\alpha, x_{\min}) = 1. \tag{6}$$

Due to the difficulty in calculations, the sums of power functions are approached by integral. Thus in most cases, random variables are treated as continuous.
The average value of a random variable X with power-law distribution is equal to the integral.

$$< x > = \int_{x_{\min}}^{\infty} xP(x)dx \tag{7}$$

The expression is divergent for $\alpha \le 2$ the function distributions have infinite expected value, which means that none of the experimentally determined average values will be true. Non-scalability of studied systems will be demonstrated by the lack of repeatability and large fluctuations in the results.
Also for $m \ge \alpha - 1$ any moments $< x^m >$ of examined distributions are divergent.

Therefore, in real systems beside the small and medium-sized events, there will appear extreme events (international armed conflicts, people with incomes comparable to the revenue of whole societies, global financial crises).

For the power-law distribution the characteristic exponent can be determined using the formula:

$$\alpha = 1 + n\,[\sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}}]^{-1} \quad . \tag{8}$$

Where x_i for $i = 1, 2, \ldots, n$ represent the values of observed events. Parameter $x_{\min}$ means the smallest value of the variable x_i about which it may be said that examined distribution has a power-law character for all $x_i \geq x_{\min}$.

Power-law distribution can be found in the literature under the names of non-scalable distribution, Pareto distribution, Zipf distribution [5, 9, 10]. Scientific results from the traditions and theories from which they arise. Consequence of the power law is the Pareto rule, which says that in a free market economy 80% of goods is held approximately by 20% of the richest members of the population. It is universal, which means that market mechanisms and free competition are not conducive to the equitable distribution of wealth.

Equally, you can meet the application of Zipf law, which in the original version was related to the words occurrence in a natural language, and the frequency is inversely proportional to the ranking. Application of this law refers to the distribution of GDP, or the study of population in urban areas.

The economic systems are complex systems constantly undergoing self-organization, providing the basis for creating a free market and competition. The basic step is the knowledge of the mechanisms governing the system at the local level, because the self-organization takes place in the direction of local activities to global consequences [6]. Power law is one of the common signatures of a nonlinear dynamical process, chaotic process, which is at a point self-organized. With power laws it is possible to express self-similarity of the large and small, i.e. to unite different sizes and lengths. In fractal and, for example, there are many more small structures then large ones. Their respective numbers are represented by a power law distribution. A common power law for all sizes demonstrates the internal self-consistency of the fractal and its unity across all boundaries. The power law distributions result from a commonality of laws and process at all scales [7].

3. Power laws in income distribution

In studies of complex systems , in which there are power law distributions, there have been examples of economic issues. Of interest are the results of research on the Japanese economy, as compared to the Italian economy, where there were studied business incomes of various industries [8], as well as research on the sizes of cities [1, 2].

To present the occurrence of power-law distribution in the distribution of national income we used data including selected countries , the European Union and Polish sub-regions. Research was related to the degree of similarity of the phenomena occurring, regardless of the spatial dimension. Polish sub-regions were selected as territorial units NUTS 3 in territorial classification of the European Union.

Calculations and graphs were prepared using the Statistica. The variable [v] shows the share of GDP of countries in the world, the European Union and the Polish sub-regions involved in the country's GDP.

Databases have been obtained from the analysis of the International Monetary Fund (report of April 2011, for the year 2010). Data for the Polish sub-regions derived from information portal of GUS (The Polish Central Statistical Office) from the document "Gross Domestic Product of regions and sub-regions in 2000-2010". The country or sub-region with the highest income received position (rank) 1. To present power laws there have been used two graphs: a scatter diagram and graph in the double logarithmic scale, which allows discovering the nature of power law distribution. They show how the distribution of the fluctuations depends on the number of cases.

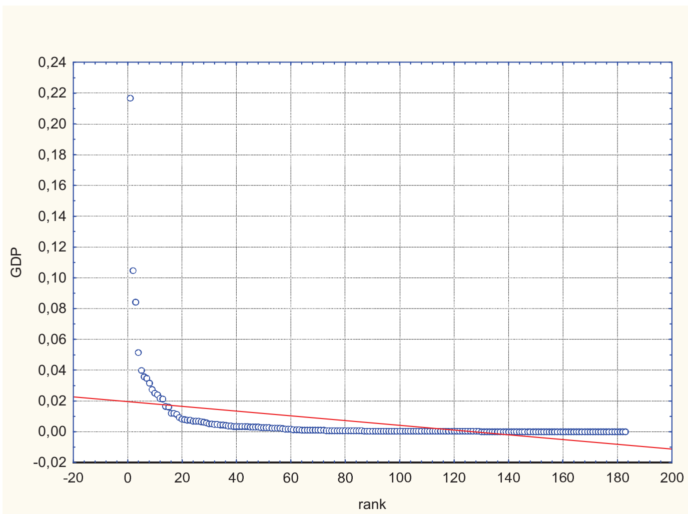

Figure 1. The share of 182 countries in the world in nominal GDP generated in 2010

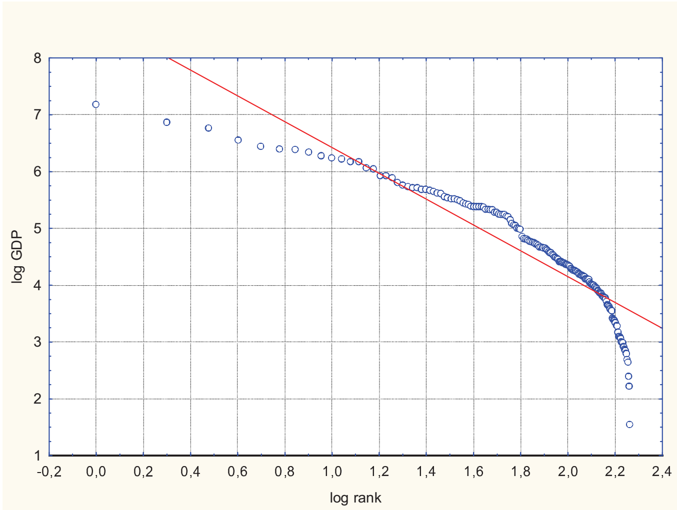

Figure 2. The share of 182 countries in the world in nominal GDP produced on a logarithmic scale in 2010

Table 1. Descriptive statistics regarding the participation of 182 countries in the world in nominal GDP generated in 2010

Descriptive statistics					
	N important	Mean	Minimum	Maximum	St. deviation
v	183	0,005468	0,000001	0,216683	0,019893

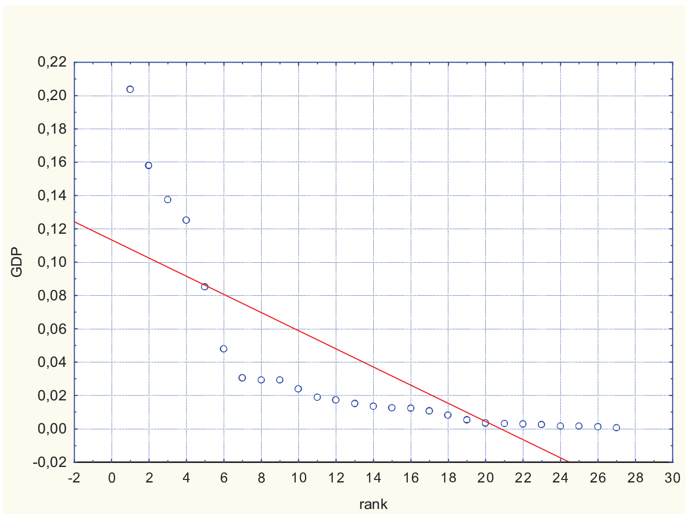

Figure 3. The share of the 27 countries of the European Union in nominal GDP in 2010

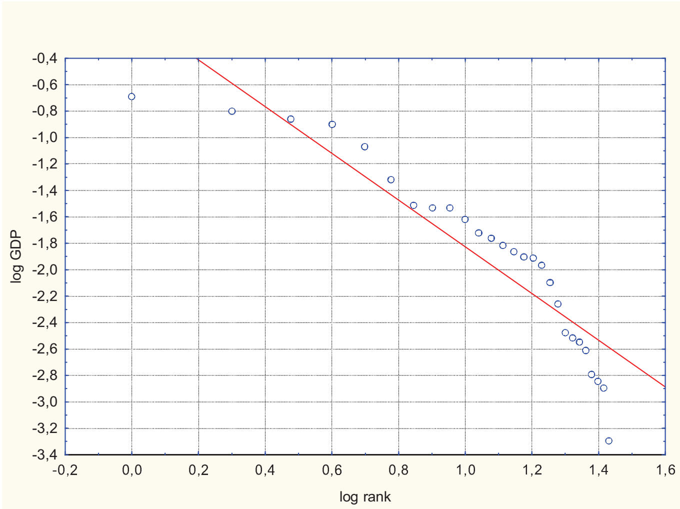

Figure 4. The share of EU-27 in nominal GDP shown on a logarithmic scale in 2010

Table 2. Descriptive statistics regarding the participation of 27 countries of the European Union in the resulting nominal GDP in 2010

Descriptive statistics					
	N important	Mean	Minimum	Maximum	St. deviation
v	27	0,037037	0,000506	0,203498	0,054853

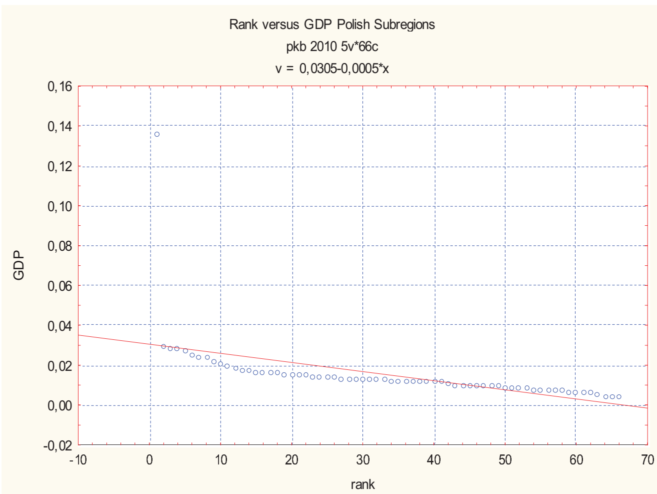

Figure 5. The share of 66 Polish sub-regions in nominal GDP in 2010

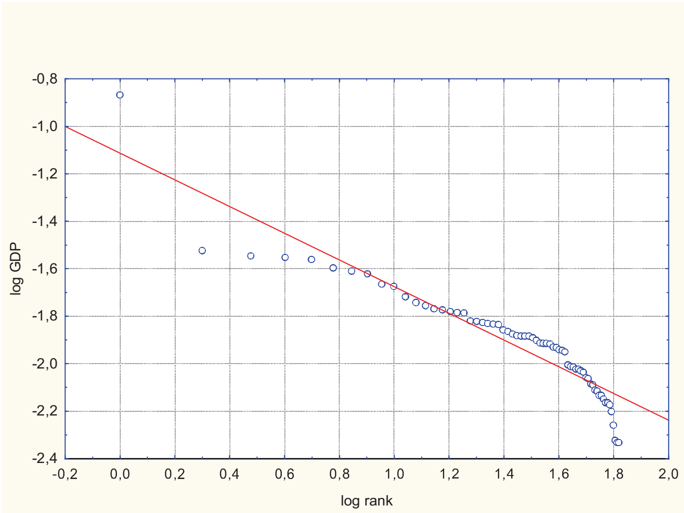

Figure 6. The share of 66 Polish sub-regions in nominal GDP on a logarithmic scale in 2010

Table 3. Descriptive statistics regarding the participation of 66 Polish sub-regions of the resulting nominal GDP in 2010

Descriptive statistics					
	N important	Mean	Minimum	Maximum	St. deviation
v	66	0,015152	0,004644	0,135372	0,016218

4. Summary

The charts that were used to visualize the given problem, suggest, due to the existing self-similarity, overlapping the power laws in the distribution of national income, and thus confirm the existence of these laws in economic systems. Non-scalability resulting from the graphs confirmed their presence at different levels of the studied phenomenon.

The values for the advanced, "rich" countries or sub-regions, in the logarithmic scale graphs normally form a line graph. For other cases, there is no such possibility. This shows that the existing inequalities in the distribution of national income at different levels cannot be avoided. It seems that on a global basis – as the most numerous, inequality will be exacerbated. It is a feature of the occurrence of power laws' distributions, where beside small and medium-sized cases, there must appear extreme events. Non-scalability is manifested by a lack of repeatability and large fluctuations in the results.

As a result, the rich countries of the world will grow faster than the others. What supports the hypothesis is also a significant share of the rich sub-EU countries in obtaining EU funds, and the development of large urban agglomerations. Interesting seem the programs aimed at reducing social inequalities that by means of the funding countries or 'poor' regions will compensate for the differences. However, a large part of funds obtained in such way anyway goes to the "rich" who are the owners of such new technologies and who thus will continue to increase their resources.

A very interesting phenomenon to be observed when examining economic systems, in the context of current power laws, are the results obtained for industries where there is a lack of competition, and even monopoly. These examples are characterized by the absence or low occurrence of power laws. This includes industries where there is poor competitiveness. An example is the energy, and in some countries, extractive industries controlled by the government.

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