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SCATTERING OF INTERFACE WAVE BY BOTTOM UNDULATIONS IN THE PRESENCE OF THIN SUBMERGED VERTICAL WALL WITH A GAP

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In this paper, the problem of interface wave scattering by bottom undulations in the presence of a thin submerged vertical wall with a gap is investigated. The thin vertical wall with a gap is submerged in a lower fluid of finite depth with bottom undulations and the upper fluid is of infinite height separated by a common interface. In the method of solution, we use a simplified perturbation analysis and suitable applications of Green's integral theorem in the two fluid regions produce first-order reflection and transmission coefficients in terms of integrals involving the shape function describing the bottom undulations and solution of the scattering problem involving a submerged vertical wall present in the lower fluid of uniform finite depth. For sinusoidal bottom undulations, the first-order transmission coefficient vanishes identically. The corresponding first-order reflection coefficient is computed numerically by solving the zero-order reflection coefficient and a suitable application of multi-term Galerkin approximations. The numerical results of the zero-order and first-order reflection coefficients are depicted graphically against the wave number in a number of figures. An oscillatory nature is observed of first-order reflection coefficient due to multiple interactions of the incident wave with bottom undulations, the edges of the submerged wall and the interface. The first-order reflection coefficient has a peak value for some particular value of the ratio of the incident wavelength and the bottom wavelength. The presence of the upper fluid has some significant effect on the reflection coefficients.

Key words: submerged vertical wall, perturbation analysis, interface wave scattering, sinusoidal bottom, Galerkin approximations, zero-order reflection coefficient, first-order reflection coefficient.

1. Introduction

Breakwaters are generally constructed to protect a harbor from the open sea. The thin vertical barriers are generally used as simple models of breakwaters to protect a sheltered area by reflecting back the incident waves into the rough sea. The problems of water wave diffraction by thin rigid barriers of various configurations are well studied in the literature by employing a number of mathematical techniques (cf. Dean [1], Ursell [2], Evans [3], Porter [4], Mandal and Kundu [5], Mandal and Dolai [6] etc.). The problems of water wave scattering by an irregular bottom have received some considerable interest in the literature on linearised theory of water waves due to their importance in finding the effects of naturally occurring bottom obstacles such as sand ripples on the wave motion. There exists only one explicit solution for the two-dimensional problem of wave propagation over a particular bottom topology considered by Roseau [7]. For general bed forms, various approximate numerical methods have been utilized in the literature. Such as the conformal mapping by which the undisturbed fluid region with a variable bottom is transformed into a uniform strip (cf. Kreisel [8], Fitz-Gerald [9], Hamilton [10]), integral equation formulation to study surface wave propagation over variable bottom topology (cf. Newman [11], Miles [12]).

Research on this class of problems was aimed at investigating the mechanism of wave-induced mass transformed that forms sand ripples and these ripples produce reflected waves thus providing a model of break water to protect the offshore area. Davies [13] considered the problem of wave reflection by sinusoidal bottom undulations using the linear perturbation theory followed by an application of Fourier transform. He obtained the reflection and transmission coefficients from the behaviour of the velocity field at infinity and observed that when the wavelength of the sinusoidal undulations is half that of the incident wave, a significant amount of wave reflection occurs. Davies and Heathershaw [14] confirmed this theoretical result by conducting experiments in a water tank.

In the aforesaid problems, the bottom irregularity is the only hindrance to the propagation of surface gravity waves. Mandal and Gayen [15] investigated the problem of water wave scattering by bottom undulations in the presence of a thin partially immersed barrier. They used a perturbation method to the governing partial differential equation producing a series of boundary value problems (BVPs) for potential functions of increasing orders, of which they considered only the first two BVPs for the zero-order and first-order. The solution for the zero-order problem was known in the literature (cf. Losada *et al.* [16], Mandal and Dolai [6], Porter and Evans [17]). The first-order potential function satisfied a radiation problem in water of uniform finite depth involving first-order reflection and transmission coefficients in the radiation condition. Analytical expressions for these first-order coefficients were obtained by using Green's integral theorem in terms of integrals involving the shape function describing the bottom topology and the solution of the zero-order problem. The first-order reflection coefficient was obtained numerically by using multi-term Galerkin approximations and depicted graphically.

The above mentioned problems involve only a single fluid. However, interface wave problems arising out of two superposed immiscible homogeneous fluids which have been investigated in the literature are very few, may be due to the complexity in handling the coupled interface conditions. Staziker *et al.* [18] considered the problem of wave scattering by undulations of arbitrary shape at the bottom connecting two fluid regions of same uniform depth by employing the integral equation method. Maiti and Mandal [19] investigated the problem of scattering of waves obliquely incident on small cylindrical undulations at the bottom of a two-layer fluid wherein the upper layer has a free surface and the lower layer has a undulating bottom. A simplified perturbation analysis was used to obtain first-order reflection and transmission coefficients. Recently, Dolai and Dolai [20] considered the problem of interface wave diffraction by bottom undulations in the presence of a thin plate submerged in the lower fluid.

The problem of interface wave scattering by bottom undulations in the presence of a vertical wall with a gap is investigated in this paper. The wall with a gap is submerged in the lower fluid of finite depth with bottom undulations and the upper fluid is of infinite height separated by a common interface. In the method of solution, we use a simplified perturbation analysis and suitable applications of Green's integral theorem in the two fluid regions produce first-order reflection and transmission coefficients in terms of integrals involving the shape function describing the bottom undulations and solution of the scattering problem involving a submerged vertical wall present in the lower fluid of uniform finite depth. For sinusoidal bottom undulations, the first-order transmission coefficient vanishes identically. The corresponding first-order reflection coefficient is computed numerically by solving the zero-order reflection coefficient and suitable application of multi-term Galerkin approximations. The numerical results of zero-order and first-order reflection coefficients are depicted graphically against the wave number in a number of figures and an oscillatory nature of first-order reflection coefficient is observed due to multiple interaction of the incident wave with bottom undulations, the edges of the submerged wall and the interface. The first-order reflection coefficient has peak value for some particular value of the ratio of the incident wavelength and the bottom wavelength. The presence of the upper fluid has some significant effect on the reflection coefficients.

Lamb [21] reported that in the mouths of some of the Norwegian fjords there exist layers of fresh water over salt water. This remark forms a basis for the practical interest in the problem considered here, wherein a two-fluid model is constructed to investigate the effect of the upper fluid on reflection and transmission coefficients for the classical problem of water wave scattering by a submerged vertical wall and bottom undulations. Also, this model may be used for practical purposes to find the effect of air on the reflection and transmission coefficients by interpreting the two fluid problems as an atmosphere-ocean system. However, as the ratio of the densities of air and water is *0.0013*, this is too small to produce any appreciable effect on the reflection coefficients.

2. Formulation of the problem

We consider two dimensional irrotational motions in two immiscible, inviscid, homogeneous and incompressible superposed fluids, the lower fluid is of finite depth having small undulations at the bottom and the upper fluid is of infinite height. A Cartesian co-ordinate system is chosen in which the y-axis is taken vertically downwards into the lower fluid and the plane $y = 0$ is the undisturbed position of the interface. Let ρ_1 be the density of the lower fluid occupying the region $0 \le y \le h + \varepsilon c(x)$ and $\rho_2 (< \rho_1)$ be the density of the upper fluid occupying the region $y \le 0$. Here $c(x)$ is a continuous bounded function describing the shape of the bottom, $c(x) \to 0$ as $|x| \to \infty$ and ε is a very small dimensionless parameter giving a measure of smallness of the bottom undulations. Let a thin vertical wall with a gap be submerged into the lower fluid below the mean interface. The position of the wall is described by $x = 0, y \in L$ with $L = (a,b) \cup (d,h)$ and very long in the z direction, so that the problem of ensuing motion is two dimensional and depends on x, y only. A simple sketch of the problem is given in Fig.1.

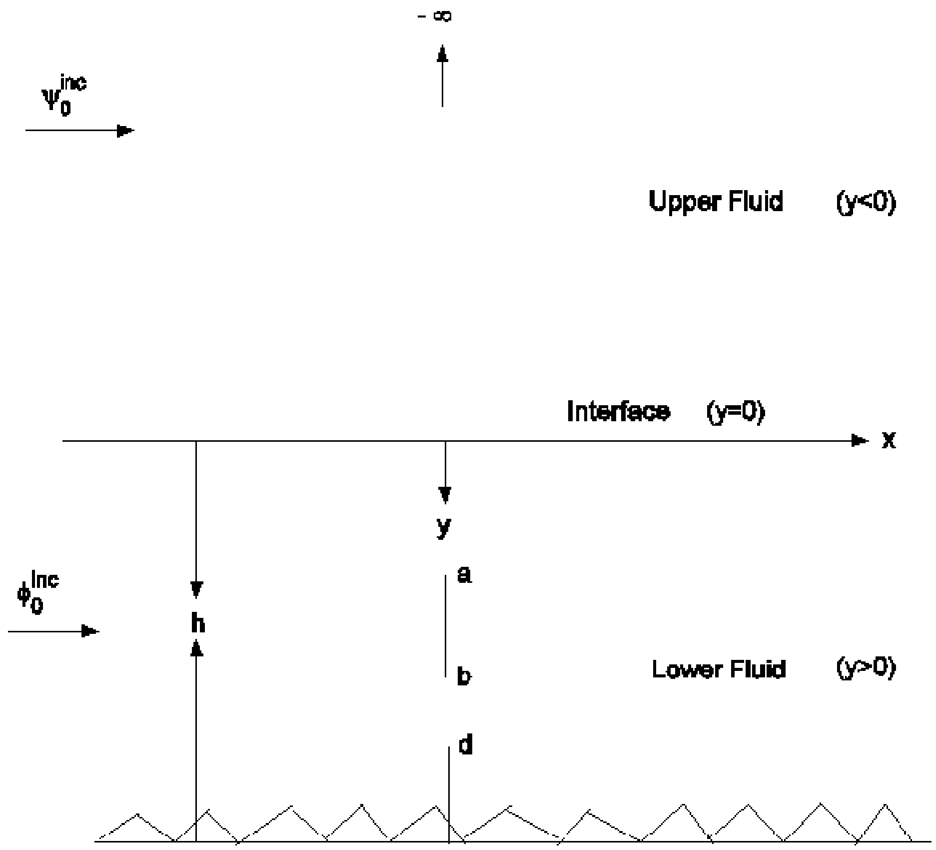

Fig.1. Geometry of the problem.

The incident wave train propagating at the interface from negative infinity is partially reflected by the wall and bottom undulations and transmitted through the gaps. Assuming the linear theory, the time harmonic progressive waves from negative infinity can be represented by the velocity potentials $\mathrm{Re}\{\phi^{\mathrm{inc}}(x,y)\exp(-i\sigma t)\}$ in the lower fluid and $\mathrm{Re}\{\psi^{\mathrm{inc}}(x,y)\exp(-i\sigma t)\}$ in the upper fluid, where

$$\phi^{\text{inc}}(x,y)=\frac{\cosh k_0(h-y)}{\sinh k_0 h}\exp(ik_0x), \qquad y>0,$$

$$\psi^{\text{inc}}(x,y)=-\exp(k_0y+ik_0x), \qquad y<0, \tag{2.1}$$

k_0 being the real positive root of the transcendental equation

$$\{k-s(k+K)\}\tanh kh = K, \tag{2.2}$$

with $K=\sigma^2/g$, $s=\rho_2/\rho_1$ $(0\le s<1)$, σ being the frequency of the incoming waves and g being the gravity. Due to the presence of the wall, the incident wave train is partially reflected by the wall and bottom undulations and transmitted through the gaps. If the resulting motion is described by the velocity potentials $\operatorname{Re}\{\phi(x,y)\exp(-i\sigma t)\}$ and $\operatorname{Re}\{\psi(x,y)\exp(-i\sigma t)\}$ in the lower and upper fluids, respectively, then ϕ and ψ satisfy

$$\nabla^2\phi=0 \text{ in the lower fluid region,}$$

$$\nabla^2\psi=0 \text{ in the upper fluid region,} \tag{2.3}$$

the interface condition

$$K\phi+\phi_y=s(K\psi+\psi_y), \qquad \phi_y=\psi_y \qquad \text{on} \qquad y=0, \tag{2.4}$$

the condition on the wall

$$\phi_x=0 \qquad \text{on} \qquad x=0, \qquad y\in L, \tag{2.5}$$

the bottom condition

$$\phi_n=0 \qquad \text{on} \qquad y=h+\varepsilon c(x), \tag{2.6}$$

n denote the normal derivative, the edge condition

$$r^{1/2}\nabla\phi \qquad \text{is bounded as} \qquad r\to 0, \tag{2.7}$$

r is the distance from the submerged ends of the wall, and the infinity conditions

$$\begin{bmatrix}\phi(x,y)\\ \psi(x,y)\end{bmatrix}\to\begin{cases} T\begin{bmatrix}\phi^{\text{inc}}(x,y)\\ \psi^{\text{inc}}(x,y)\end{bmatrix} & \text{as} \qquad x\to\infty,\\ \begin{bmatrix}\phi^{\text{inc}}(x,y)\\ \psi^{\text{inc}}(x,y)\end{bmatrix}+R\begin{bmatrix}\phi^{\text{inc}}(-x,y)\\ \psi^{\text{inc}}(-x,y)\end{bmatrix} & \text{as} \qquad x\to-\infty\end{cases} \tag{2.8}$$

where T and R are, respectively, the unknown transmission and reflection coefficients and are to be determined.

The bottom condition (2.6) can be expressed approximately as

$$\phi_y - \varepsilon \frac{d}{dx}\{c(x)\phi_x\} + O\left(\varepsilon^2\right) = 0 \qquad \text{on} \qquad y = h. \tag{2.9}$$

This suggests that a perturbation technique can be employed to solve the BVP described by Eqs (2.3) to (2.8) approximately. This is described in the next section.

3. Method of solution

The approximate boundary condition (2.9) suggests that ϕ, ψ, R, T can be expressed in terms of ε as given by

$$\begin{aligned}
&\phi(x,y;\varepsilon) = \phi_0(x,y) + \varepsilon\phi_1(x,y) + O\left(\varepsilon^2\right),\\
&\psi(x,y;\varepsilon) = \psi_0(x,y) + \varepsilon\psi_1(x,y) + O\left(\varepsilon^2\right),\\
&R(\varepsilon) = R_0 + \varepsilon R_1 + O\left(\varepsilon^2\right),\\
&T(\varepsilon) = T_0 + \varepsilon T_1 + O\left(\varepsilon^2\right).
\end{aligned} \tag{3.1}$$

Substituting the expressions (3.1) in Eqs (2.3) to (2.5) and (2.7) to (2.9), we find after equating the coefficients of ε^0 and ε from both sides, that the functions $\phi_0(x,y), \psi_0(x,y), \phi_1(x,y)$ and $\psi_1(x,y)$ satisfy the following BVPs:

BVP-I: The functions $\phi_0(x,y)$ and $\psi_0(x,y)$ satisfy

$$\nabla^2\phi_0 = 0, \qquad 0 < y < h,$$

$$\nabla^2\psi_0 = 0, \qquad y < 0,$$

$$K\phi_0 + \phi_{0y} = s\left(K\psi_0 + \psi_{0y}\right), \qquad \phi_{0y} = \psi_{0y} \qquad \text{on} \qquad y = 0,$$

$$\phi_{0x} = 0 \qquad \text{on} \qquad x = 0, \qquad y \in L,$$

$$\phi_{0y} = 0 \qquad \text{on} \qquad y = h,$$

$$r^{1/2}\nabla\phi_0 \qquad \text{is bounded as} \qquad r \to 0,$$

$$\begin{bmatrix}\phi_0(x,y)\\ \psi_0(x,y)\end{bmatrix}\to\begin{cases}T_0\begin{bmatrix}\phi^{\text{inc}}(x,y)\\ \psi^{\text{inc}}(x,y)\end{bmatrix} & \text{as}\quad x\to\infty,\\ \begin{bmatrix}\phi^{\text{inc}}(x,y)\\ \psi^{\text{inc}}(x,y)\end{bmatrix}+R_0\begin{bmatrix}\phi^{\text{inc}}(-x,y)\\ \psi^{\text{inc}}(-x,y)\end{bmatrix} & \text{as}\quad x\to-\infty.\end{cases}$$

BVP-II: The functions $\phi_1(x,y)$ and $\psi_1(x,y)$ satisfy

$$\nabla^2\phi_1=0,\qquad 0<y<h,$$

$$\nabla^2\psi_1=0,\qquad y<0,$$

$$K\phi_1+\phi_{1y}=s\left(K\psi_1+\psi_{1y}\right),\qquad \phi_{1y}=\psi_{1y}\qquad \text{on}\qquad y=0,$$

$$\phi_{1x}=0\qquad \text{on}\qquad x=0,\qquad y\in L,$$

$$\phi_{1y}=\frac{d}{dx}\{c(x)\phi_{0x}\}\qquad \text{on}\qquad y=h,$$

$$r^{1/2}\nabla\phi_1\qquad \text{is bounded}\qquad \text{as}\qquad r\to 0,$$

$$\begin{bmatrix}\phi_1(x,y)\\ \psi_1(x,y)\end{bmatrix}\to\begin{cases}T_1\begin{bmatrix}\phi^{\text{inc}}(x,y)\\ \psi^{\text{inc}}(x,y)\end{bmatrix} & \text{as}\quad x\to\infty,\\ R_1\begin{bmatrix}\phi^{\text{inc}}(-x,y)\\ \psi^{\text{inc}}(-x,y)\end{bmatrix} & \text{as}\quad x\to-\infty.\end{cases}$$

It may be noted that the BVP-I corresponds to the problem of interface wave scattering by a thin vertical wall with a gap submerged in the lower fluid of uniform finite depth *h*. In the absence of the upper fluid, this problem has been solved in the literature approximately in the sense that numerical estimates for R_0 and T_0 have been obtained (cf. Banerjea *et al.* [22]).

The BVP-II is a radiation problem in a two-fluid medium in which the bottom condition involves ϕ_0 the solution of BVP-I. Without solving $\phi_1(x,y),\psi_1(x,y)$ explicitly, R_1 and T_1 can be determined in terms of integrals involving the shape function $c(x)$ and $\phi_{0x}(x,h)$.

Applying Green's integral theorem to the functions $\phi_0(x,y)$ and $\phi_1(x,y)$ in the region bounded by the lines $y=0,-X\le x\le X; x=X,0\le y\le h; y=h,0<x\le X;$ $x=0-,d\le y\le h; x=0+,d\le y\le h; y=h,$ $-X\le x<0; x=-X,0\le y\le h$ and a closed contour around $x=0,a\le y\le b$, where X is large and positive, and ultimately makes $X\to\infty$, produces

$$iR_1\frac{2k_0h+\sinh 2k_0h}{2\sinh^2 k_0h}=\int_{-\infty}^{\infty}c(x)\phi_{0x}^2(x,h)\,dx+\int_{-\infty}^{\infty}\left(\phi_0\phi_{1y}-\phi_1\phi_{0y}\right)_{y=0}dx\,. \tag{3.2}$$

Again, applying Green's integral theorem to the functions $\psi_0(x,y)$ and $\psi_1(x,y)$ in the region bounded by the lines $y=0, -X \le x \le X; x=\pm X, -Y \le y \le 0; y=-Y, -X \le x \le X$, where Y is large and positive and make $X, Y \to \infty$. We find

$$iR_1 = \int_{-\infty}^{\infty} \left(\psi_0\psi_{1y} - \psi_1\psi_{0y}\right)_{y=0} dx . \tag{3.3}$$

Multiplying (3.3) by s and subtracting from Eq.(3.2), we find

$$\begin{aligned} iR_1\left[\frac{2k_0h + \sinh 2k_0h}{2\sinh^2 k_0h} - s\right] &= \int_{-\infty}^{\infty} c(x)\phi_{0x}^2(x,h)\,dx + \\ &+ \int_{-\infty}^{\infty} \left[\phi_0\phi_{1y} - s\psi_0\psi_{1y} - \phi_1\phi_{0y} + s\psi_1\psi_{0y}\right]_{y=0} dx. \end{aligned} \tag{3.4}$$

The interface conditions satisfied by ϕ_0, ψ_0 and ϕ_1, ψ_1 imply

$$\begin{aligned} \phi_0\,\phi_{1y} - s\,\psi_0\,\psi_{1y} &= \frac{s-1}{K}\phi_{0y}\,\phi_{1y} \quad \text{on} \quad y=0, \\ \phi_1\,\phi_{0y} - s\,\psi_1\,\psi_{0y} &= \frac{s-1}{K}\phi_{0y}\,\phi_{1y} \quad \text{on} \quad y=0, \end{aligned} \tag{3.5}$$

so that the term in the square bracket in the right side of Eq.(3.4) vanishes identically. Thus, Eq.(3.4) produces

$$iR_1 = N_0 \int_{-\infty}^{\infty} c(x)\phi_{0x}^2(x,h)\,dx \tag{3.6}$$

where

$$N_0 = \frac{2\sinh^2 k_0h}{2k_0h + \sinh 2k_0h - 2s\sinh^2 k_0h} .$$

Similarly, applying Green's integral theorem to $\phi_0(-x,y)$ and $\phi_1(x,y)$ in the lower region, $\psi_0(-x,y)$ and $\psi_1(x,y)$ in the upper region described above, we find

$$iT_1 = -N_0 \int_{-\infty}^{\infty} c(x)\phi_{0x}(x,h)\,\phi_{0x}(-x,h)\,dx . \tag{3.7}$$

Thus, both R_1 and T_1 are derived in terms of integrals involving the shape function $c(x)$ and zero-order potential function $\phi_0(x,y)$. Unfortunately $\phi_0(x,y)$ cannot be obtained analytically. However, it can be expressed as

$$\phi_0(x,y)=\begin{cases}\left\{\exp(ik_0x)+R_0\exp(-ik_0x)\right\}\dfrac{\cosh k_0(h-y)}{\sinh k_0 h}+\sum\limits_{n=1}^{\infty}A_n\cos k_n\left(h-y\right)\exp(k_nx), & x<0,\\ T_0\exp(ik_0x)\dfrac{\cosh k_0(h-y)}{\sinh k_0h}+\sum\limits_{n=1}^{\infty}B_n\cos k_n(h-y)\exp(-k_nx), & x>0\end{cases}\tag{3.8}$$

where $k_n\ (n=1,2,\cdots\cdots)$ are real roots of $k\tan kh+K=0$ and $A_n, B_n\ (n=1,2,\cdots\cdots)$ are unknown constants. R_0, T_0, A_n, B_n can be estimated numerically by using multi-term Galerkin approximations employed by Porter and Evans [17]. The details are given in the Appendix.

Thus, R_1 and T_1 can be computed numerically once the shape function $c(x)$ is known. We consider sinusoidal undulations at the bottom so that $c(x)$ can be taken in the form

$$c(x)=\begin{cases}c_0\sin\lambda x,\ -l\le x\le l,\ l=\dfrac{m\pi}{\lambda},\\ 0,\ \text{otherwise}\end{cases}\tag{3.9}$$

where m is a positive integer. Thus there exists m number of sinusoidal ripples at the bottom with wave number λ. In this case T_1 vanishes identically, and R_1 is given by

$$\begin{aligned}R_1=&\frac{N_0c_0k_0^2(R_0-1)}{\sinh^2 k_0h}\left\{\frac{\sin(\lambda-2k_0)l}{\lambda-2k_0}-\frac{\sin(\lambda+2k_0)l}{\lambda+2k_0}\right\}+\\&+\frac{iN_0c_0k_0^2R_0}{\sinh^2k_0h}\left\{\frac{2(1-\cos\lambda l)}{\lambda}-\frac{2\lambda}{\lambda^2-4k_0^2}+\frac{\cos(\lambda-2k_0)l}{\lambda-2k_0}+\frac{\cos(\lambda+2k_0)l}{\lambda+2k_0}\right\}+\\&+\frac{2iN_0c_0k_0}{\sinh k_0h}\sum_{n=1}^{\infty}k_nA_n\left[\frac{k_n}{k_n^2+(\lambda-k_0)^2}-\frac{k_n}{k_n^2+(\lambda+k_0)^2}+\left\{\frac{(\lambda-k_0)\sin(\lambda-k_0)l-k_n\cos(\lambda-k_0)l}{k_n^2+(\lambda-k_0)^2}+\right.\right.\\&\left.\left.-\frac{(\lambda+k_0)\sin(\lambda+k_0)l-k_n\cos(\lambda+k_0)l}{k_n^2+(\lambda+k_0)^2}\right\}\exp(-k_nl)\right].\end{aligned}$$

4. Numerical results

For numerical computations of R_0 and R_1, we need to evaluate the constants $A_n\ (n=1,2,\cdots\cdots)$ associated with the solution $\phi_0(x,y)$ of the BVP-I. These are evaluated numerically by using multi-term Galerkin approximations. In the numerical computations we take at most five terms to produce fairly accurate numerical estimates for R_0 and R_1. The zero-order reflection coefficient $|R_0|$ corresponds to the interface wave scattering due to the submerged wall with a gap in lower fluid of uniform finite depth h is plotted in Fig.2 against the wave number *Kh*. The first-order reflection coefficient $|R_1|$ corresponds to the interface wave scattering due to the submerged wall in the lower fluid with bottom undulations as shown in Figs 3 to 9 against *Kh* and in Fig.10 against $\alpha=\dfrac{2k_0}{\lambda}$.

In Fig.2, $|R_0|$ is plotted against the wave number *Kh* for $a/h=.2, b/h=.4, d/h=.6, s=0,.1,.3$. It is observed that for $s=0$ (single fluid), $|R_0|$ coincides with the known results in the literature evaluated by other methods (cf. Banerjea *et al.* [22]). It is also observed that $|R_0|$ first increases then decreases with *Kh*. Again, as s increases, $|R_0|$ increases for some *Kh*.

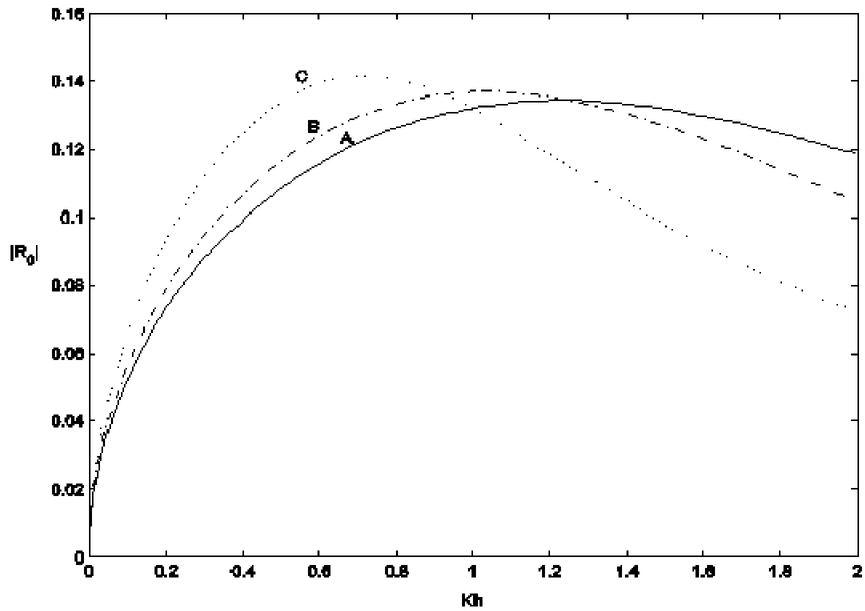

Fig.2. Profiles of $|R_0|$ for $a/h=.2, b/h=.4, d/h=.6, s=0(A), .1(B), .3(C)$.

Figure 3 depicts $|R_1|$ against *Kh* due to bottom undulations without any wall for $c_0/h=0.1, \lambda h=1.0, m=1$ (single ripple), $s=0, 0.1, 0.3$. It is observed that $|R_1|$ is oscillatory in nature with a peak value. Also, as s increases, the number of zeros of $|R_1|$ increases. This behaviour is quite expected due to multiple interactions of the incident wave with bottom undulations and the interface. Again, $|R_1| \to 0$ for large *Kh* which is quite obvious because waves are confined at the interface for large *Kh* and there will be no impact on bottom undulations.

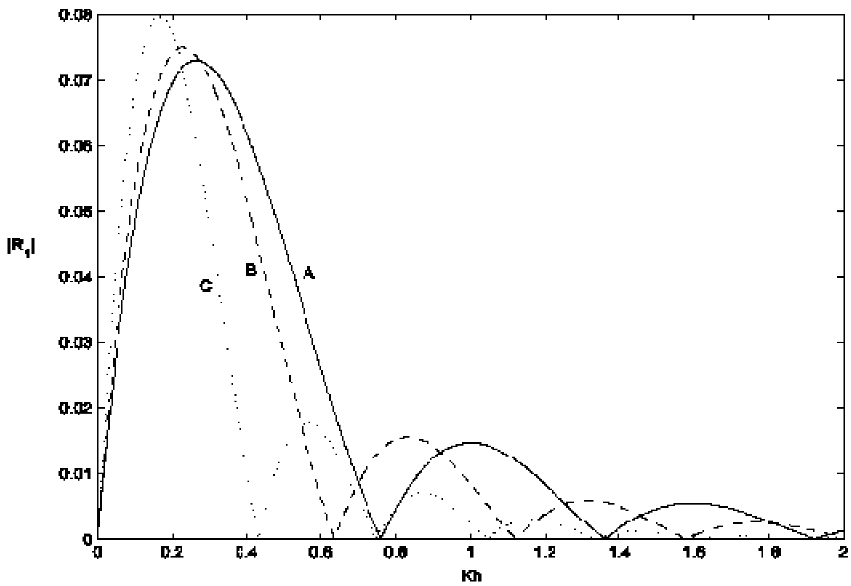

Fig.3. Profiles of $|R_1|$ for $c_0/h=.1, \lambda h=1, m=1, s=0(A), .1(B), .3(C)$.

Figures 4 depicts $|R_1|$ against Kh due to bottom undulations and submerged vertical wall with a gap in the lower fluid for $c_0 / h = 0.1, \lambda h = 1.0, m = 1$, $a / h = .2, b / h = .4, d / h = .6$, $s = 0, 0.1, 0.3$. It is observed that $|R_1|$ is oscillatory in nature. Again, due to the presence of the upper fluid and submerged vertical wall in the lower fluid, as s increases, $|R_1|$ increases for some Kh and zeros of $|R_1|$ are shifted towards the origin.

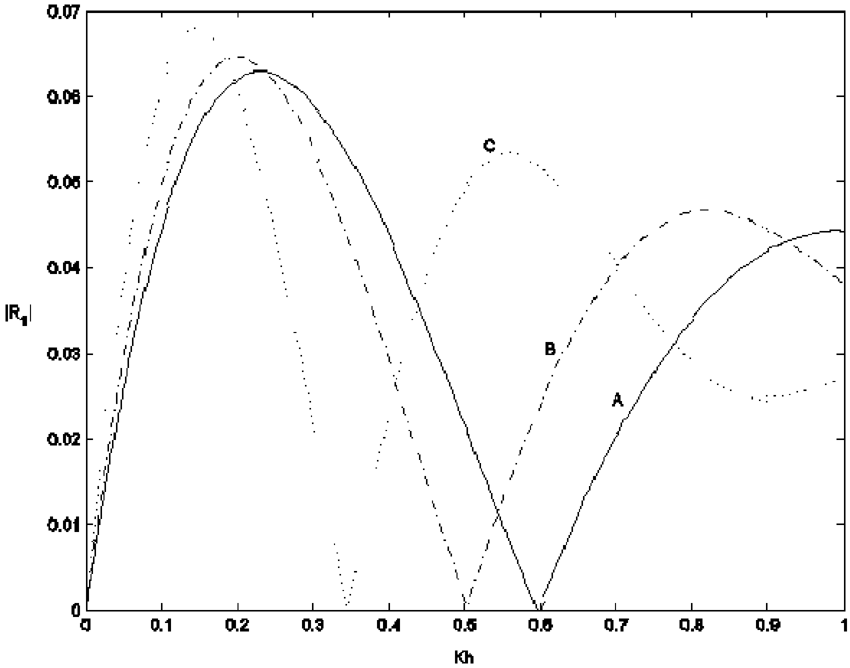

Fig.4. Profiles of $|R_1|$ for $c_0 / h = .1, \lambda h = 1, m = 1, a / h = .2, b / h = .4, d / h = .6, s = 0(A), .1(B), .3(C)$.

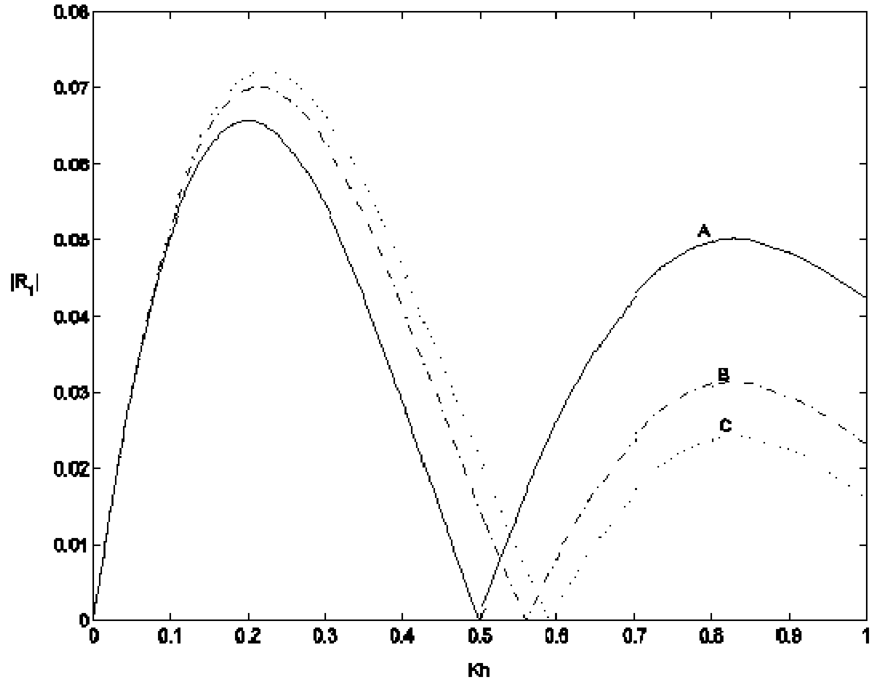

Fig.5. Profiles of $|R_1|$ for $c_0 / h = .1, \lambda h = 1, m = 1, s = .1, b / h = .6, d / h = .8, a / h = .1(A), .3(B), .5(C)$.

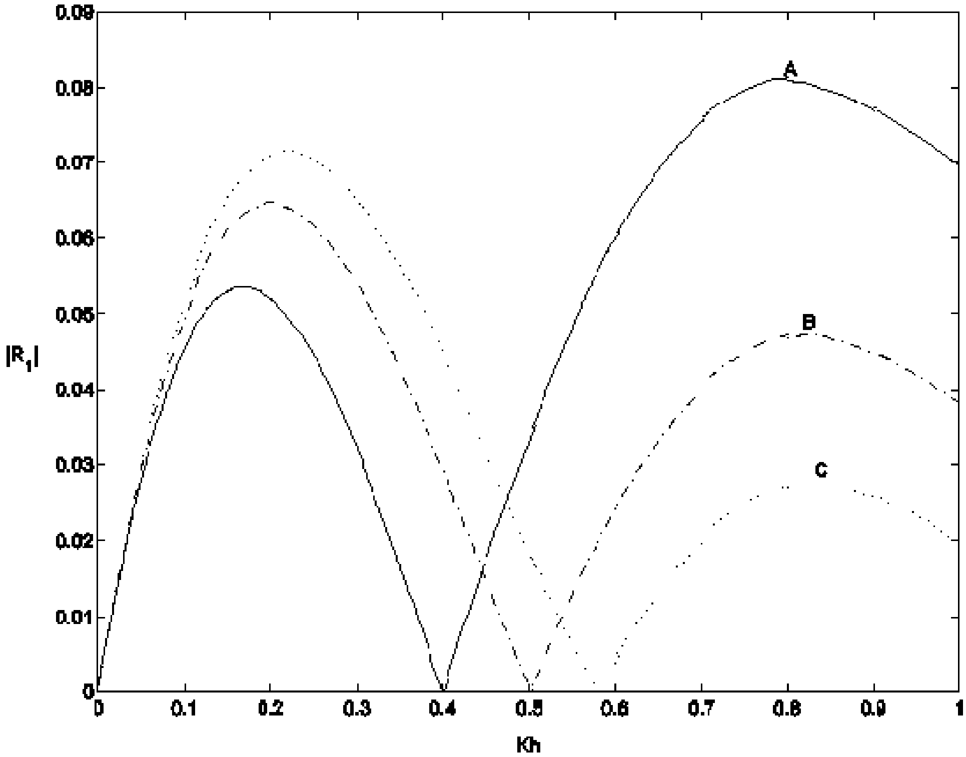

Fig.6. Profiles of $|R_1|$ for $c_0/h=.1, \lambda h=1, m=1, s=.1, a/h=.1, b/h=.3, d/h=.4(A), .6(B), .8(C)$.

Figures 5 and 6 depict $|R_1|$ against Kh for $c_0/h=.1, \lambda h=1, m=1, s=.1, b/h=.6, d/h=.8$, $a/h=.1\ .3,.5$ and $c_0/h=.1$, $\lambda h=1$, $m=1$, $s=.1$, $a/h=.1$, $b/h=.3$, $d/h=.4.6,.8$. The oscillatory behaviours of $|R_1|$ are observed. Also, $|R_1|$ increases for small Kh and decreases for large Kh as the depth of the submerged gaps increases.

Figure 7 depicts $|R_1|$ against Kh for $c_0/h=.1, \lambda h=1, a/h=.2, b/h=.4, d/h=.6$, $s=.1$, $m=2,4,6$ As m increases, $|R_1|$ also increases, becomes more oscillatory and the number of zeros also increases. This is due to multiple interactions of the incident wave between the ripple tops, the submerged wall and the interface.

Figure 8 shows the effect of c_0/h (non-dimensional ripple amplitude) on $|R_1|$ for $\lambda h=1, m=1, a/h=.2, b/h=.4, d/h=.6, s=.1, c_0/h=.2,.4,.6$. As c_0/h increases, $|R_1|$ also increases and the zeros of $|R_1|$ remain unchanged with the change of c_0/h.

Figure 9 shows the effect of λh (wavelength of the ripple) on $|R_1|$ for $c_0/h=.1$, $m=2, a/h=.2, b/h=.4, d/h=.6, s=.1, \lambda h=.2,.6,.8$. As λh increases, $|R_1|$ also increases, becomes less oscillatory and the number of zeros decreases

It is known that in the absence of the barrier in a single fluid, $|R_1|$ has a peak value when $\alpha=\frac{2k_0}{\lambda}\approx 1$ (twice the ratio of incident wavelength to the bottom wavelength). A similar feature of $|R_1|$ is also observed in Fig.10 depicting $|R_1|$ against α for $c_0/h=.1, m=2, Kh=1.5$, $s=.1; a/h=0$, $b/h=0, d/h=1$; $a/h=.1, b/h=.4$, $d/h=.8$; $a/h=.4, b/h=.6$, $d/h=.8$. However, the value of α for which $|R_1|$ attains its peak, is somewhat more than unity and the peak value reduces as the depth of the submerged wall increases.

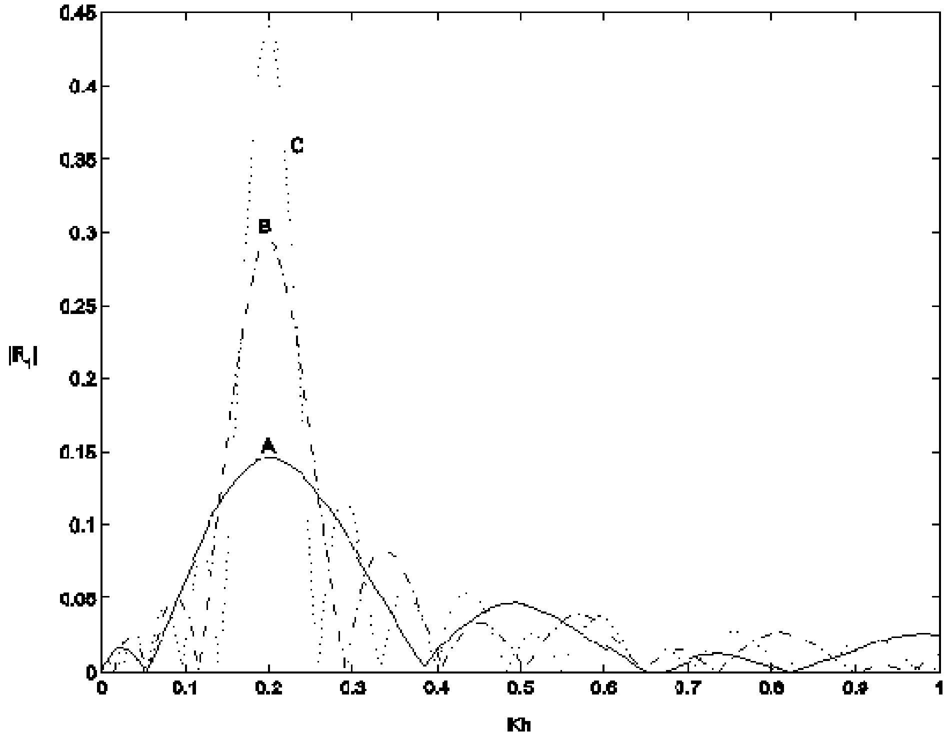

Fig.7. Profiles of $|R_1|$ for $c_0/h=.1, \lambda h=1, s=.1, a/h=.2, b/h=.4, d/h=.6, m=2(A), 4(B), 6(C)$.

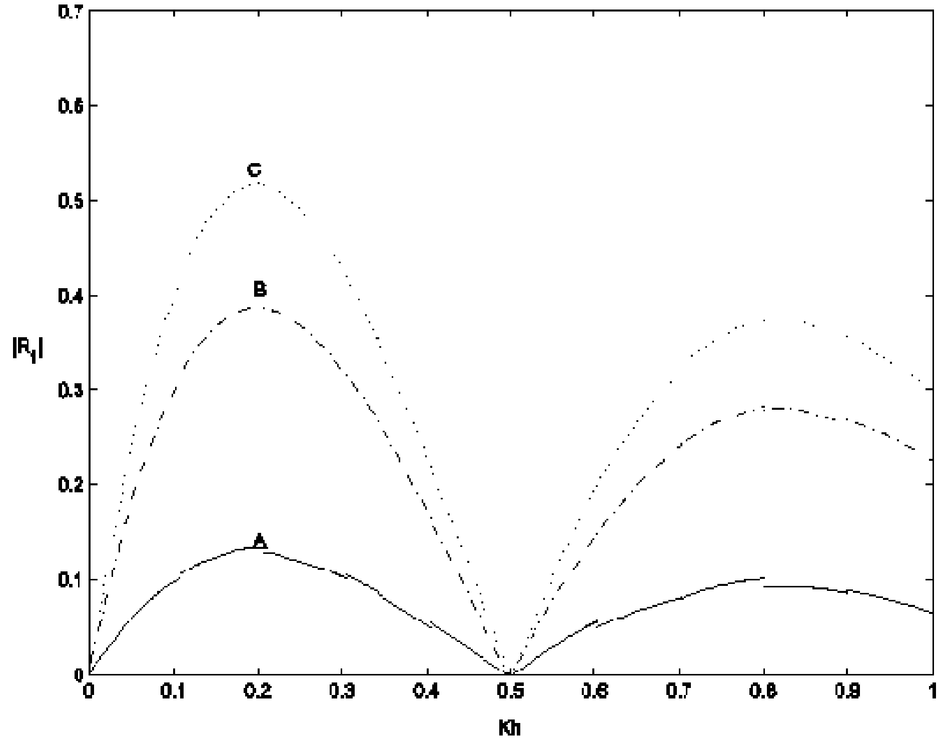

Fig.8. Profiles of $|R_1|$ for $\lambda h=1, m=1, s=.1, a/h=.2, b/h=.4, d/h=.6, c_0/h=.2(A), .4(B), .6(C)$.

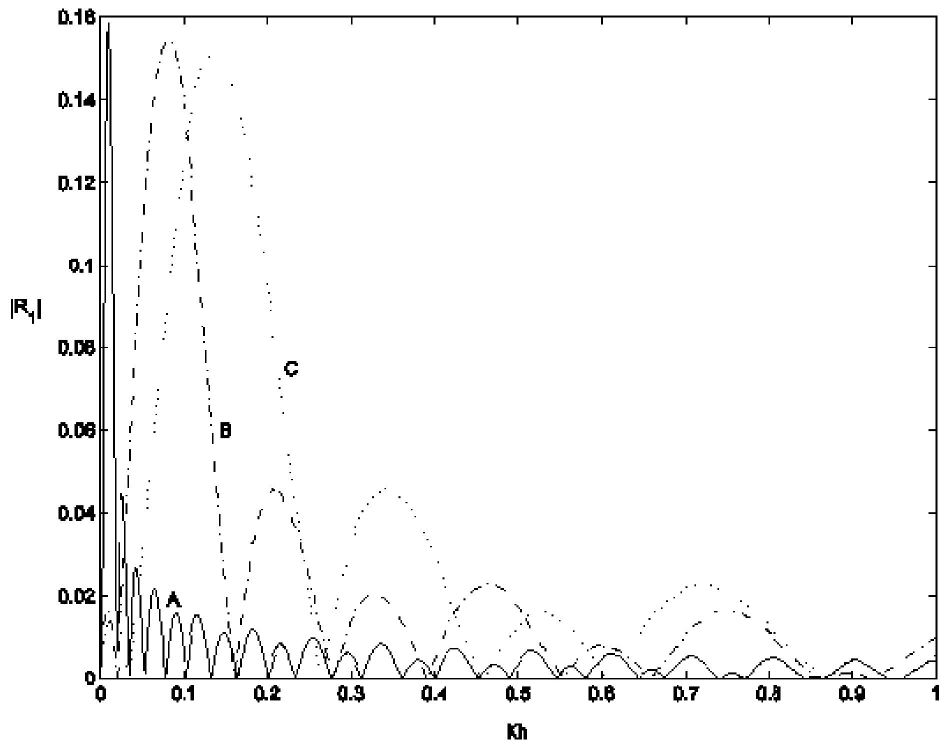

Fig.9. Profiles of $|R_1|$ for $c_0/h=.1, m=2, s=.1, a/h=.2, b/h=.4, d/h=.6, \lambda h=.2(A), .6(B), .8(C)$.

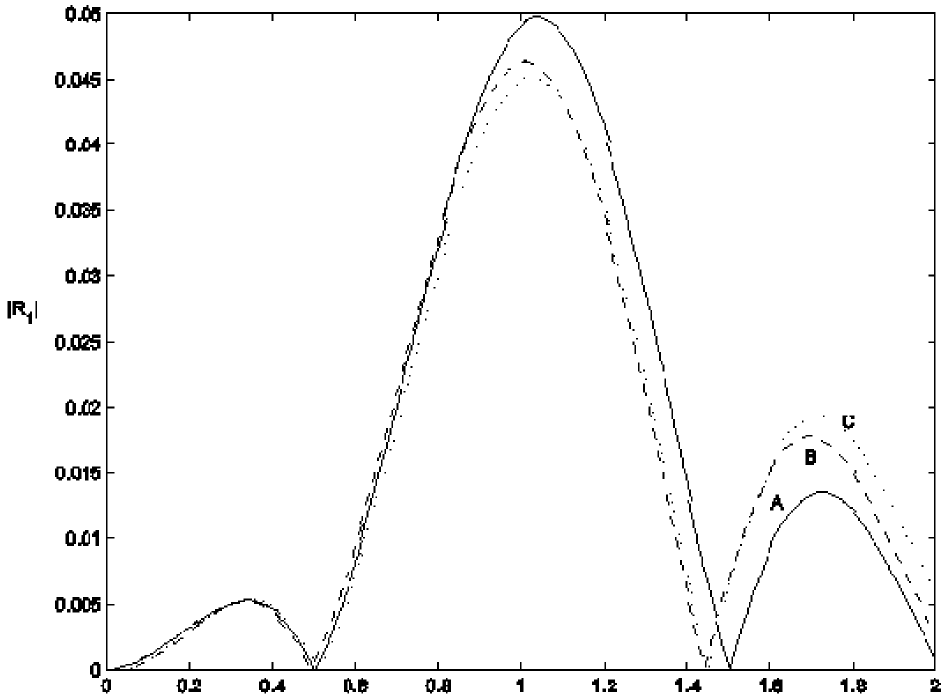

Fig.10. Profiles of $|R_1|$ for $c_0/h=.1, m=2, Kh=1.5, s=.1$, $a/h=0, b/h=0, d/h=1(A)$, $a/h=.1$, $b/h=.4$, $d/h=.8(B), a/h=.4, b/h=.6, d/h=.8(C)$.

5. Conclusion

The problem of interface wave scattering by bottom undulations in the presence of a thin vertical submerged wall with a gap in the lower fluid is considered here by employing a simplified perturbation analysis. By suitable applications of Green's integral theorem in the two-fluid regions, the first-order reflection and transmission coefficients R_1 and T_1 are obtained in terms of integrals involving the shape function describing the bottom undulations and the solution of the corresponding scattering problem for fluid of uniform finite depth. For sinusoidal ripples at the bottom, the zero-order and first-order reflection coefficients $|R_0|$ and $|R_1|$, respectively, are depicted in a number of figures for various values of the parameters. As a function of the wave number Kh, $|R_1|$ is oscillatory in nature due to multiple interaction of the incident wave with bottom undulations, the edges of the submerged wall and the interface. Due to the presence of the upper fluid, as the density ratio s of the superposed fluids increases, $|R_0|$ and $|R_1|$ also increases for moderate Kh. Also, $|R_1|$ has a peak value for some particular value of the ratio of the incident wavelength and the bottom wavelength. The overall value of $|R_1|$ are somewhat decreased due to the presence of the submerged wall compared to the case when there is no wall.

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Nomenclature

a,b,d – depth of the edges of the submerged barrier
g – gravity
h – depth of the lower fluid
K – wave number
R_0 – zero-order reflection coefficient
R_1 – first-order reflection coefficient
s – density ratio
T_0 – zero-order transmission coefficient
T_1 – first-order transmission coefficient
t – time
x – horizontal distance
y – vertical distance
ε – very small dimensionless parameter
ρ_1 – density of the lower fluid
ρ_2 – density of the upper fluid
σ – wave frequency
ϕ – velocity potential in the lower fluid
ψ – velocity potential in the upper fluid

References

[1] Dean W.R. (1945): *On the reflection of surface waves by a submerged plane barrier.* – Proc. Camb. Phil. Soc., vol.41, pp.231-238.

[2] Ursell F. (1947): *The effect of a fixed barrier on surface waves in deep water.* – Proc. Camb. Phil. Soc., vol.43, pp.374-382.

[3] Evans D.V. (1970): *Diffraction of water waves by a submerged vertical plate.* – J. Fluid Mech., vol.40, pp.433-451.

[4] Porter D. (1972): *The transmission of surface waves through a gap in a vertical barrier.* – Proc. Camb. Phil. Soc., vol.71, pp.411-422.

[5] Mandal B.N. and Kundu P.K. (1987): *Scattering of water waves by vertical barriers and associated mathematical methods.* – Proc. Indian Natn. Sci. Acad., vol.53, pp.514-530.

[6] Mandal B.N. and Dolai D.P. (1994): *Oblique water wave diffraction by thin vertical barriers in water of uniform finite depth.* – Appl. Ocean Res., vol.16, pp.195-203.

[7] Roseau M. (1976): *Asymptotic wave theory*. – North Holland, pp.311-347.

[8] Kreisel G. (1949): *Surface waves.* – Quart. Appl. Math., vol.7, pp.21-44.

[9] Fitz-Gerald G.F. (1976): *The reflection of plane gravity waves traveling in water of variable depth.* – Phil. Trans. Roy. Soc. Lond., vol.34, pp.49-89.

[10] Hamilton J. (1977): *Differential equations for long period gravity waves on fluid of rapidly varying depth.* – J. Fluid Mech., vol.83, pp.289-310.

[11] Newman J.N. (1965): *Propagation of water waves over an infinite step.* – J. Fluid Mech., vol.23, pp.399-415.

[12] Miles J.W. (1967): *Surface wave scattering matrix for a shelf.* – J. Fluid Mech., vol.28, pp.755-767.

[13] Davis A.G. (1982): *The reflection of wave energy by undulations on the seabed.* – Dyn. Atmos. Oceans, vol.6, pp.121-123.

[14] Davis A.G. and Heathershaw A.D. (1984): *Surface wave propagation over sinusoidally varying topology.* – J. Fluid Mech., vol.144, pp.419-443.

[15] Mandal B.N. and Gayen, Rupanwita (2006): *Water wave scattering by bottom undulations in the presence of a thin partially immersed barrier.* – Appl. Ocean Res. vol.28, pp.113-119.

[16] Losada I.J., Losada M.A. and Roldan A.J. (1992): *Propagation of oblique incident waves past rigid vertical thin barriers.* – Appl. Ocean Res., vol.14, pp.191-199.

[17] Porter R. and Evans D.V. (1995): *Complementary approximations to waves scattering by vertical barriers.* – J. Fluid Mech., vol.294, pp.160-186.

[18] Staziker D.J., Porter D. and Stirling D.S.G. (1996): *The scattering of surface waves by local bed elevations.* – Appl. Ocean Res., vol.18, pp.283-291.

[19] Maiti, Paramita and Mandal B.N. (2006): *Scattering of oblique waves by bottom undulations in a two-layer fluid.* – J. Appl. Math. and Computing, vol.22, pp.21-39.

[20] Dolai D.P. and Dolai P. (2010): *Interface wave diffraction by bottom undulations in the presence of a thin plate submerged in lower fluid.* – Int. J. Appl. Mech. and Engg., vol.15, pp.1017-1036.

[21] Lamb H. (1932): *Hydrodynamics*. – Cambridge University Press, p.371.

[22] Banerjea S., Kanoria M., Dolai D.P. and Mandal B.N. (1996): *Oblique wave scattering by submerged thin wall with gap in finite depth water.* – Appl. Ocean Res., vol.18, pp.319-327.

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Appendix

A brief outline for numerical evaluation of R_0 and A_n $(n=1,2,\cdots\cdots)$ is given here.

Let

$$\left.\begin{aligned} f(y) &= \phi_0(+0,y)-\phi_0(-0,y), \\ g(y) &= \frac{\partial\phi_0(\pm 0,y)}{\partial x}, \end{aligned}\right\} 0<y<h. \tag{A1}$$

Then

$$\begin{aligned} f(y) &= 0 \quad \text{for} \quad y\in L'=(0,a)\cup(b,d), \\ g(y) &= 0 \quad \text{for} \quad y\in L=(a,b)\cup(d,h). \end{aligned} \tag{A2}$$

Using Eq.(3.8) in the definition of $g(y)$ in Eq.(A1), we find

$$\left.\begin{aligned} g(y) &= ik_0(1-R_0)\frac{\cosh k_0(h-y)}{\sinh k_0 h}+\sum_{n=1}^{\infty}k_n A_n\cos k_n(h-y), \\ g(y) &= ik_0 T_0\frac{\cosh k_0(h-y)}{\sinh k_0 h}-\sum_{n=1}^{\infty}k_n B_n\cos k_n(h-y), \end{aligned}\right\} 0<y<h. \tag{A3}$$

The use of Havelock inversion theorem produces

$$\begin{aligned} ik_0(1-R_0) &= ik_0 T_0 = \frac{4k_0\sinh k_0 h}{2k_0 h+\sinh 2k_0 h}\int_{L'} g(y)\cosh k_0(h-y)\,dy, \\ k_n A_n &= -k_n B_n = \frac{4k_n}{2k_n h+\sin 2k_n h}\int_{L'} g(y)\cos k_n(h-y)\,dy. \end{aligned} \tag{A4}$$

Again, using Eq.(3.8) in the definition of $f(y)$ in Eq.(A1), we find

$$f(y) = \left(T_0-1-R_0\right)\frac{\cosh k_0(h-y)}{\sinh k_0 h}+\sum_{n=1}^{\infty}\left(B_n-A_n\right)\cos k_n(h-y), \qquad 0<y<h. \tag{A5}$$

The use of Havelock inversion theorem produces

$$\begin{aligned} 2R_0 &= -\frac{4k_0\sinh k_0 h}{2k_0 h+\sinh 2k_0 h}\int_{L} f(y)\cosh k_0(h-y)\,dy, \\ 2A_n &= -\frac{4k_n}{2k_n h+\sin 2k_n h}\int_{L} f(y)\cos k_n(h-y)\,dy. \end{aligned} \tag{A6}$$

If we define

$$F(y)=\frac{\sinh^2 k_0h}{2k_0h+\sinh 2k_0h}\frac{2f(y)}{ik_0(1-R_0)},\ y\in L,$$

$$G(y)=-\frac{\sinh^2 k_0h}{2k_0h+\sinh 2k_0h}\frac{4g(y)}{R_0},\ y\in L', \tag{A7}$$

then $F(y)$ and $G(y)$ satisfy the integral equations

$$\int_L F(t)\mathrm{K}_F(y,t)\,dt=\frac{\cosh k_0(h-y)}{\sinh k_0h}, \qquad y\in L, \tag{A8}$$

$$\int_{L'} G(t)\mathrm{K}_G(y,t)\,dt=\frac{\cosh k_0(h-y)}{\sinh k_0h}, \qquad y\in L' \tag{A9}$$

where

$$\mathrm{K}_F(y,t)=\frac{2k_0h+\sinh 2k_0h}{\sinh^2 k_0h}\sum_{n=1}^{\infty}\frac{k_n^2\cos k_n(h-y)\cos k_n(h-t)}{2k_nh+\sin 2k_nh}, \qquad y,t\in L,$$

$$\mathrm{K}_G(y,t)=\frac{2k_0h+\sinh 2k_0h}{\sinh^2 k_0h}\sum_{n=1}^{\infty}\frac{\cos k_n(h-y)\cos k_n(h-t)}{2k_nh+\sin 2k_nh}, \qquad y,t\in L', \tag{A10}$$

together with

$$\int_L F(y)\frac{\cosh k_0(h-y)}{\sinh k_0h}dy=\frac{1}{k_0^2C},$$

$$\int_{L'} G(y)\frac{\cosh k_0(h-y)}{\sinh k_0h}dy=C \tag{A11}$$

where

$$C=i\left(1-\frac{1}{R_0}\right). \tag{A12}$$

It may be noted that the functions $F(y),G(y)$ and the constant C are all real. The integral Eqs (A8) and (A9) are to be solved by $(N+1)$ multi-term Galerkin approximations of $F(y)$ and $G(y)$ in terms of Chebyshev polynomials given by (cf. Porter and Evans [17])

$$F(y)=\begin{cases}\sum_{n=0}^{N}a_n^{(1)}f_n^{(1)}(y), & a<y<b,\\ \sum_{n=0}^{N}a_n^{(2)}f_n^{(2)}(y), & d<y<h,\end{cases} \tag{A13}$$

$$G(y)=\begin{cases}\sum_{n=0}^{N} b_n^{(1)} g_n^{(1)}(y), & 0<y<a,\\ \sum_{n=0}^{N} b_n^{(2)} g_n^{(2)}(y), & b<y<d\end{cases} \tag{A14}$$

where

$$f_n^{(1)}(y)=\frac{2\left[(y-a)(b-y)\right]^{1/2}}{\pi(n+1)(b-a)h}U_n\left(\frac{2y-a-b}{b-a}\right), \qquad a<y<b, \tag{A15}$$

$$f_n^{(2)}(y)=\frac{2(-1)^n\left[(y-d)(2h-d-y)\right]^{1/2}}{\pi(2n+1)(h-d)h}U_{2n}\left(\frac{h-y}{h-d}\right), \qquad d<y<h, \tag{A16}$$

$$g_n^{(1)}(y)=\frac{d}{dy}\left[\exp(-Ky)\int_y^a u_n^{(1)}(t)\exp(Kt)\,dt\right], \qquad 0<y<a, \tag{A17}$$

with

$$\begin{aligned} u_n^{(1)}(y)&=\frac{2(-1)^n}{\pi(a^2-y^2)^{1/2}}T_{2n}\left(\frac{y}{a}\right), \quad 0<y<a,\\ g_n^{(2)}(y)&=\frac{2}{\pi\left[(y-b)(d-y)\right]^{1/2}}T_n\left(\frac{2y-b-d}{d-b}\right), \quad b<y<d.\end{aligned} \tag{A18}$$

U_n and T_n being the Chebyshev polynomials of second and first kinds respectively. The unknown coefficients $a_n^{(1)}, a_n^{(2)}, b_n^{(1)}, b_n^{(2)}$ $(n=0,1,2,\cdots\cdots,N)$ are obtained by solving the system of linear equations

$$\begin{aligned} \sum_{n=0}^{N} a_n^{(1)} \mathrm{K}_{mn}^{F_1^{(1)}} + \sum_{n=0}^{N} a_n^{(2)} \mathrm{K}_{mn}^{F_1^{(2)}} &= F_m^{(1)}, \qquad m=0,\ 1,\ 2,\cdots\cdots,N,\\ \sum_{n=0}^{N} a_n^{(1)} \mathrm{K}_{mn}^{F_2^{(1)}} + \sum_{n=0}^{N} a_n^{(2)} \mathrm{K}_{mn}^{F_2^{(2)}} &= F_m^{(2)}, \qquad m=0,\ 1,\ 2,\cdots\cdots,N,\end{aligned} \tag{A19}$$

$$\begin{aligned} \sum_{n=0}^{N} b_n^{(1)} \mathrm{K}_{mn}^{G_1^{(1)}} + \sum_{n=0}^{N} b_n^{(2)} \mathrm{K}_{mn}^{G_1^{(2)}} &= G_m^{(1)}, \qquad m=0,\ 1,\ 2,\cdots\cdots,N,\\ \sum_{n=0}^{N} b_n^{(1)} \mathrm{K}_{mn}^{G_2^{(1)}} + \sum_{n=0}^{N} b_n^{(2)} \mathrm{K}_{mn}^{G_2^{(2)}} &= G_m^{(2)}, \qquad m=0,\ 1,\ 2,\cdots\cdots,N\end{aligned} \tag{A20}$$

where

$$\mathrm{K}_{mn}^{F_1^{(1)}}=\frac{2k_0h+\sinh 2k_0h}{\sinh^2 k_0h}\sum_{l=1}^{\infty}\frac{k_l^2}{2k_lh+\sin 2k_lh}\left(\int_a^b \cos k_l(h-y)f_m^{(1)}(y)dy\right)\left(\int_a^b \cos k_l(h-t)f_n^{(1)}(t)dt\right),$$

$$\mathrm{K}_{mn}^{F_1^{(2)}} = \frac{2k_0h + \sinh 2k_0h}{\sinh^2 k_0h} \sum_{l=1}^{\infty} \frac{k_l^2}{2k_lh + \sin 2k_lh} \left(\int_a^b \cos k_l(h-y) f_m^{(1)}(y)dy \right) \left(\int_d^h \cos k_l(h-t) f_n^{(2)}(t)dt \right),$$

$$\mathrm{K}_{mn}^{F_2^{(1)}} = \frac{2k_0h + \sinh 2k_0h}{\sinh^2 k_0h} \sum_{l=1}^{\infty} \frac{k_l^2}{2k_lh + \sin 2k_lh} \left(\int_d^h \cos k_l(h-y) f_m^{(2)}(y)dy \right) \left(\int_a^b \cos k_l(h-t) f_n^{(1)}(t)dt \right),$$

$$\mathrm{K}_{mn}^{F_2^{(2)}} = \frac{2k_0h + \sinh 2k_0h}{\sinh^2 k_0h} \sum_{l=1}^{\infty} \frac{k_l^2}{2k_lh + \sin 2k_lh} \left(\int_d^h \cos k_l(h-y) f_m^{(2)}(y)dy \right) \left(\int_d^h \cos k_l(h-t) f_n^{(2)}(t)dt \right),$$

$$\mathrm{K}_{mn}^{G_1^{(1)}} = \frac{2k_0h + \sinh 2k_0h}{\sinh^2 k_0h} \sum_{l=1}^{\infty} \frac{1}{2k_lh + \sin 2k_lh} \left(\int_0^a \cos k_l(h-y) g_m^{(1)}(y)dy \right) \left(\int_0^a \cos k_l(h-t) g_n^{(1)}(t)dt \right),$$

$$\mathrm{K}_{mn}^{G_1^{(2)}} = \frac{2k_0h + \sinh 2k_0h}{\sinh^2 k_0h} \sum_{l=1}^{\infty} \frac{1}{2k_lh + \sin 2k_lh} \left(\int_0^a \cos k_l(h-y) g_m^{(1)}(y)dy \right) \left(\int_b^d \cos k_l(h-t) g_n^{(2)}(t)dt \right),$$

$$\mathrm{K}_{mn}^{G_2^{(1)}} = \frac{2k_0h + \sinh 2k_0h}{\sinh^2 k_0h} \sum_{l=1}^{\infty} \frac{1}{2k_lh + \sin 2k_lh} \left(\int_b^d \cos k_l(h-y) g_m^{(2)}(y)dy \right) \left(\int_0^a \cos k_l(h-t) g_n^{(1)}(t)dt \right),$$

$$\mathrm{K}_{mn}^{G_2^{(2)}} = \frac{2k_0h + \sinh 2k_0h}{\sinh^2 k_0h} \sum_{l=1}^{\infty} \frac{1}{2k_lh + \sin 2k_lh} \left(\int_b^d \cos k_l(h-y) g_m^{(2)}(y)dy \right) \left(\int_b^d \cos k_l(h-t) g_n^{(2)}(t)dt \right),$$

$$F_m^{(1)} = \int_a^b \frac{\cosh k_0(h-y)}{\sinh k_0h} f_m^{(1)}(y)dy,$$

$$F_m^{(2)} = \int_d^h \frac{\cosh k_0(h-y)}{\sinh k_0h} f_m^{(2)}(y)dy,$$

$$G_m^{(1)} = \int_0^a \frac{\cosh k_0(h-y)}{\sinh k_0h} g_m^{(1)}(y)dy,$$

$$G_m^{(2)} = \int_b^d \frac{\cosh k_0(h-y)}{\sinh k_0h} g_m^{(2)}(y)dy.$$

Once $a_n^{(1)}, a_n^{(2)}, b_n^{(1)}, b_n^{(2)}$ $(n = 0,1,2,\cdots\cdots,N)$ are solved, the real constant C can be determined by using any one of the equations in Eq.(A11) after substituting from Eqs (A13) or (A14). Then R_0 can be found using Eq.(A12).

To find the constants A_n, we use either the second relation in Eq.(A4) or in Eq.(A6). Noting the relations in Eq.(A7) and the multi-term expansions Eqs (A13) or (A14), A_n is ultimately approximated as

$$A_n = -ik_0(1-R_0)\frac{2k_0h+\sinh 2k_0h}{\sinh^2 k_0h}\frac{k_n}{2k_nh+\sin 2k_nh}\sum_{l=0}^{N}\left(a_l^{(1)}\int_a^b \cos k_n(h-y)f_l^{(1)}(y)dy + \right.$$
$$\left. + a_l^{(2)}\int_d^h \cos k_n(h-y)f_l^{(2)}(y)dy\right), \tag{A.21}$$

or

$$A_n = -R_0\frac{2k_0h+\sinh 2k_0h}{\sinh^2 k_0h}\frac{1}{2k_nh+\sin 2k_nh}\sum_{l=0}^{N}\left(b_l^{(1)}\int_0^a \cos k_n(h-y)g_l^{(1)}(y)dy + \right.$$
$$\left. + b_l^{(2)}\int_b^d \cos k_n(h-y)g_l^{(2)}(y)dy\right). \tag{A.22}$$

In the numerical computations for R_1, both the sets of multi-term Galerkin approximations for $F(y)$ and $G(y)$ have been used. Almost the same numerical results for R_1 are obtained.