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SIMULATION OF DELAYED BUSES IN A DUMPLING STATE AND ANALYSIS OF MAXIMUM WAITING TIME USING LOGISTIC REGRESSION

Summary. As cities become larger and societies become more complicated, the corresponding transportation systems also become more complicated. Thus far, many important transportation models have been investigated and applied to societies. In this work, we analyze a bus transportation model that includes high randomness. By strengthening the viewpoint of the users, the bunching of buses is further explored and considered as “the dumpling bus state,” referring to cases when the next scheduled buses closely run behind a delayed bus for a while. It is described that waiting people are split into winners (people with shorter waiting times) and losers (people with longer waiting times). Waiting time is also analyzed using logistic regression to obtain the probability of people who continue to wait.

1. INTRODUCTION

A bus system is one of the most important transportation systems that is used daily by people, whether they are living in large cities or small towns [1-3]. Many cities and towns run bus transportation systems as a basic element of their social services. In contrast to trains, which can run on definite routes and stay on time, buses are frequently delayed because they run in complicated traffic situations. Bus delays are caused by a variety of factors such as bad weather and traffic jams involving cars. Although frequent delays in bus arrivals are expected, they still induce frustration among people who have to decide whether to continue waiting for the bus or change their route and select another mode of transportation such as a taxi.

Many cities have provided solutions for dealing with bus delays based on using smartphones with an internet connection [4–9]. These smart systems provide information about the length of the delays and the alternate routes available to get to the required destinations. For example, in Tokyo, mobility as a service systems [1] optimize personal transportation by linking information on several transportation routes through smartphones with destination services in the vicinity of stations and along railway lines. In these systems, a GPS is the major tool used to provide real-time traffic information and help the system calculate the shortest route for people to reach their destinations. For the bus system, monitoring each bus audio-visually from a remote office supports advanced bus operations. Data are collected using adaptive video distribution and abnormal sound detection technologies to ensure the safety of the bus. These methods focus more on observing the buses than describing their transportation patterns precisely.

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Thus, there is a gap between research on cars and buses: the construction of the transportation models of buses has been more difficult than that of cars.

Many traffic congestion simulations have been carried out since the 1960s [10]. Car-related traffic congestion on highways has been investigated most intensively [11–14]. On highways, there are no traffic lights or stop signs; therefore, slow traffic can be mainly attributed to cars that have mutual interactions. The mathematical formulation of car flows on highways is relatively easier than it is in the city, where there are many complicated factors that influence the formulation, such as traffic lights, pedestrians, and abundant city facilities. City buses have more complicated factors to deal with than personal cars do. Buses stop at specified bus stops, and the number of people using these buses cannot be exactly predicted, even for the same bus stops with the usual traffic lights and stop signs. These unstable bus transportation cases have been investigated as a bunching bus phenomenon [4,15-17]. Regarding the theoretical traffic model, asymmetric simple exclusion process (ASEP) models have been developed to describe the phenomenon of bunching buses. These models are based on stochastic cellular automaton models and have succeeded in describing various traffic jams. Most of the cases treated in ASEP models are described in one-dimensional structures, and periodic patterns are treated as an extension of the one-dimensional model. Recently, automated driving systems have been discussed as a development in neural network models. Kerner discussed the free-flow-to-synchronized-flow transition [18]. Baghbani et al. analyzed short-term passenger flow using a bus network graph and showed that their bus model is scalable [19].

However, as the models include many of the factors mentioned above, the corresponding mathematical models become complicated in large cities. Buses are systematically controlled by companies under the regulations of the public sectors in the cities in which they run. This enables smoother bus transportation, but it induces another complexity in bus transportation. These matters imply that when describing bus services, the presence of high randomness should be considered.

In this study, we focus on buses that periodically run in a definite closed loop. We use a simple model in which a complicated mathematical formulation is not required. Based on the perspective of running in a loop, Sugiyama et al. [13] conducted a simulation of cars running in a circle. They discussed the bottlenecks in traffic congestion cases and found that congestion occurred when the traffic flow exceeded the bottleneck limit. They investigated this traffic problem as a dynamic phenomenon in multi-particle systems. When the car density was below the critical value, a constant flow was maintained. On the other hand, when it was beyond the critical value, the flow was disrupted. Note that buses cannot be described by a single parameter, such as density. Since buses frequently stop owing to complicated traffic conditions, it is better to describe their stops purely as random events.

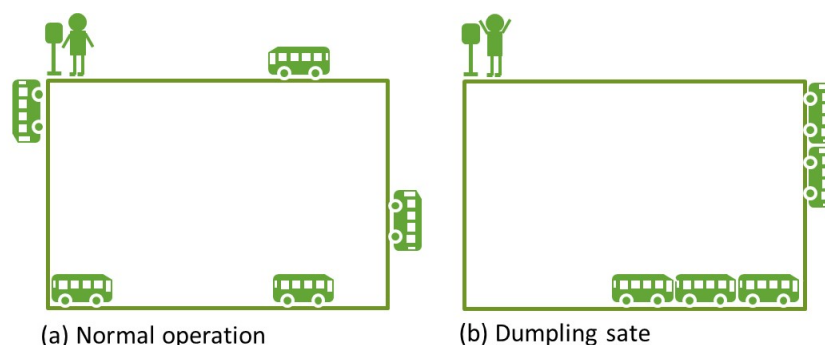


Fig. 1. A bus simulation model of buses periodically running around a rectangular route. The number of buses is given by N_{bus} . The arrival time of the buses in the upper left corner of the route has been recorded, and the difference in the arrival time is defined as the waiting time. The base time interval without any randomness between the buses in the upper left corner is set to 10 min. (a) Buses run smoothly without any delays. (b) Owing to the random stops, the buses arrived one by one in series; this state is called the “dumping bus state”

The purpose of this study is to construct a transportation model of buses. We modeled the buses such that they periodically went around a closed route, stopping many times depending on different

randomness values that described their stops, as shown in Fig.1. Sometimes, when a delayed bus arrived, the next scheduled bus immediately followed the delayed bus. In general, this phenomenon is called the bunching bus state. However, in the worst case, more than two buses came continuously one after another for a long time. From the viewpoint of the people waiting for the buses, the bunching buses look like a train or dumplings. Thus, we call this state the “dumpling bus state,” as shown in Fig.1(b). The relation between the running time and the dumpling state of the buses is analyzed.

As the waiting time becomes longer, the people waiting become irritated [20]. The purpose of riding buses is to reach one’s destination. When buses come later than expected and when there are other available modes of transportation, such as taxis, waiting people must judge whether they should continue to wait or not. The randomness of bus arrival times frequently makes people insecure and increases their irritation. Thus, how long people continue to wait at a bus stop can be treated as a probabilistic event and is mainly determined by the distributions of the interval between the bus arrivals at the bus stop. Thus, it is possible to connect the distribution of the interval of the bus arrival time to the waiting time of irritated people. Here, we analyzed the calculated bus arrival time by using the logistic regression, and we regarded the probability function derived from the logistic regression as the waiting time of the irritated people. We inferred that the frustration of people who had been waiting a long time became the strongest when they saw buses in the dumpling state.

We believe that the analysis of bus delays based on the dumpling bus states and the influence they have on people’s frustration is an important topic to improve smart bus transport systems. Learning about this in detail can support future research on improving transportation facilities so that dumpling bus states can be avoided, consequently reducing the frustration of people who experience bus delays.

The remainder of this paper is organized as follows. Section 2 describes our proposed transportation bus model. Section 3 shows the numerical results based on the proposed model. Section 4 explains the analysis of frustration using logistic regression. Section 5 concludes our work.

2. BUS TRANSPORTATION MODEL

Here, we would like to describe our bus transportation model used in detail. The basic idea is to treat bus operations as random events. Usually, the locations of traffic lights and bus stops are fixed. However, even if a bus encounters the same traffic lights, the location where it needs to stop changes depending on the number of cars that stop between the traffic lights and buses. Additionally, even if a bus arrives at a regular bus stop, the time it spends at the stop depends on the number of people waiting and getting off. In daily life, people often experience that the city buses they ride randomly stop in the middle of cities. Based on this fact, to simplify our calculations, we randomly set the locations where the buses stopped. It is considered that buses ran around the rectangular route shown in Fig. 1, and we focused on the arrival times of the buses at the left upward corner. As mentioned above, we use the term “dumpling state” to refer to cases when buses run one after the other, as shown in Fig.1(b). In the simulation program, we set a time of 1 min to move one frame of the bus distributions. For each frame (1 min), random numbers R_0 were independently generated for every bus. If R_0 was less than the given value R_{ran} (i.e., $R_0 < R_{\text{ran}}$), the corresponding bus stopped for one frame; otherwise (i.e., if $R_0 > R_{\text{ran}}$), the bus moved for one frame. We set the basic setup such that the buses arrived every 10 min in the top left corner without any delay.

It is assumed that the buses were scheduled to run at a constant speed along the rectangle shown in Fig. 1(a). The number of buses is N_{bus} , and the random number of the stopping probability of the buses is R_{ran} . We set the number of buses N_{bus} at 5, 10, and 15 and 20, 22, 24, and 30 for $R_{\text{ran}} = 0.2$ and 0.4, respectively.

The algorithm is described as follows:

1. Input the number of buses N_{bus} and the random number R_{ran} .
2. Generate a random number R_0 (≤ 1) every minute. For a given value of R_{ran} , if $R_0 < R_{\text{ran}}$, the bus stops for 1 min.
3. Count the time interval at the upper left corner of the route.

This model not only visually shows the dumpling state, but it also records the elapsed time of the bus at the top left corner. As mentioned above, the simulation proceeded with each step corresponding to one frame (1 min) of the movement of the buses. Here, it is assumed that the time between the bus's arrival at the upper left corner was 10 min if there were no bus stops. Then, $R_{ran} = 0.2$ and 0.4 indicates that the buses had probabilities of stopping for 2 and 4 min, respectively, every 10 min. The initial condition is that the buses start to depart every 10 min from the top left corner of the route. The bus delay problem was simulated using these data.

In addition to these parameter settings, we could determine the distance the bus needed to cover to go around the rectangle using the average bus velocity v_{bus} [km/h]. When there are N_{bus} buses, the time for one bus to go around the route is $10 N_{bus}$ min because the time interval between the buses is set at 10 min. For the average bus velocity v_{bus} without any random stops, the average bus speed was reduced to $v_{bus}/(1+R_{ran})$ for the finite R_{ran} . Then, the bus mileage around the route is given by:

$$\text{Bus mileage} = N_{bus} v_{bus}/(1+R_{ran})/6 \text{ [km]}. \tag{1}$$

Generally, if $v_{bus} = 18$ km/h when $N_{bus} = 5$ and $R_{ran} = 0.2$ and 0.4 , the average bus speeds are 15 and 12.9 km/h, respectively, and the bus mileages are 12.5 and 10.7 km, respectively. For $v_{bus} = 24$ km/h, the average bus speeds are 20 and 17.1 km/h for $R_{ran} = 0.2$ and 0.4 , respectively, and the corresponding mileages are 16.7 and 14.3 km, respectively.

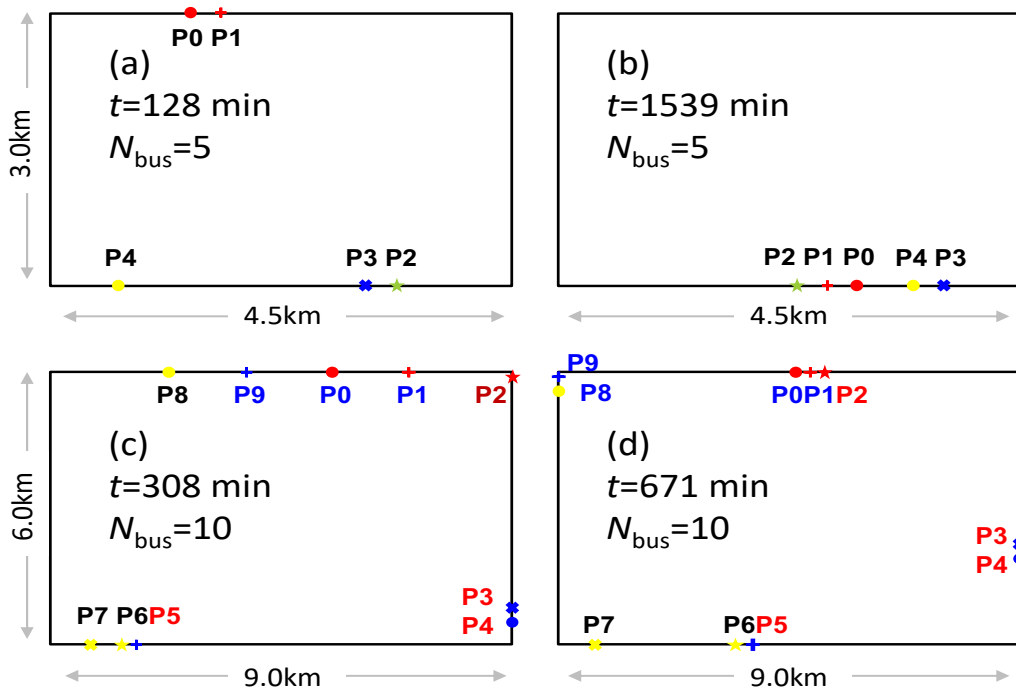


Fig. 2. Examples of buses in dumpling states. “P0-P9” indicates the bus position in the routes for $R_{ran} = 0.2$. (a) and (b) $N_{bus} = 5$ and (c) and (d) $N_{bus} = 10$. The distances were calculated assuming an average speed of 18 km/h without any stops

3. NUMERICAL RESULTS

Fig. 2 shows examples of running bus states for which $N_{bus} = 5$ and 10. We frequently observed clusters of the dumpling bus state. As time passed, the size of the dumpling bus state increased. We rarely observed the perfect dumpling bus state (i.e., a case in which all the buses were connected). In most cases, we observed buses in partial dumpling states. The typical pattern of the actual bunching

buses consists of two buses. For example, in the rain, two buses frequently arrived at once after a long waiting time, and passengers felt very tired. Fig. 2 reproduces this pattern in which the most frequent pattern of the dumping state consists of two buses; even the dumping state of two buses increased the waiting time and irritated the people who were waiting.

Fig. 3 shows the comprehensive numerical results of the waiting time of the buses at the upper left corner of the operating route shown in Fig. 1, where five different simulations are illustrated based on different random numbers. The dots indicate the people at the top left corner of the bus route. The elapsed time was counted starting from the arrival of the first bus at the upper left corner. The buses were randomly distributed. We now examine each case in more detail. In Fig. 3(a), the dark circles start appearing below 10 min, while the yellow circles start appearing close to 30 min. The waiting close to 0 min shows that the next bus comes soon after the bus before it. This result also means that buses are inclined to enter the dumping state. As time goes by, the distribution of the waiting time becomes larger, and in all cases, as time elapses, some people must wait for one hour or more. This tendency was observed more for cases where $R_{\text{ran}} = 0.4$ than when $R_{\text{ran}} = 0.2$. As N_{bus} increases, the distribution becomes slightly larger. This means that people need to wait slightly longer when N_{bus} is large. The green lines show the average waiting times, and these times do not change much. Note that not many bus lines run for 24 hours. In addition, the interval of buses becomes much longer than 10 min in the morning and at night. We carried out the simulations in order to observe the characteristics of the model.

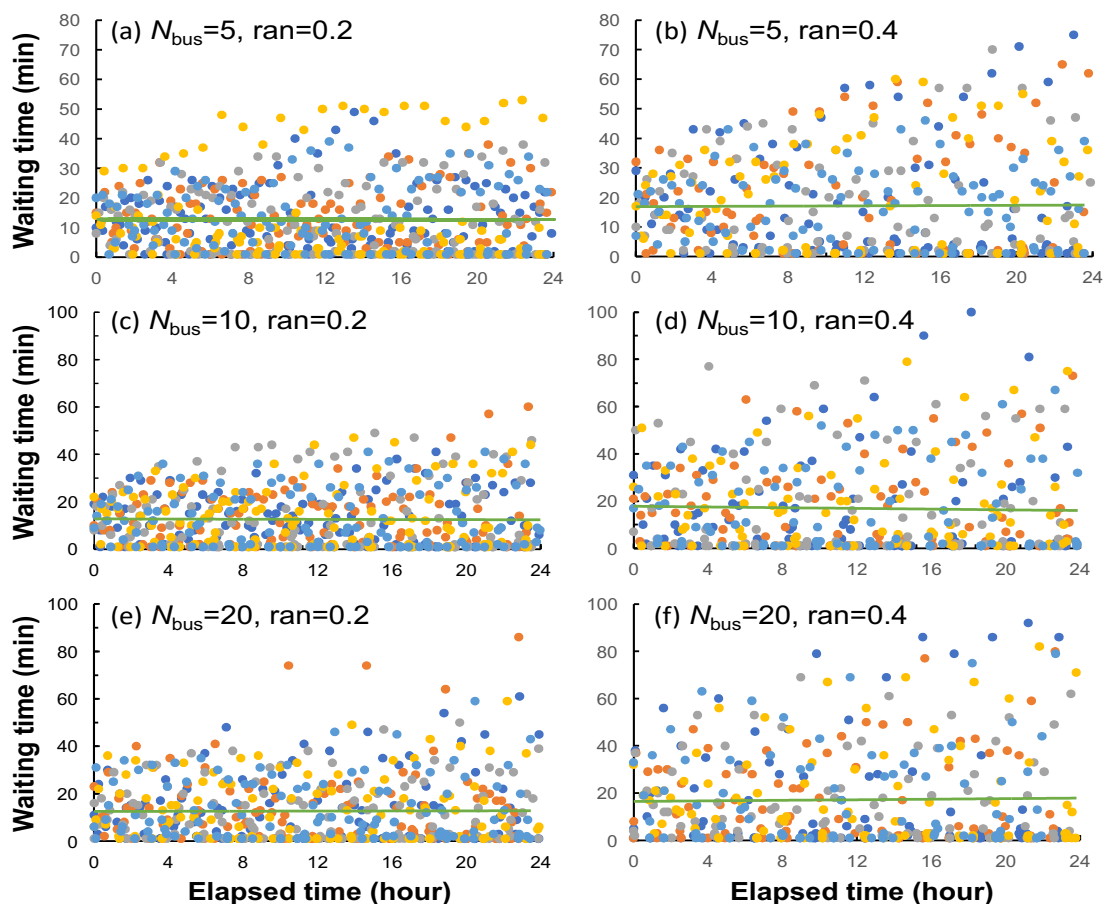


Fig. 3. Simulation results of the bus model. The dots indicate the people at the top left corner of the bus route. The different colors correspond to different random number selections. The buses run around the route. The waiting time is the time interval between the arrival of buses at the upper left corner. (a) $N_{\text{bus}} = 5$ and $R_{\text{ran}} = 0.2$, (b) $N_{\text{bus}} = 5$ and $R_{\text{ran}} = 0.4$, (c) $N_{\text{bus}} = 10$ and $R_{\text{ran}} = 0.2$, (d) $N_{\text{bus}} = 10$ and $R_{\text{ran}} = 0.4$, (e) $N_{\text{bus}} = 20$ and $R_{\text{ran}} = 0.2$, and (f) $N_{\text{bus}} = 20$ and $R_{\text{ran}} = 0.4$

Table 1 shows the statistical results of the simulations presented in Fig. 3. The average waiting time was added after introducing the effects of the randomness parameter R_{ran} . This table shows that the average waiting times were almost the same for all cases. For simplicity, we defined the number of people as 1 for each dot in Fig. 3. Thus, the total number of people corresponds to the total number of dots in Fig. 3. In the rightmost two columns of Table 1, the waiting time is divided into times above (long) and below (short) the average time. According to these waiting times, people in traffic congestion can be split into two groups, namely, “winners” and “losers.” The waiting time for winners is about 4 min, and that for losers is more than 20 min. It was found that more than half of the people in this study were losers. This indicates that the randomness of the bus system placed more than half of the waiting people in an unhappy state. Moreover, as the N_{bus} increased, the average waiting time of the winners became shorter and the average waiting time of the losers became longer. This gap between the winners and losers became larger as the randomness R_{ran} increased. These findings reflect the wider distributions of the bus arriving times for larger values of N_{bus} and R_{ran} in Fig. 3. Because it is assumed that a person waits for each bus, here, the sum of the winners and the losers equals the number of arriving buses.

In Fig. 4, the total time between the arrival of the first bus and the last (N_{bus} -th) bus at the upper left corner indicates the dumpling bus state as a function of the elapsed time. As more time goes by, the buses have a greater tendency to enter the dumpling state. The buses in the $N_{\text{bus}} = 5$ case have a higher probability of entering the dumpling state than those in the $N_{\text{bus}} = 10$ case do. Additionally, as the randomness becomes larger in the case of $R_{\text{ran}} = 0.4$, the distribution of the total bus length becomes larger. Particularly, in the $N_{\text{bus}} = 5$ case, when the buses run for longer than half a day, perfect dumpling states (surrounded in red dashed lines in Fig. 4) can be observed. These results show that the dumpling state strongly affects the nonuniformity of the bus arriving when the N_{bus} is small. As can be seen from the number of dots in Fig. 4, most dumpling states consisted of a small number of buses, and the probability of the perfect dumpling state was low. This provides some relief for both bus companies and users.

Table 1

Statistical summary of Fig. 3. The number of people equals the number of arriving buses. The total number of people is the sum of the “long waiting people” and “short waiting people.” For example, for $R_{\text{ran}} = 0.2$ and $N_{\text{bus}} = 5$, the total number of people is $486 + 659 = 1145$, which equals the total number of arriving buses

R_{ran}	N_{bus}	Average waiting time [min]	Std dev [min]	Average of long waiting time [min] (No. of people)	Average of short waiting time [min] (No. of people)
0.2	5	12.7	11.7	23.9 (486)	4.3 (659)
	10	12.7	12.3	24.9 (473)	4.1 (678)
	15	12.6	13	24.6 (484)	3.9 (663)
	20	12.6	13	25.7 (459)	4.0 (694)
0.4	5	17	16.2	32.2 (377)	5.1 (482)
	10	17	19.5	38.0 (289)	4.2 (475)
	15	17.2	20.3	38.1 (328)	4.4 (535)
	20	16.9	19.5	37.7 (327)	4.2 (536)

In reality, more than one person waits for each bus arrival. In addition, less than 100 buses run in a day, even in large city centers. Thus, the total number of counts in this study (about 1000) is unrealistically large. However, these simulations still allow us to describe the characteristics of the randomness of bus transportation.

4. ANALYSIS OF FRUSTRATION

Next, we investigate the effect of the randomness of bus patterns on the mental state of people by carrying out the simulation of frustration. It is assumed that frustration is closely related to long waiting times, and we consider that longer waiting times lead to more frustration. There are many kinds of origins of frustration in daily life, including conflicts, pressure, reactions, ignorance, an inability to achieve goals and objectives, illness, and a lack of resources. In the case of bus congestion, people know that buses are often delayed; nevertheless, as the waiting time becomes longer, the probability of them reaching their destination on time decreases, and they become irritated, and they may even feel unsafe in some cases. They must decide whether to continue to wait for the next bus or choose another method of transportation, such as a taxi. Although the actual limit of the waiting time depends on each person's characteristics (e.g., age, job, and gender), here, we use the simplest case and correlate the data of the interval between buses to frustration.

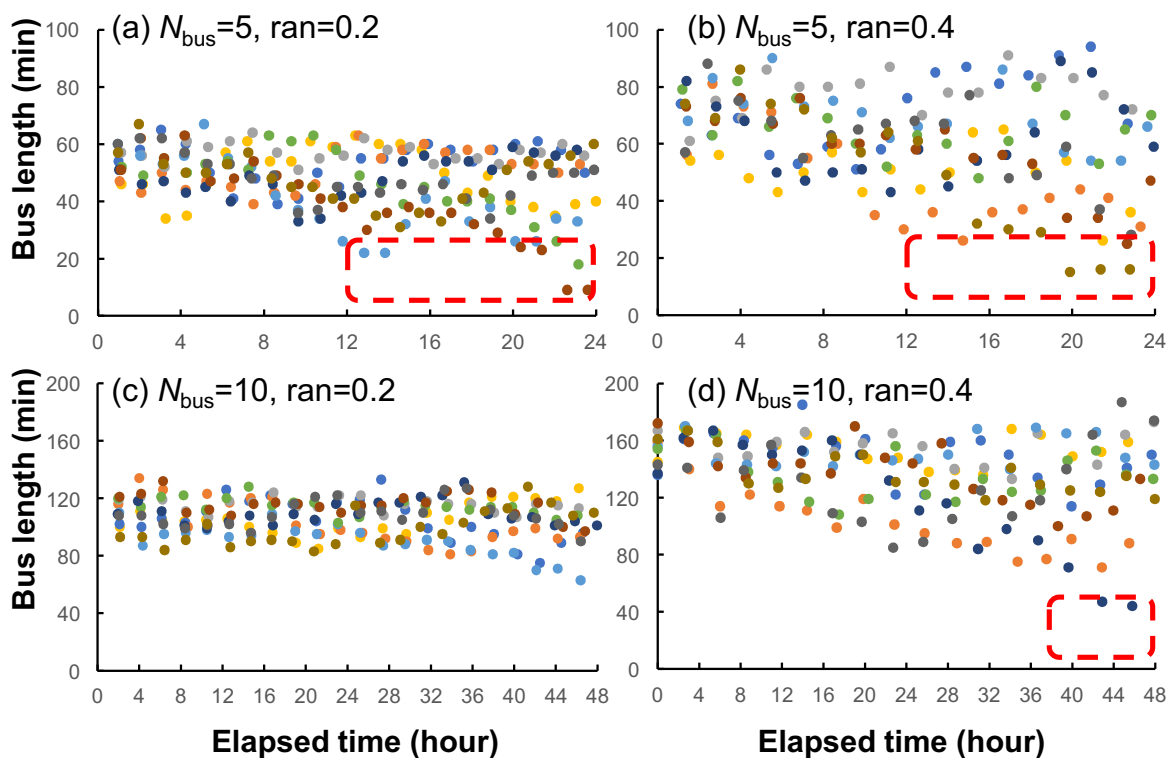


Fig. 4. Time between the arrival of the first bus and N_{bus} -th bus at the upper left corner as a function of the time elapsed over 10 simulations. For the case of N_{bus} , the average time is given by $10 N_{\text{bus}}$ because the average initial bus interval is set to 10 min. The different colors correspond to different random number selections. As time goes by, the perfect dumping state (surrounded in red dashed lines) appears; in these cases, all the buses come serially in 10 min. This is the typical dumping bus state. (a) and (b) $N_{\text{bus}} = 5$, and (c) and (d) $N_{\text{bus}} = 10$. (a) and (c) $R_{\text{ran}} = 0.2$, and (b) and (d) $R_{\text{ran}} = 0.4$

Generally, it is appropriate to determine the irritation time statistically. Some consider a waiting time of 10 min too long, while others do not. For the same person at the same bus stop, the irritation time changes depending on the daily life of people. Thus, the irritation time is subjective and should be treated statistically. In every case, it can be said that as the waiting time increases, so does one's level of irritation. Thus, we can assume that the irritation probability is inferred from $P(t) = \text{sigmoid}(Wt + b)$, (2) where t is the waiting time, which is the interval between bus arrivals, and W and b are the weights in the probability. We calculated this probability in the framework of the logistic regression problem; the TensorFlow library was used. We set several critical irritation times and examined the statistical

expectation by using a machine learning method from the TensorFlow library. Usually, TensorFlow is used to solve classification problems, such as distinguishing between a dog or cat in a given photograph. However, here, TensorFlow was used to obtain the probability density function as a function of the waiting time in the framework of the logistic regression. First, we set the time limit of the irritation, t_{limit} , the time above which people quit waiting for the bus. For the given data set simulated above, we chose the training data from the first 75-80% of the data set and labeled the waiting time t_{wait} such that label = 0 for $t_{\text{wait}} < t_{\text{limit}}$ and label = 1 for $t_{\text{wait}} \geq t_{\text{limit}}$. Next, TensorFlow was used to obtain the probability density function. More specifically, we used a single layer and tried some activation functions in the logistic regression mode. A success rate of more than 80% was obtained around epoch = 3000. The activation function tanh was better than ReLU and sigmoid, and the final results were transferred from the tanh form to the sigmoid form. We also calculated the case of the three labels such that the waiting time would be distinguished into (i) $t_{\text{wait}} < t_{\text{limit}}$, (ii) $t_{\text{limit}} - 5 \leq t_{\text{wait}} < t_{\text{limit}}$, and (iii) $t_{\text{limit}} \leq t_{\text{wait}}$. In this case, we tried two layers in the TensorFlow algorithm. However, the success rate was greatly reduced, and therefore, the single-layer cases are shown in Fig. 5.

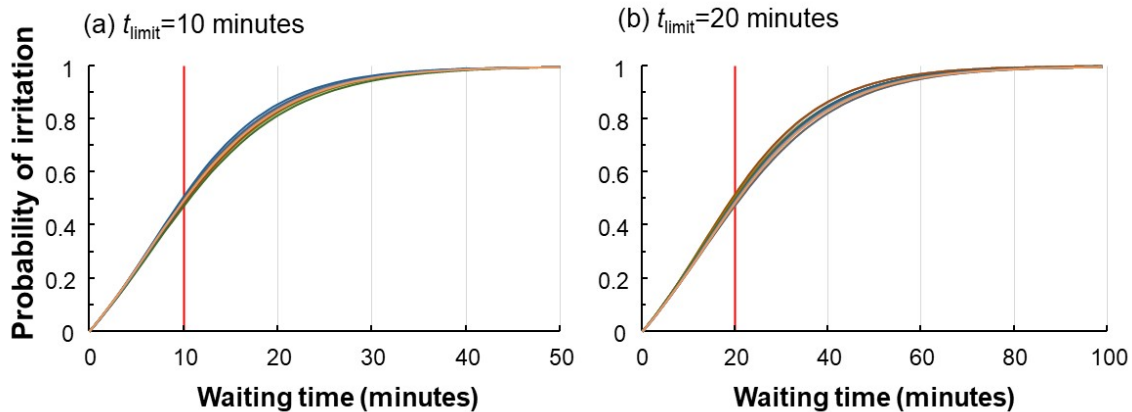


Fig. 5. Results of logistic regression using TensorFlow. The irritation limit is given by (a) $t_{\text{limit}} = 10$ min and (b) $t_{\text{limit}} = 20$ min (red lines). The results include all the cases calculated above from $N_{\text{bus}} = 5, 10, 15, 22, 24,$ and 30 and both $R_{\text{ran}} = 0.2$ and $R_{\text{ran}} = 0.4$

Figs. 5(a) and (b) show the results for $t_{\text{limit}} = 10$ min and $t_{\text{limit}} = 20$ min, respectively. There were initially offset values of the probabilities for the waiting time $t_{\text{wait}} = 0$. This is because the boundary of t_{limit} is too close to the point of $t_{\text{wait}} = 0$ and most data exist in the above t_{limit} region. Therefore, we transformed the calculated results such that the minimum value of the probability at $t_{\text{wait}} = 0$ was set to 0. The results can be interpreted as follows. In our analysis, the boundary t_{limit} is regarded as the time limit above which people quit waiting for the bus. From Fig. 5(a) and (b), it can be seen that the probability at t_{limit} is around 50%, which can be interpreted as the percentage of people who stopped waiting after $t = t_{\text{limit}}$. According to Fig. 5(a), it is probable that more than 10% of people continue to wait around 20 min. However, after around 30 min, most people stop waiting. On the other hand, the results show that 20% of people stopped waiting after just 5 min. In the case of $t_{\text{limit}} = 20$ min, Fig. 5(b) shows that it is probable that 10% of people continue to wait for around 40 min and 10% of people stop waiting after only 5 min. At around 60 min, most people stop waiting.

The case for $t_{\text{limit}} = 10$ min in Fig. 5(a) is considered to correspond to bus transport in large cities where people only choose buses when they follow the shortest path to their destination, when the buses are on schedule, and when people have other ways to commute. The result of $t_{\text{limit}} = 20$ min in Fig. 5(b) is considered to correspond to the case in which buses are the main transportation method and other transport choices are limited to only taxis, such as in rural districts. Although Fig. 3 shows that as the elapsed time increases, the waiting time varies as N_{bus} increases, no such clear relationship is shown in Fig. 5. Therefore, a more detailed model may be needed to analyze this logistic regression in the future.

5. CONCLUSIONS

As the simplest approach to describing bus transportation in complicated traffic situations, we introduced a bus model in which the buses stop randomly, mainly depending on random numbers. Specifically, classifying dumpling states using cluster analysis was required. Here, we simulated many buses (800-1000) during long hours (24 hours) in order to see the general characteristics of our model. It was found that the buses soon entered dumpling states. In addition, it was found that the most frequent pattern was the dumpling of a small number of buses (two or three), which is considered to reproduce the actual transportation patterns of buses. These dumpling states generated a nonuniform waiting time, allowing people to be categorized as “winners” and “losers.” It was also found that dumpling states can be observed for all numbers of buses with randomness. Future studies are required to provide more detailed analyses of dumpling states. Future studies should also use more realistic numbers of buses than the present work used.

Additionally, we carried out simulations of frustration among bus users and correlated the waiting time for delayed buses with the frustration of people by using simple logistic regression. By setting the target time limit of waiting, we could obtain the probability of the percentage of waiting people. However, the probabilities only weakly depend on the number of buses and randomness. This might be because of the present simple analysis. More advanced analyses of these variables using machine learning will be required in the future. Moreover, we will also need to combine our simulation results with the transportation patterns of actual buses by monitoring the operations of the actual buses and the waiting people. It will also be beneficial for the waiting people to know the probability of irritation calculated in this work. These are additional issues to be addressed in future research.

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