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Nonhomogenous Poisson process application to modelling accidents number at Baltic Sea waters and ports

Keywords

Poisson process, safety characteristics.

Abstract

The stochastic processes theory provides concepts and theorems that allow to build probabilistic models concerning incidents or (and) accidents. Counting processes are applied for modelling accidents number in Baltic Sea region in the given time interval. A crucial role in construction of the models plays a Poisson process and its extensions especially a nonhomogeneous Poisson process. The models of accidents number in the sea and seaports are here presented. Moreover some procedures of the model parameters identification are presented in the paper. Estimation of model parameters was made based on data from reports of HELCOM (2014) and Interreg project Baltic LINes (2016-2019).

1. Introduction

Parameters of the constructed models were estimated on the basis of the available data coming from reports [9] , [10] and [11] . So, the main issues of the work will be preceded by the background, based on these reports. To monitor shipping safety and maintain the safety at sea HELCOM annually compiles a report on shipping accidents in the Baltic Sea. "The Interreg project Baltic LINes (2016-2019) seeks to increase transnational coherence of shipping routes and energy corridors in Maritime Spatial Plans to prevent cross-border mismatches and secure transnational connectivity as well as efficient and sustainable use of Baltic Sea space". The report [9] was developed under Work Package (WP) 2.1. Screen and analyse available information. This report contains the data and maps related to shipping in the Baltic Sea. It represents a first synthesis of available literature on the topic of shipping in the Baltic Sea and related past, present and future developments relevant for MSP. We should add that data mainly concern events in the Baltic Sea and relatively little information on accidents in the ports.

2. Statistical analysis

Annual report on shipping accidents in the Baltic Sea in 2013 HELCOM (2014) provides a lot of important information relevant to our problem. From the date coming from report we can asses some parameters in our models.

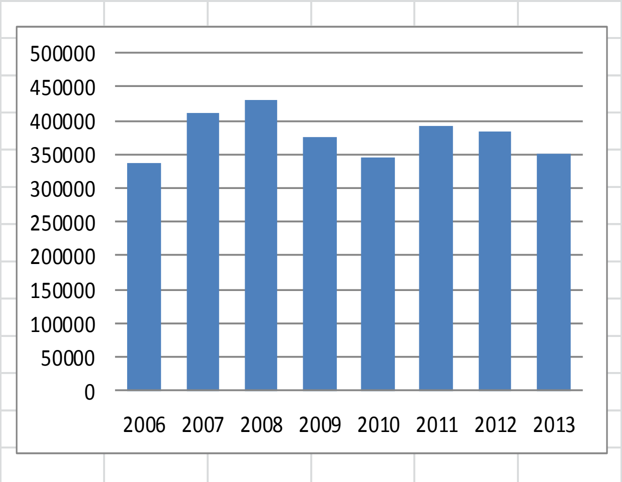

Figure 1. Total number of ships crossing in the Baltic Sea during 2006-2013

Table 1 and *Figure 1* show a total number of ships crossing in the Baltic Sea during 2006-2013. The minimum number of ships crossing amounted

376671 in 2006 and maximum was 430064 in 2008. From 2009 to 2013 the number of ships crossing was in interval 342754 – 391699.

Table 1. Total number of ships crossing all fixed HELCOM AIS lines in the Baltic Sea during 2006-2013

Year	Passenger	Cargo	Tanker	Other	No info	Total
2006	42731	226855	67458	39627	0	376671
%	11	60	18	11	0	100
2007	43998	237740	69281	53225	8204	412448
%	11	58	17	13	2	100
2008	43060	206755	60746	104814	14689	430064
%	10	48	14	24	3	100
2009	37994	198427	68008	61014	9234	374677
%	10	53	18	16	2	100
2010	30471	181932	59409	46950	23028	342754
%	9	53	17	14	7	100
2011	35398	207273	64957	60123	23948	391699
%	9	53	17	15	6	100
2012	33193	207056	66524	54627	22959	384359
%	9	54	17	14	6	100
2013	31329	182770	61193	57959	17141	350392
%	9	52	17	17	5	100

Table 1 shows in detail the number and types of ships crossing in the years 2006 -2013 in the Baltic Sea.

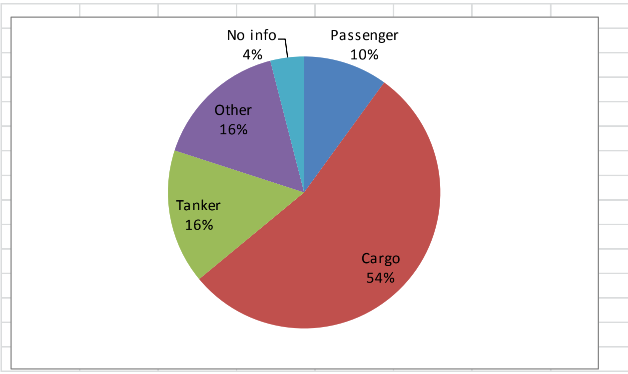

Figure 2 . Different types of cruises in ships crossing in the Baltic Sea during 2006-2013

Based on data coming from[10] we have drawn up *Figure 3* that shows a total number of shipping accidents in the Baltic Sea during 2006-2013.

According to the reports 149 ship accidents occurred in the Baltic Sea area in 2013, which is the highest recorded number in the last ten years (*Figure 3*) . The number of accidents in the Baltic Sea has shown a slight increase in the last three years. Compared to 2010 the total number of accidents increased by 17% in 2013.

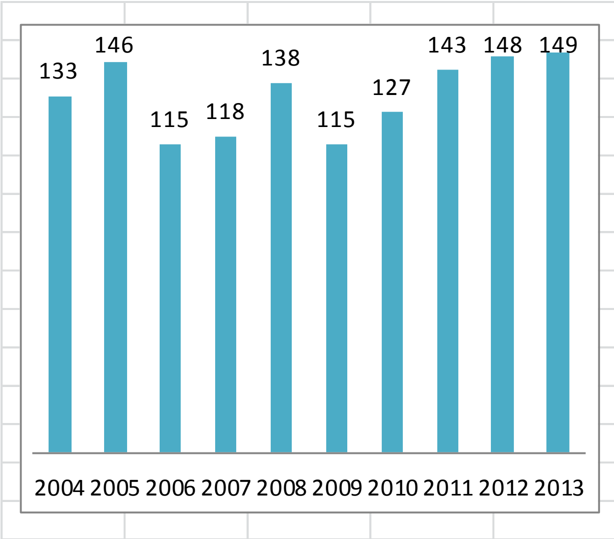

Figure 3. Total number of reporting ship accidents in the Baltic Sea during 2004-2013

Using date form *Table 1* and *Figure 1* we can compute the indicators of shipping accidents intensity in relation to the ships crossing number. We define indicators $\gamma = \frac{NSC}{NA}$ and $\alpha = \frac{NA}{NSC}$, where

$NSC -$ number of ship crossing ,
$NA -$ number of shipping accidents.

Table 2. Indicators of shipping accidents intensity in relation to the ships crossing number.

Year	$\gamma = \frac{NSC}{NA}$	$\alpha = \frac{NA}{NSC}$
2006	3275,40	0,000305
2007	3495,32	0,000286
2008	3130,84	0,000320
2009	3258,06	0,000306
2010	2698,85	0,000370
2011	2739,15	0,000365
2012	2596,97	0,000385
2013	2351,62	0,000425

From the *Figure 4* it follows that the indicator α of shipping accidents intensity with respect to the ships crossing number in the Baltic Sea during 2006-2013 has a growing trend over time.

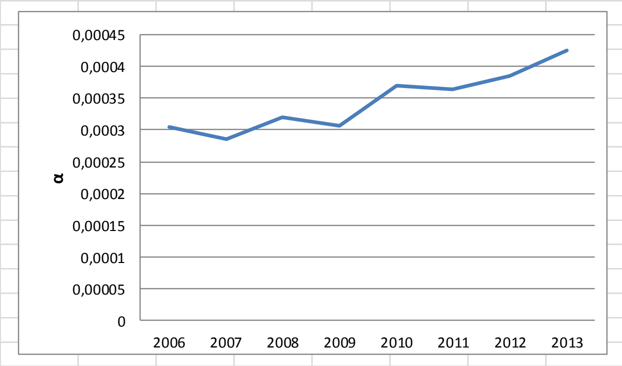

Figure 4. Indicator α of shipping accidents intensity with respect to the ships crossing number in the Baltic Sea during 2006-2013

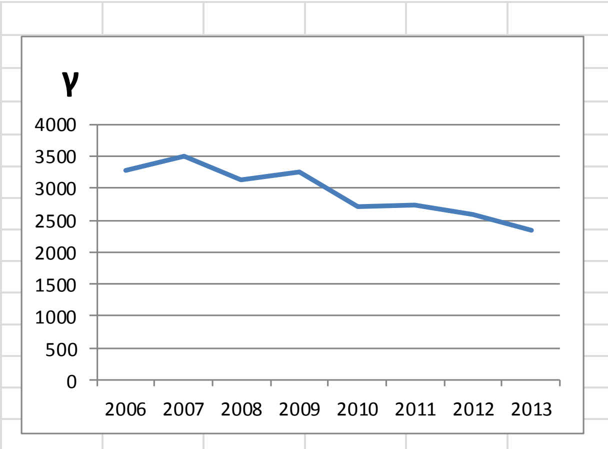

Figure 5. Indicator γ of ships crossing number with respect to the shipping accidents in the Baltic Sea during 2006-2013

The indicator γ represents the average number of ships crossing on one accident. From *Figure 5* it follows that the indicator has a decreasing trend over time. This number can be interpreted as the average number of ships crossing between accidents.

The number of shipping accidents increased over the past years, with cargo ships most frequently involved followed by passenger ships and tankers [9],[10],[11]. Human error is the main cause for accidents and is mainly related to unintentional action. However, 17% of the accidents occurred after intentional decisions against common rules and plans [9]. The number of collisions with other vessels and contacts to fixed or floating objects has south-western Baltic Sea is the main hotspot for these types of accidents. The offshore wind energy sector will have high spatial demands in the future especially when ample safety distances are assigned to all components and additional space is when ample safety distances are assigned to all components and additional space is reserved for the related service traffic. The expected increase in leisure traffic will also demand more space which should be possible dedicated to an expansion of safety distances to keep the commercial shipping possible dedicated to an expansion of safety distances to keep the commercial shipping [9].

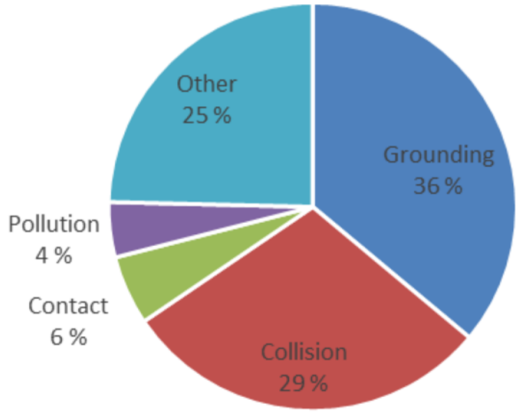

Figure 6. [10] Types of accidents in the Baltic Sea in 2004-2013

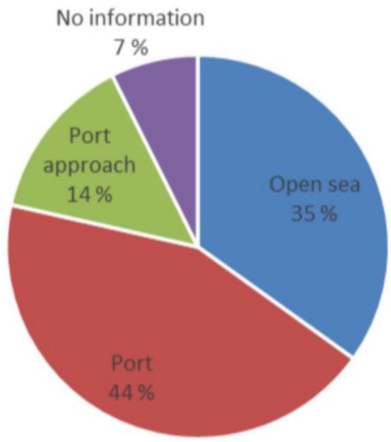

Figure 7. [11] Location of accidents in the Baltic Sea in 2012

3. Homogenuous Poisson process

The dangerous events and accidents number in Baltic Sea region during given time are the randomly changing quantities. The theory of stochastic (random) processes allows the modelling of the random evolution of systems through the time. A crucial role in construction of the models plays a Poisson process and its extensions. They allow to construct the models of the dengerous incidents and accidents number in the sea and seaports of the Baltic region.

A random process $\{X(t)\colon t \geq 0\}$ is said to be *process with independent increments* if for all $t_1, \dots, t_n$ such that $0 < t_1 < t_2 < \cdots < t_n$ the random variables

$X(0),\ X(t_1) - X(0), \dots, X(t_n) - X(t_{n-1})$ are mutually independent. If the increments $X(s) - X(t)$ and $X(s+h) - X(t+h)$ for all $t, s,\ h > 0$, $s > t$ have the identical probability distributions then $\{X(t)\colon t \geq 0\}$ is called a process with the *stationary independent increments* (SII). It is proved,that for the SII processes such that $X(0) = 0$ an expectation and a variance are

$$E[X(t)] = m_1\, t, \qquad V[X(t)] = \sigma_1^2\, t, \tag{1}$$

where

$$m_1 = E[X(1)] \text{ and } \sigma_1^2 = V[X(1)]. \tag{2}$$

An example of a SII random process is a Poisson process.

A stochastic process $\{X(t);\ t \geq 0\}$ taking values on $S = \{0,1,2,\dots\}$, with the right continuous trajectories is said to be a Poisson process with parameter $\lambda > 0$ if:

1. $X(0) = 0$,
2. $\{X(t)\colon t\ \geq 0\}$ *is the process with the stationary independent increments,*
3. *For all* $t > 0,\ h \geq 0$,

$$P(X(t+h) - X(t) = k) = \frac{(\lambda\, h)^k}{k!}\, e^{-\lambda\, t}, \tag{3}$$

$$k \in S$$

For $t = 0$ we get a first order distribution of the Poisson process:

$$p_k(h) = P(X(h) = k) = \frac{(\lambda\, h)}{k\,!}\, e^{-\lambda\, h}, \quad k \in S \tag{4}$$

For $h = 1$ we obtain the Poisson distribution with parameter λ. Hence $E[X(1)] = \lambda$ and $V[X(1)] = \lambda$. Therefore, from (1) and (2), we obtain the expectation and the variance of the Poisson process:

$$E[X(t)] = \lambda\, t, \qquad V[X(t)] = \lambda\, t, \quad t \geq 0.$$

For a fixed t this formula determines the Poisson distribution with parameter $\Lambda = \lambda\, t$:

$$p(k) = P(X = k) = \frac{(\Lambda)^k}{k\,!}\, e^{-\Lambda}, \quad k \in S.$$

Let $0 < \tau_1, < \tau_2, \dots$ represent the consecutive instants of the state changes (jumps) in the Poisson process or another process with the right continuous, nondegreasing and picewise constant trajectories. The random variables $\vartheta_1 = \tau_1,\ \vartheta_2 = \tau_2 - \tau_1, \dots$ denote the sojourn times of the states $0,\ 1, \dots$. Let us notice that

$$\tau_0 = \vartheta_0 = 0, \tau_n = \vartheta_1 + \vartheta_2 + \cdots + \vartheta_n,\ n \in \mathbb{N},$$
$$\tau_\infty = \lim_{n\to\infty} \tau_n = \sup\{\tau_n\colon n \in \mathbb{N}_0\}.$$

A stochastic process $\{N(t)\colon t \geq 0\}$ defined by the formula

$$N(t) = \sup\{n \in \mathbb{N}_0\colon \tau_n \leq t\}$$

is called a *counting process* corresponding to a random sequence $\{\tau_n : \in \mathbb{N}_0\}$.

For the Poisson process with parameter λ the random variables $\vartheta_1, \vartheta_2, \dots, \vartheta_n,\ \ n = 2,3, \dots$ are mutually independent and *exponentially distributed with the identical parameter* λ.

The Poisson process is a *counting process* which is genereted by the random sequence $\{\tau_n : \in \mathbb{N}_0\}$, where $\tau_n = \vartheta_1 + \vartheta_2 + \cdots + \vartheta_n,\ n \in \mathbb{N}$.

4. Nonhomogeneous Poisson process

Let $\{N(t);\ t \geq 0\}$ be a stochastic process taking values on $S = \{0,1,2,\dots\}$, value of which represents the number of events in a time interval $[0, t]$.

A *counting process* $\{N(t)\colon t \geq 0\}$ is said to be *nonhomogeneous Poisson process* (NPP) with an intensity function $\lambda(t),\ t \geq 0$ such that $\lambda(t) \geq 0$ for $t \geq 0$, if

1. $P(N(0) = 0) = 1$;
2. *The process* $\{N(t)\colon t \geq 0\}$ *is the stochastic process with independent increments* and the right continuous trajectories;
3. $P(N(t+h) - N(t) = k) =$ (5)

$$= \frac{\left(\int_t^{t+h} \lambda(x)dx\right)^k}{k\,!}\, e^{-\int_t^{t+h} \lambda(x)dx};$$

From this definition it follows that the one dimensional distribution of NPP is given by the rule

$$P(N(t) = k) = \frac{\left(\int_0^t \lambda(x)dx\right)^k}{k\,!}\, e^{-\int_0^t \lambda(x)dx}, \tag{6}$$

$$k = 0,1,2,\dots$$

The expectation and variance of NPP are the functions

$$\Lambda(t) = E[N(t)] = \int_0^t \lambda(x)dx \ , \quad t \geq 0, \qquad (7)$$

$$\text{V(t)} = V[N(t)] = \int_0^t \lambda(x)dx \, , \quad t \geq 0.$$

The corresponding standard deviation is

$$\text{D(t)} = \sqrt{V[N(t)]} = \sqrt{\int_0^t \lambda(x)dx} \, , \quad t \geq 0.$$

The expected value of the increment

$N(t+h) - N(t)$

$$\Delta(t; h) = E\big(N(t+h) - N(t)\big)$$
$$= \int_t^{t+h} \lambda(x)dx$$
(8)

The corresponding standard deviation is

$$\sigma(t; h) = \sigma\big(N(t+h) - N(t)\big)$$
$$= \sqrt{\int_t^{t+h} \lambda(x)dx} \qquad (9)$$

An nonhomogeneous Poisson process with $\lambda(t) = \lambda, \ t \geq 0$ for each t ≥ 0, is a regular Poisson process.

The increments of an nonhomogeneous Poisson process are independent, but not necessarily stationary.

A nonhomogeneous Poisson process is a Markov process.

4.1.Illustrative example

The number of accidents during one year can be describe by a nonhomogeneous Poisson process with an intensity function given by

$$\lambda(t) = \begin{cases} 0{,}42 & \text{for } t \in [0, 121) \\ 0{,}35 & \text{for } t \in [121, 244) \\ 0{,}40 & \text{for } t \in [244, 365) \end{cases}$$

Using (7) we compute the expected value of the process. We obtain

$$\Lambda(t) = \begin{cases} 0{,}38\, t & \text{for } t \in [0, 121) \\ 0{,}35t + 45{,}98 & \text{for } t \in [121, 244) \\ 0{,}40t + 85{,}4 & \text{for } t \in [244, 365) \end{cases}$$

The number of dangerous events between May 1 and September 30 is a random variable having Poisson distribution with parameter $\Delta(t; h)$ given by (8),
where $t = 121$ [day] and $t + h = 275$ [day].

Hence, from (8) and (9) we obtain the parameter of the Poisson distribution

$$\Lambda = E(N(275) - \text{E}(N(121)) = 195{,}4 - 85{,}91 = 109{,}49.$$

The corresponding to it standard deviation is

$$\sigma = \sqrt{109{,}49} = 10{,}46.$$

Probability that the number of dangerous events between May 1 and September 30 is not greater than 100 is

$$P_{\leq 100} = 1 - \sum_{k=0}^{100} \frac{(\Lambda)^k}{k\,!} \, e^{-\Lambda}, \qquad \text{where} \qquad \Lambda = 109{,}49$$

Applying approximation by normal distribution we get

$$P_{\leq 100} = \Phi\left(\frac{100 - 109{,}49}{10{,}46}\right) = \Phi(-0{,}907) = 0{,}183$$

5. Models of accidents number in the Baltic Sea and Seaports

Let $\{N(t);\ t \geq 0\}$ be a stochastic process taking values on $S = \{0,1,2, \dots\}$, value of which represents the number of accidents in the Baltic Sea in a time interval $[0, t]$. Due to the nature of these events, pre-assumption that it is a nonhomogeneous Poisson process with some parameter $\lambda(t) > 0$, seems to be justified. The expected value of increment of this process is given by (8) while its one dimensional distribution is determined by (6). We can use practically these rules if will know the intensity function $\lambda(t) > 0$. To define this function we utilize information presented above in Statistical Analysis.

5.1. Estimation of models parameters

Dividing the number of accidents in each year, that are shown in figure 3, by 365 or 366 we get the intensity in units of [1 / day]. The results are shown in Table 3. Figure 8 shows the empirical intensity of accidents in the Baltic Sea and Seaports.

As a parameters of the models we will approximate the empirical intensity by a linear regression function $\boldsymbol{y = ax + b}$ that satisfied condition

$$S(a, b) = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2 \to min$$

Recall that parameters a and b are given by the rules

$$a = \frac{\mu_{11}}{\mu_{20}} \qquad b = m_{01} - am_{10}$$
(10)

or

$a = r\frac{s_Y}{s_X} \quad b = \bar{y} - a\bar{x},$

where

$$\bar{x} = m_{10} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad \bar{y} = m_{01} = \frac{1}{n}\sum_{i=1}^{n} y_i,$$

$$m_{11} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i, \ \mu_{11} = m_{11} - m_{10}\, m_{01},$$

$$r = \frac{\mu_{11}}{\sqrt{\mu_{20}\,\mu_{02}}}.$$

Table 3. The empirical intensity of accidents in the Baltic Sea and Seaports

Year	Interval	Center of interval	Number of accidents	Intensity [1/day]
2004	[0, 366)	183	133	0,36338
2005	[366,731)	731,5	146	0,40000
2006	[731, 1096)	913,5	115	0,31506
2007	[1096, 1461)	1278,5	118	0,32328
2008	[1461, 1827)	1644	138	0,37704
2009	[1827, 2192)	2009,5	115	0,31506
2010	[2192, 2557)	2374,5	127	0,34794
2011	[2557, 2922)	2374,5	143	0,39178
2012	[2922, 3288)	3105	148	0,40437
2013	[3288, 3653)	3470,5	149	0,40821

Applaying the rules (10) for the data from *Table 3* and using Excel we obtain the linear intensity of accidents

$$\lambda(x) = 0.0000147564\, x + 0.3379257233, \quad x \geq 0\,. \tag{11}$$

From (7) we have

$$\Lambda(t) = \int_0^t (0{,}0000147564\, x + 0{,}3379257233)dx.$$

Hence we obtain

$$\Lambda(t) = 0{,}0000073782\, t^2 + 0{,}3379257233\, t, \quad t \geq 0. \tag{12}$$

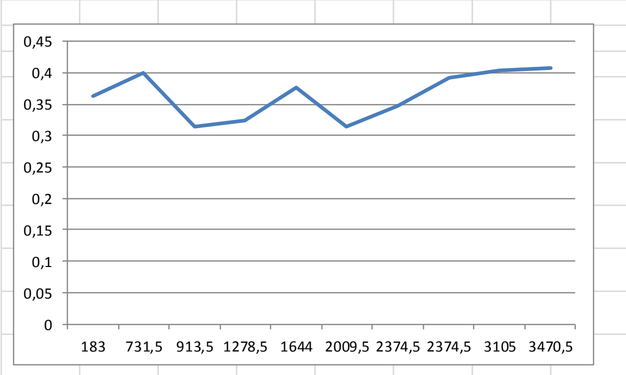

Figure 8. The empirical intensity of accidents in the Baltic Sea and Seaports

From (6) and (7) we obtain one dimensional distribution of NPP

$$P(N(t) = k) = \frac{(\Lambda(t))^k}{k\,!} e^{-\Lambda(t)}, \quad k = 0,1,2,\ldots, \tag{13}$$

where $\Lambda(t)$ is given by (9).
Finnally we can say that the model of the accident number in the Baltic Sea and Seaport is the nonhomogeneous Poisson process with the parameter $\Lambda(t), \quad t \geq 0$ determines by (9).

5.2. Anticipattion of the accident number

From (6) we get

$$P(N(t+h) - N(t) = k) = \frac{[\Lambda(t+h) - \Lambda(t)]^k}{k\,!} e^{-[\Lambda(t+h)-\Lambda(t)]}; \tag{14}$$

It means that we can anticipate number of accidents at any time interval with a length of *h*. The expected value of the increment $N(t+h) - N(t)$ is defined by (8).
For the function $\Lambda(t)$ given by (9) we obtain the expeted value of the accidents at time interval $[t, t+h)$

$$\Delta(t;h) = a\,h^2 + b\,h + 2a\,th\,, \tag{15}$$

where

$a = 0.0000073782$ and b= 0.3379257233

The corresponding standard deviation is

$$\sigma(t;h) = \sqrt{a\,h^2 + b\,h + 2\,a\,th}\,. \tag{16}$$

Example 1
We want to predict the number of accidents from

June 1 of 2017 to September 30 of 2017. We also want to calculate the probability of a given number of accidents.

First we have to determine parameters t and h. To do it we extent part of *Table 9*.

Table 4. Extensions of part of *Table 9*

2013	[3288, 3653)
2014	[3653, 4018)
2015	[4018, 4383)
2016	[4383, 4749)
2017	[4749, 5114)

From January 1 of 2017 to June 1 of 2017 have pased 151 days. Hence $t = 4749 + 151 = 4900$.
From June 1 to September 30 have passed h =122 days. For these parameters we obtain

$$\Delta(t; h)$$
$$\sigma(t; h)$$

This means that the average predicted number of accidents between 1 June 2017 and 30 September 2017 is about 50 with a standard deviation of about 7.

Probability that the number of accidents in the Baltic Sea and Seaports between June 1, 2017 and September 30 , 2017 is not greater than ***d***=60 and not less that ***c***=40 is

$$P_{40\le k\le 60} = P(40 \le N(t+h) - N(t) \le 60) = $$
$$= \sum_{k=60}^{k=40} \frac{50{,}16^k}{k\,!} e^{-50{,}16};$$

Applying approximation by normal distribution we get

$$P_{40\le k\le 60} = \Phi\left(\frac{60-50{,}16}{7{,}08}\right) - \Phi\left(\frac{40-50{,}16}{7{,}08}\right) =$$
$$= 0.84206$$

5.3. Models describing total sum of accidents in the in the Baltic See Region.

From the so-called theorem on adding of the random variables with Poisson distributions it follows that *the sum of* n independent nonhomogeneous Poisson processes $N_1(\mathrm{t}), N_2(t), \dots, N_n(t)$ with parameters $\lambda_1(t), \lambda_2(t), \dots, \lambda_n(t)$, *is the nonhomogeneous Poisson process with parameter* $\lambda(t) = \lambda_1(t) + \lambda_2(t) + \cdots + \lambda_n(t),\ t \ge 0$. It means that the process $\{N(t);\ t \ge 0\}$, $N(t) = N_1(\mathrm{t}) + N_2(t) + \cdots + N_n(t)$ is the nonhomogenous Poisson. This process counts total number of accidents in a Baltic Sea Region in a time interval $[0, t]$ while the processes $N_1(\mathrm{t})$, $N_2(t), \dots, N_n(t)$ denote different kind of accidents due the partition of them according to different criteria.
A first order distribution of this process is given by the rule

$$P(N(t) = k) = \frac{\left(\int_0^t \lambda(x)dx\right)^k}{k\,!} e^{-\int_0^t \lambda(x)dx},$$
$$k = 0, 1, 2, \dots,$$

where

$$\lambda(x) = \lambda_1(x) + \lambda_2(x) + \cdots + \lambda_n(x),\ \ x \ge 0.$$

It is also true to say that every nonhomogeneous Poisson process can be represented as a sum of independent Poisson processes with parameters $\lambda_1(x), \lambda_2(x), \dots, \lambda_n(x)$, such that $\lambda_1(x) + \lambda_2(x) + \cdots + \lambda_n(x) = \lambda(x),\ \ t \ge 0$.

5.4. Models describing different kinds of number of accidents in the Baltic See due to location

Due to the location of the accident, based on the data presented in Figure 7, the nonhomogeneous Poisson process can be represented as a sum

$$N(t) = N_1(\mathrm{t}) + N_2(t) + N_3(t) + N_4(t),$$

where

$N_1(\mathrm{t})$ – number of accidents in the ports,
$N_2(\mathrm{t})$ – number of accidents in the oppen sea,
$N_3(\mathrm{t})$ – number of acc. in the port approches,
$N_4(\mathrm{t})$ – number of other accidents.

The corresponding to these processes parameters are:

$\lambda_1(x)$=0,44 $\lambda(x)$, (17)
$\lambda_2(x)$=0,35 $\lambda(x)$, $\lambda_3(x) = 0{,}14\,\lambda(x)$,
$\lambda_4(x)$=0,07 $\lambda(x)$.

5.5. Models describing number of accidents in the Baltic See Ports

According to (12) and (17) the expectation of the process $N_1(\mathrm{t})$ describing number of accidents in the Baltic Sea Ports is given by

$\Lambda_1(t) =$
$0{,}44\,(0{,}0000073782\,t^2 +$

$0{,}3379257233\, t),$ (18)

The expected value of the accidents at time interval $[t, t+h)$ is

$\Delta_1(t;h) = a_1\, h^2 + b_1\, h + 2a_1 t\, h\,,$ (19)

where

$a_1 = 0{,}000003246408$, (21)

$b_1 = 0{,}148687318252.$ (22)

The corresponding standard deviation is

$\sigma_1(t;h) = \sqrt{a_1\, h^2 + b_1 h + 2\, a_1\, t}\ .$ (23)

Example 2
We want to anticipate the number of accidents in the Baltic Sea Ports from June 1 , 2017 to September 30, 2017. We also want to calculate the probability of a given number of that kind of accidents. Parameters t and h are the same like in example l, parameters a_1 and b_1 are given by (21) and (22). Using the formula (19) we obtain the expected value of the accidents in the Baltic Sea Ports in time interva $[t, t+h)$-between June 1, 2017 to September 30, 2017;

$\Delta_1(t;h) = 22.07.$

Applying formula (23) we get the corresponding to that one standard deviation

$\sigma_1(t;h) = 4.67.$

Probability that the number of accidents in the Baltic Sea Ports in that interval of time is not greater than ***d***=30 and not less that ***c***=15 is approximately equal to

$$P_{15\le k\le 30} = \Phi\left(\frac{30-22.07}{4.67}\right) - \Phi\left(\frac{15-22.07}{4.67}\right) = $$
$$= 0.8902.$$

6. Conclusions

The random processes theory deliver concepts and theorems that enable to construct stochastic models concerning the incidents or (and) accidents. Counting processes and processes with independent increaments are the most appropriate for modelling number of the dangerous events and accidents number in Baltic Sea and Baltic Seaports in specified period of time. A crucial role in the models construction plays a nonhomogeneous Poisson process . Models of accidents number in the Baltic Sea and Seaports are constructed. Moreover some procedures of the model parameters identification are presented in the paper. Estimation of model parameters was made based on data from reports of HELCOM (2014) and Interreg project Baltic LINes (2016-2019). To select the most appropriate model one should verify the models using appropriate statistical tests. But it requires a large number of data.

Acknowledgements

The paper presents the results developed in the scope of the HAZARD project titled “Mitigating the Effects of Emergencies in Baltic Sea Region Ports” that has received funding from the Interreg Baltic Sea Region Programme 2014-2020 under grant agreement No #R023. https://blogit.utu.fi/hazard/

"Scientific work granted by Poland’s Ministry of Science and High Education from financial resources for science in the years 2016-2019 awarded for the implementation of an international co-financed project."

I would like to thank Professor Lauri Ojala for pointing to the reports Baltic LINes (2016) [9] and HELCOM (2014) [10], from which the data allowed me to estimate the models parameters.

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