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## NUMERICAL REPRESENTATION OF POST-CRITICAL DEFORMATIONS IN THE PROCESSES OF DETERMINING STRESS DISTRIBUTIONS IN CLOSED MULTI-SEGMENT THIN-WALLED AIRCRAFT LOAD-BEARING STRUCTURES

### NUMERYCZNE ODWZOROWANIE DEFORMACJI ZAKRYTYCZNYCH W PROCESACH OKREŚLANIA ROZKŁADÓW NAPRĘŻEŃ W ZAMKNIĘTYCH WIELOSEGMENTOWYCH CIENKOŚCIENNYCH LOTNICZYCH STRUKTURACH NOŚNYCH\*

*The study presents results of a research work on the problem of obtaining reliable results of nonlinear FEM analyses of thin-walled load-bearing structures subjected to post-critical loads. Consistency of numerical simulations results and actual stress distribution states depends on the correct numerical reproduction of bifurcations that occur during advanced deformations processes.*

**Keywords:** shell, torsion, bifurcation, experiment, load-bearing structures, nonlinear numerical analysis.

*Opracowanie przedstawia wyniki badań związanych z problemem uzyskiwania wiarygodnych wyników nieliniowych analiz numerycznych, w ujęciu metody elementów skończonych, cienkościennych struktur nośnych poddawanych obciążeniom zakrytycznym. Zgodność wyników numerycznych symulacji z rzeczywistymi dystrybucjami naprężeń uzależniona jest od poprawnego numerycznego odwzorowania bifurkacji zachodzących w procesach zaawansowanych deformacji.*

**Słowa kluczowe:** powłoka, skręcanie, bifurkacja, eksperyment, struktury nośne, nieliniowa analiza numeryczna.

#### Introduction

Thin-walled load-bearing systems are widely applied in modern aviation structures. The strict requirements with regard to the levels of transferred loads and the need to minimise the total mass of the structure is often the cause for which it becomes necessary to accept occurrence of physical phenomena that in case of other structures are considered as inadmissible. An example of such a phenomenon is the loss of stability of shells that are parts of load-bearing structures within the range of admissible operating loads [13].

Thus, an important stage in the design work on an aircraft load-bearing structure is to determine the stress distribution in post-critical deformation state [12]. One of the numerical tools used to obtain actual displacement distributions and the resulting stress distributions is the nonlinear finite elements analysis. The assessment of the reliability of the results thus obtained is based on the solution uniqueness rule according to which a specific deformation pattern can correspond to one and only one stress state. In order to apply this rule it is necessary to obtain such displacement distributions of the numerical model that fully corresponding to actual deformations of the analysed structure.

An element decisive for a structure's deformation state is the effect of a rapid change of the structure's shape occurring when the critical load levels are exceeded [2]. From the numerical point of view, this phenomenon is interpreted as a change of the relationship between state parameters corresponding to specific degrees of freedom of the system and the control parameter related to the load [3]. This relation-

ship, defined as the equilibrium path, in the case of occurrence of the above-mentioned phenomenon, has an alternative nature usually called bifurcation. Therefore, the fact of taking a new deformation pattern by the structure corresponds to a sudden change to an alternative branch of the equilibrium path [4].

Therefore, a prerequisite for obtaining a proper form of the numerical model deformation is to retain the conformity between numerical bifurcations and those occurring in the actual structure. In order to determine such conformity it is necessary to verify the results obtained by an appropriate model experiment or by using the data obtained during the tests of the actual object [9]. Obtaining reliable results of nonlinear numerical analyses is often troublesome and it requires an appropriate selection of numerical methods depending on the type of the analysed structure and the degree of precision to which the parameters controlling the course of procedures were determined [14].

In view of the number of state parameters, the full equilibrium path should be interpreted as a hyper-surface in hyperspace of states satisfying the matrix equation for residual forces:

$$\mathbf{r}(\mathbf{u}, \Lambda) = \mathbf{0}, \quad (1)$$

where  $\mathbf{u}$  is the state vector composed of structure nodes' displacement components corresponding to current geometrical configuration,  $\Lambda$  is a matrix composed of control parameters corresponding to the current load state, and  $\mathbf{r}$  is the residual vector composed of uncompensated force components related to the current system deformation state. The

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set of control parameters can be represented by a single parameter that is a function of the load. Equation (1) takes then the form

$$\mathbf{r}(\mathbf{u}, \lambda) = \mathbf{0}, \quad (2)$$

called a monoparametric equation of residual forces. The prediction-correction methods of determining the consecutive points of the equilibrium path used in contemporary software routines contain also a correction phase based on the requirement that the system satisfies an additional equation called the increment control equation or the constraints equation [5]:

$$c(\Delta \mathbf{u}_n, \Delta \lambda_n) = 0, \quad (3)$$

where the increments

$$\Delta \mathbf{u}_n = \mathbf{u}_{n+1} - \mathbf{u}_n \quad \text{and} \quad \Delta \lambda_n = \lambda_{n+1} - \lambda_n \quad (4)$$

correspond to the transition from  $n$ -th state to  $n+1$ -th state.

Graphical interpretation of the increment control equation is shown in Figure 1.

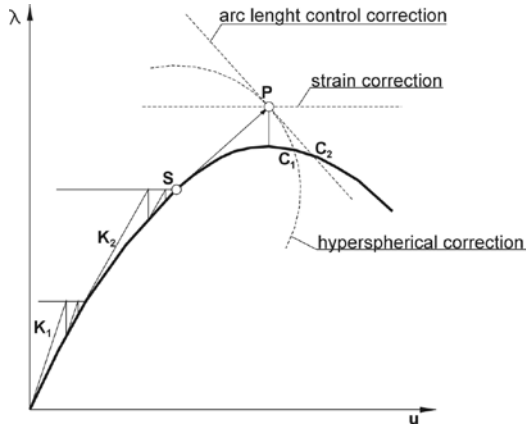


Fig. 1. Graphical representation of various correction strategies for the representative system with one degree of freedom

Because of the large number of degrees of freedom and state parameters corresponding to them, the deformation processes are represented in practice by applying substitute characteristics called the representative equilibrium paths [8]. They define relationships between a control parameter related to the load and a selected characteristic geometric value related to deformation of the structure.

## 2. Subject and scope of research

Bifurcation changes of forms observed in load-bearing structures containing shells with considerable curvature occur the more violently the higher is the ratio of the square of the smaller of dimensions of the shell segment area limited by the adjacent member frames to the value of the local radius of its curvature [6]. Semi-monocoque structures with relatively low number of the framing elements are therefore especially difficult as far as their nonlinear numerical representations are considered [7].

An example of such a structure is a closed cylindrical thin-walled shell reinforced with frames and stringers. The type of the structure itself corresponds to solutions commonly used in the aviation technology, e.g. the construction of an aircraft fuselage tail beam (Figure 2). The subject of the present research were structures a general schematic diagram of which is presented in Figure 3.

Several variants of them were analysed, differing in the number of stringers and frames. The first variant was considered a fundamental

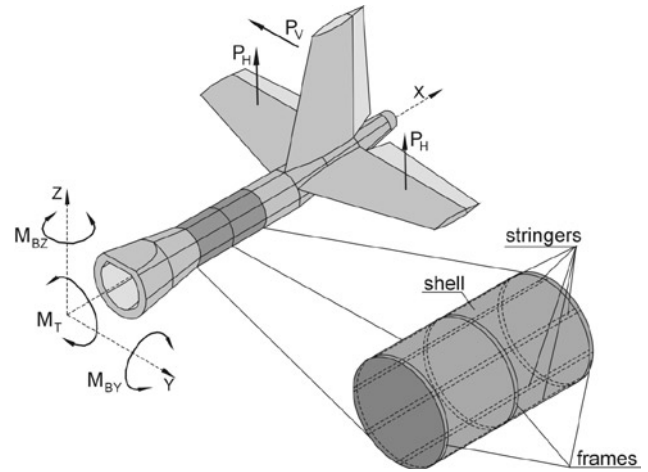


Fig. 2. Application of the examined structure in the airframe structure

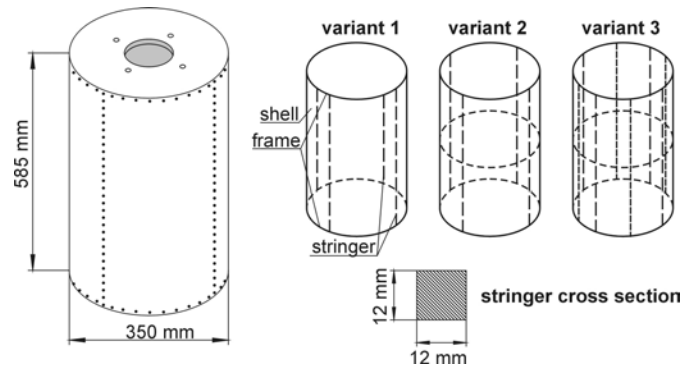


Fig. 3. Schematic drawing of the examined structure

one, with the skeleton consisting of two frames and four stringers. The second variant represented a structure with four stringers reinforced with additional frame placed in the middle of the shell length. The third variant was the structure reinforced with eight evenly distributed stringers. Models of all variants of the structure were made out of polycarbonate for which the tensile strength test was carried out and the material constants determined, yielding the Young's modulus  $E = 3000 \text{ MPa}$  and the Poisson ratio  $\nu = 0.36$ .

Figure 4 shows the characteristic of the above-mentioned material corresponding to one-dimensional tensile stress. The clearly visible elastic and inelastic deformation zones suggest the possibility to approximate the actual material characteristic by the ideal elastic-plastic model. However, due to fact that only a local elastic loss of stability of the structure is acceptable, the elastic material model was adopted in all numerical models. Moreover, because of its low elasticity modulus value (by two orders of magnitude less than this of steel) it was possible to carry out experiments at low values of external loads.

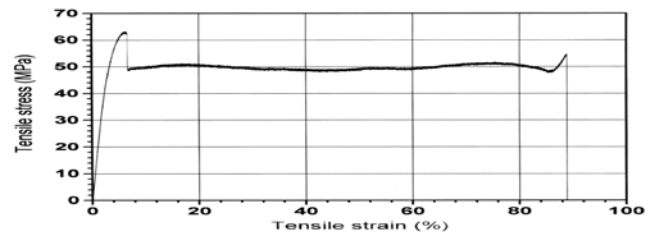


Fig. 4. The tensile stress plot for a polycarbonate sample

The choice of the material, apart from the above-mentioned physical characteristics, was also justified by its high optical activity thanks to which it became possible to obtain qualitative information about optical effect distribution in circular polarisation conditions [1]. Joints

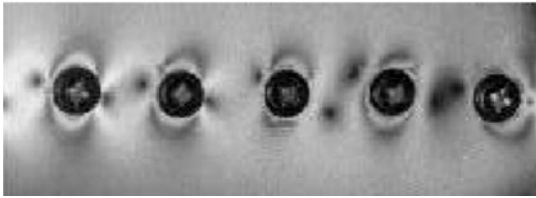


Fig. 5. Distribution of isochromatic fringe patterns in vicinity of bolts subject to control during the assembly

between the shell and the stringers were realised by means of steel bolts spaced 20 mm apart. In order to avoid possible assembly stress at bolt joints, continuous observation of isochromatic fringe pattern fields in the vicinity of each bolt was carried out throughout the whole assembly work (Figure 5).

### 3. Experimental research

The first variant of the structure's framing comprises a minimum number of crosswise elements, i.e. two closing frames and four longitudinal members. It should be emphasised, however, that the model subjected to examinations constituted a special instance of a structure of purposefully minimised number of longitudinal members. The actual solutions are usually based on more extended framings corresponding to the consecutive two variants.

The examined structure was subjected to constrained torsion using the experimental set-up depicted in Figure 6.

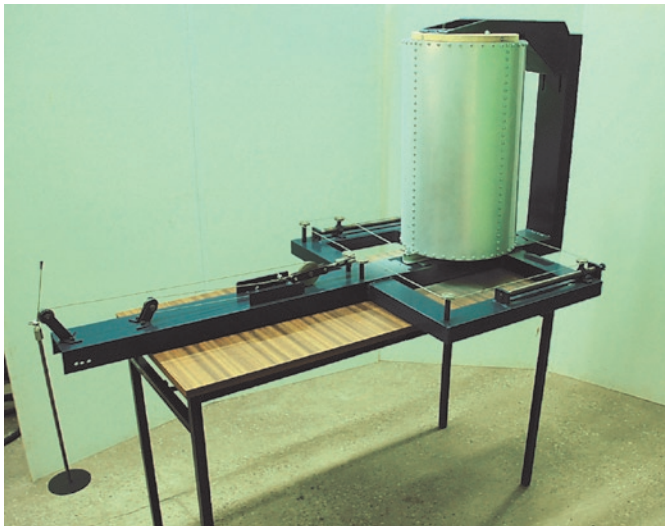


Fig. 6. Experimental set-up

According to the expectations, post-critical deformations occurred in a violent way. Due to application of gravitational load, the measurement of the relationship between the torsion angle and the torque moment, assumed to be the representative equilibrium path, corresponded to the steady states [9] (Figure 16).

With this measurement method, the representative characteristics does not reflect bifurcation points in an apparent way, but attention should be drawn to the occurrence of its horizontal section. It corresponds to the phase of the experiment in which a sudden change occurred in the structure state while the load level remained constant. The nature of the equilibrium path curve corresponds to the unstable bifurcation type typical for shell structures.

With regard to the symmetry, the deformed structure revealed four characteristic wrinkles in all the shell segments (Figure 7). During the experiment, the surface geometry was registered using the projection moiré method. Atos scanner by German company GOM Optical Measuring Techniques was used for this purpose.

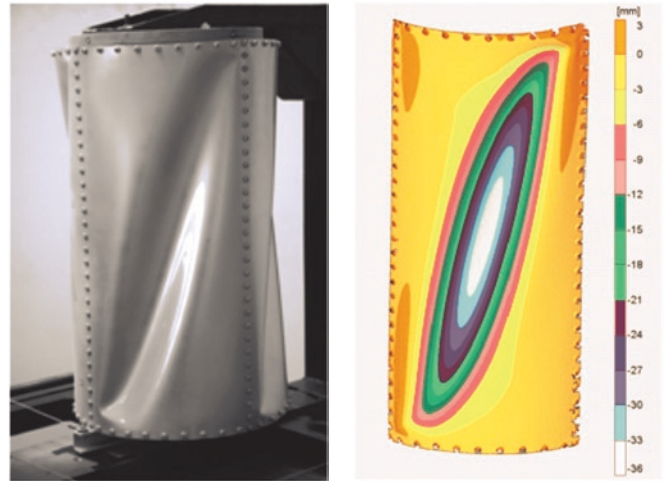


Fig. 7. Advanced post-critical deformation of the examined structure (left) and the distribution of contour lines representing magnitude of deformation with the use of the projection moiré method (right) – variant 1

Due to large dimensions of the shell segments, the loss of stability resulted in this case in an abrupt significant change of the total torsion angle of examined structure. As it has been already mentioned above, such a property makes it impossible to apply such technical solution in actual aircraft structures [13]. However, analyses of similar special cases allow to elaborate FEM numerical models and represent a significant research potential.

In the next step, another variant of the structure (Figure 3, variant 2) with an additional frame was examined. The change of dimension ratios characterising shell segments limited with skeleton components of the structure did not result in any significant change of the critical load value. However, the loss of stability revealed a more gentle nature in this case manifesting with a lack of a sudden change on the representative equilibrium path shown in Figure 17.

A large decrease of the total torsion angle value was also observed compared to the first variant. Therefore, this kind of technical solution seems to be much more useful from the practical point of view. Thanks to the lack of rapid deformation increases, there is no considerable risk that an aircraft will lose its essential aerodynamic characteristics (for instance, effectiveness of its flight control surfaces).

Figure 8 presents distribution of deformations in an isolated shell segment of the examined structure and the related displacement distribution obtained by means of the optical scanner.

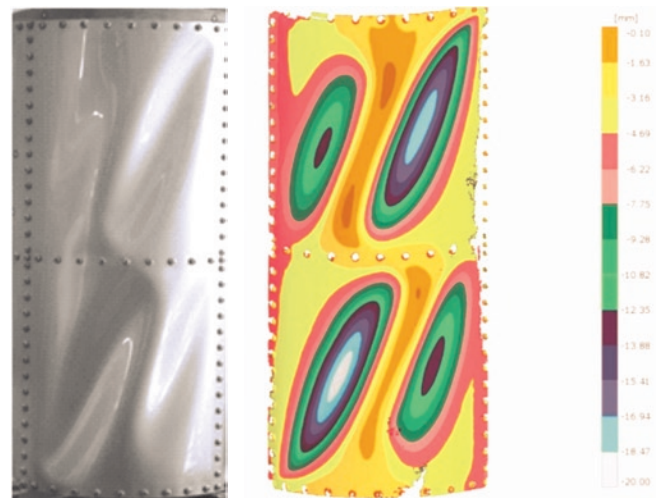


Fig. 8. Advanced post-critical deformation of the examined structure (left) and the distribution of contour lines representing magnitude of deformation with the use of the projection moiré method (right) – variant 2



In the last version of the examined structure, four additional stringers were applied (Figure 3, variant 3). Distribution of post-critical deformations presented in Figure 9 and the representative equilibrium path plotted in Figure 18 allow to conclude that the observed phenomena had in this case significantly different nature than in the two previous models. First of all, the loss of stability occurred at much lower level of the load. It seems to be a result of change of the ratio of shell segments dimensions to their curvature radii [6]. Decrease of the critical load value, however, does not mean necessarily the lessening of practical value of the structure. It should be emphasised that its torsional stiffness significantly increased. Therefore, the loss of stability results in occurrence of small geometrical imperfections on the fuselage skin and the related increase of the drag coefficient, but higher stiffness guarantees that essential aerodynamic characteristics of the aircraft are maintained.

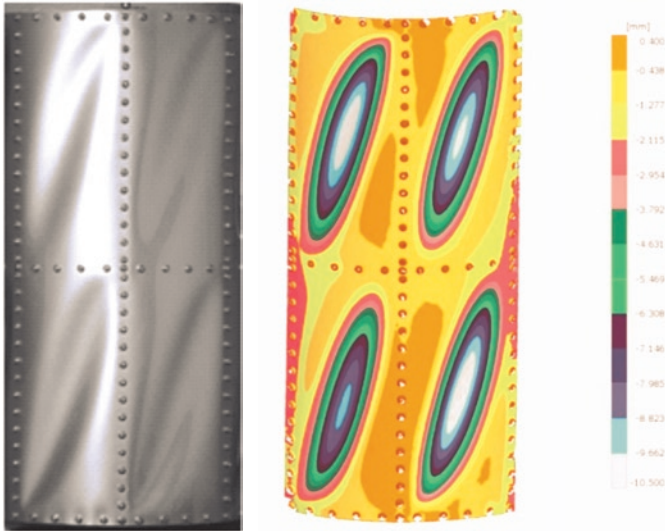


Fig. 9. Advanced post-critical deformation of the examined structure (left) and the distribution of contour lines representing magnitude of deformation with the use of the projection moiré method (right) – variant 3

#### 4. Numerical FEM analysis

The first variant of the examined structure turned out to be one of the most troublesome from the point of view of a FEM-based nonlinear numerical simulation. A number of tests performed with the use of MSC MARC software revealed the lack of effectiveness of its procedures in case of this problem, with regard to determining the appropriate post-buckling state of the structure. The algorithms used in these procedures are characterised by inability to represent the symmetry of the phenomenon. With the idealised geometric form of the model, the obtaining of the new form of the structure after crossing the critical load value occurs only in one of the segments, despite the apparently correct symmetrical initiation of the loss of stability [11]. This indicates existence of errors in algorithms used for choosing appropriate variants of the equilibrium path in case of the appearance of changes in the state parameters combinations in several of their independent subsets [8].

The situation improved when the shell imperfections were implemented by introducing forces normal to the skin and applied to central points of individual skin segments (Figure 10). The values of the forces in all cases corresponded to their highest values applied during the experiment.

However, even in the case of using this type forcing a change in the fuselage shape, it was still very difficult to obtain results fully consistent with the experiment. Assuming the use of shell elements with linear shape functions, the appropriate density of the mesh turned out

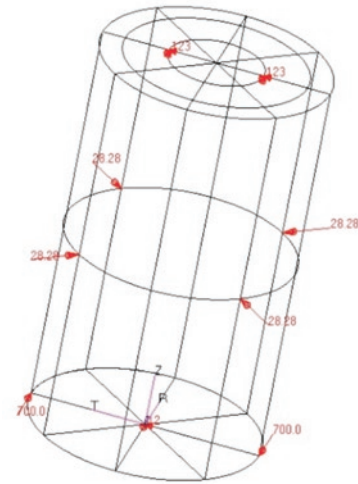


Fig. 10. A geometrical model of a structure defined in MSC PATRAN environment with boundary conditions and load (in N)

to be the key factor, but excessive density resulted in appearance of incorrect forms of post-critical deformations [14] (Figure 11).

A better result, in the case of beam elements representing stringers, was obtained with the use of a relatively low density of FE mesh. This speaks for correctness of the argument, confirmed many times in the course of numerous studies, that the decrease of the overall number of degrees of freedom corresponding to the number of state parameters, in case of nonlinear procedures used in the available commercial programs, often brings benefits that considerably exceed the deficiencies of mathematical description related to reduced number of elements [5].



Fig. 11. An incorrect form of deformation obtained in case of too many elements

The best result was obtained only after the fundamental change of FEM model concept, when a different type of finite element was applied to represent stringers (thick shell element instead of the usually recommended beam element). Although this solution is considered less correct from the point of view of mathematical description, it turned out to be much more effective in the case of relatively low value of the structure's total torsion angle. The results of analysis of this FEM model version obtained using the secant prediction method and the strain correction strategy [10] (Figure 1) are presented in Figure 12.

The strain-correction strategy turned out to be most effective in case of significant violent change of the deformation pattern when the representative equilibrium path comprised a relatively long "horizontal section".

Modelling stringers with thick-shell elements turned out to be effective also when applied to the other two variants of the model. In order to obtain the symmetry of deformations, small forces normal to the skin surface representing geometrical imperfections were applied

in all cases at central points of each of the shell segments. The way in which variants 2 and 3 of the structure were modelled is shown in Figure 13.

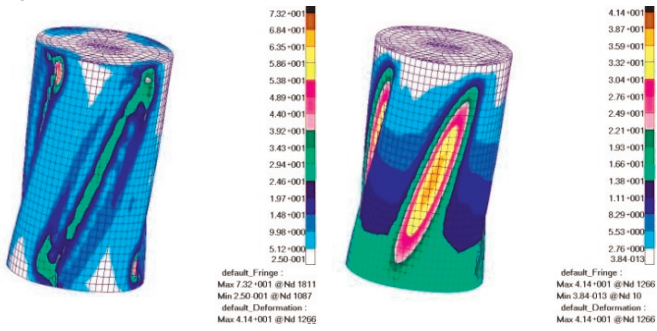


Fig. 12. Distribution of displacement (left) and reduced stress according to Huber-Mises hypothesis (right) for 100% of the maximum load (stringers modelled with thick-shell bilinear elements)

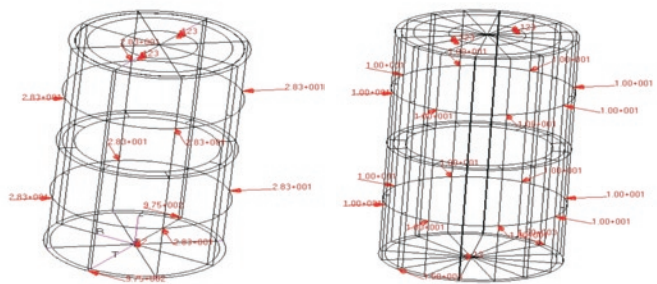


Fig. 13. Geometrical models of the 2-nd (left) and 3-rd (right) variant of a structure created in MSC PATRAN environment with boundary conditions and loads (in N)

Results of FEM-based nonlinear analyses of the above-discussed structure variants are presented in Figures 14 and 15. The representative equilibrium paths are compared in Figures 16–18.

The deformation distributions obtained numerically reveal satisfactory consistence with the above-presented results of experiments, both quantitatively and qualitatively. Plots of representative equilib-

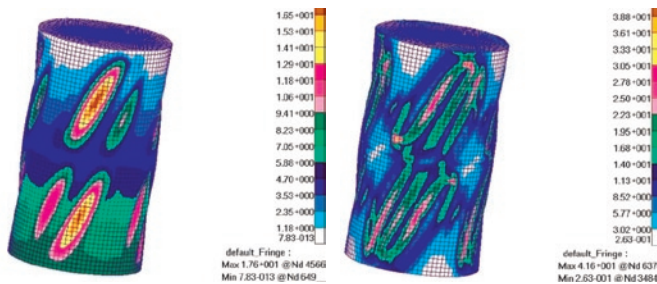


Fig. 14. Distribution of displacement (left) and reduced stress according to Huber-Mises hypothesis (right) for 100% of the maximum load (variant 2)

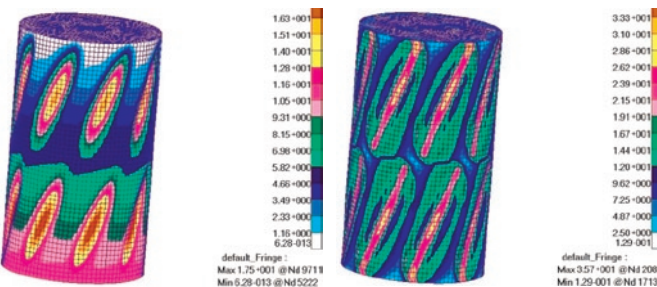


Fig. 15. Distribution of displacement (left) and reduced stress according to Huber-Mises hypothesis (right) for 100% of the maximum load (variant 3)

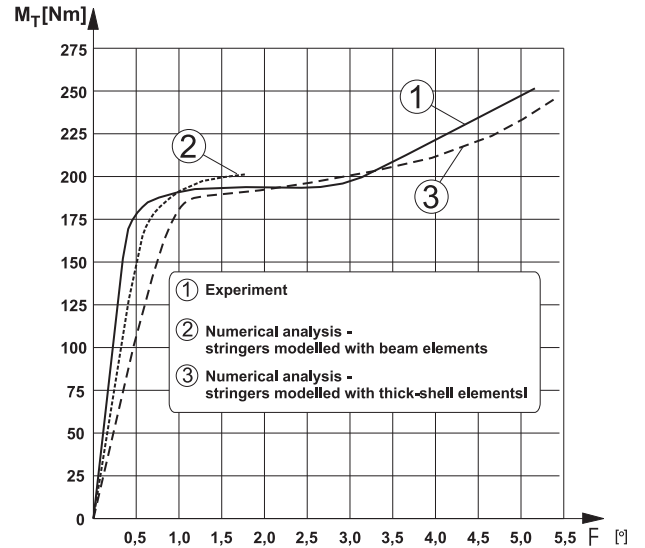


Fig. 16. A comparison of representative equilibrium paths – variant 1

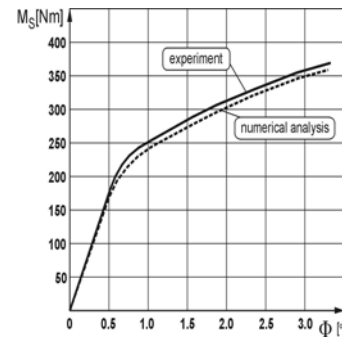


Fig. 17. A comparison of representative equilibrium paths – variant 2

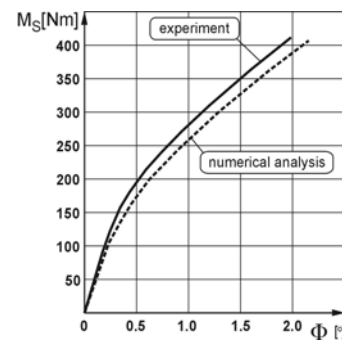


Fig. 18. A comparison of representative equilibrium paths – variant 3

rium paths are characterised by much better conformity with results of the experiment than those obtained for variant 1. This compliance is almost perfect in the subcritical range, whereas for the maximum load, the error of reproduction of the total torsion angle value does not exceed 10%. It allows to conclude that the adopted modelling method and the choice of the numerical procedures turned out to be satisfactory.

More gentle nature of equilibrium paths compared to those observed for variant 1 allows for a greater deal of freedom in selection of nonlinear procedures, e.g. application of Newton-Raphson's method or Crisfield's hyperspherical correction [8].

## 5. Conclusions

The results of numerical analyses presented above prove that thanks to the use of contemporary engineering tools, it is possible to analyse in detail both values and gradients of effective stress in thin-

walled structures subjected to complex post-critical deformations. However, to be able to accept the obtained results as reliable ones, correct numerical reproduction of bifurcations occurring in the actual hyperspace of states is necessary.

The fundamental conclusion that can be drawn from the presented research results is the absolute need for using experimental verifications with regard to FEM nonlinear numerical analyses concerning structures of this type. In fact, even in the cases where correctness of the results obtained seems to be unquestionable, they may be burdened with errors resulting from the very limited reliability of the numerical procedures used in commercial programs.

Results of the studies on certain variants of a closed, thin-walled cylindrical shell, constituting typical example of the design solution used in aviation, can be recognised as a structural standard, in qualitative meaning, in view of the fact that qualitative nature of post-critical deformations, in case of maintaining geometrical ratios and proportions of stiffness between components of the structure, will not change after applying other isotropic materials and different load values. It is confirmed by numerical tests carried out with the presented models.

Based on the nonlinear numerical analyses concerning the presented structures, and in many instances repeated a number of times, a

general recommendation can be also formulated as for the maximum possible limitation of the size of a task. Striving for improving accuracy of calculations by increasing the density of finite elements mesh, applied successfully in linear analyses, may turn out to be ineffective in case of a nonlinear analysis and lead to incorrect results or the lack of convergence of calculations.

Detailed analysis of the presented research results allows to formulate design recommendations concerning selection of most effective number of structure skeleton components and most effective ways of their arrangement. As it has been proven, by increasing number of stringers and applying additional frames which results in reducing the size of shell segments, improves stiffness of the structure. However, it is necessary to remember that this means also an increase of mass of the design solution. Comparison of representative equilibrium paths obtained for separate variants of the examined structure allows to claim that applying another stringers or frames may not bring the benefit in the form of increase of the stiffness in view of the related increase of the mass. The detailed determination of this relationship requires further experiments with the use of larger models and appropriate experimental set-up.

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