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Hourly identification and simulation of the TGE S.A. Day-Ahead Market system[1]

by

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Abstract: The paper presents selected research results concerning the identification and simulation of the TGE S.A. Day-Ahead Market (DAM) system of the day for electricity delivered and sold, listed for the following hours: 5:01-6:00, 11:01-12:00, 17:01-18:00 and 23:01-24:00 in 2019, which were obtained in the MATLAB and Simulink environment using the System Identification Toolbox. As a result of identification, four respective discrete parametric arx models were obtained, which were then subject to quality assessment. Then, a simulation model was built in the Simulink environment, which was used for simulation tests and for assessing the sensitivity of the model created using the data from 2019 as the basis and the data from 2020 for verification. The obtained results confirm the correctness of both the performed discrete parametric identification and the possibility of testing the quality of the model and its sensitivity with the use of the DAM system model in the MATLAB and Simulink environment.

Keywords: equations of state, control system, Day-Ahead Market, identification, Polish Electricity Exchange, simulation, state space

1. Introduction

At the beginning of the 21[st] century, the interest in modeling of the Electricity Market System (EMS) increased significantly (see, in particular, Chodakowska

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et al., 2005; Wnukowska, 2005), including the Polish Electricity Exchange System (PEE) as a subsystem operating on TGE S.A.[1] (see Tchórzewski and Marlęga, 2019a, 2021; Ruciński, 2022), due to the very dynamically changing international conditions, including structural changes caused by COVID, and then by the energy crisis brought about by the international conflicts over energy resources (see the Report of the President of the Energy Regulation Board, Prezes…, 2022; Wiśniewski, 2022).

The structural changes, taking place on the EMS concern, inter alia, the Day-Ahead Market System (DAM), which is a subsystem operating on PEE. In Poland, this development has continued since the establishment of TGE S.A., i.e. since 2002 (see TGE S.A., 2002-2022; Tchórzewski, 2010). Within the framework of DAM, the volume of electric power (ep) and the volume-weighted average price of ee are quoted for each hour of the day. Transactions on the DAM are concluded 24 hours before the physical delivery of electricity to the recipient (see TGE S.A., 2002-2022).

For these reasons, the development of TGE S.A., including changes to the DAM, mean that all participants, i.e. including suppliers, recipients and prosumers, as well as trade intermediaries, are forced to properly prepare for transactions (see Mielczarski, 2000). One of the ways to adequately prepare for concluding transactions is the prior simulation of the expected volume-weighted average ee prices, hence there is now an increased demand for the use of DAM system models.

Models can be obtained with the use of various methods, from analytical methods, through identification ("econometric") methods, to artificial intelligence methods, involving methodologies such as artificial neural networks, fuzzy systems, or evolutionary algorithms (see Box and Jenkins, 1983; Popławski and Weżgowiec, 2015; Tchórzewski, 2013, 2021; Trusz and Tserakh, 2017; Wesołowski, 2011; Voronin, 2013; Ziel and Weron, 2018). In the study of the DAM system model, which is reported in the present paper, two methods were used, i.e. the arx parametric method (see Ejdys, Halicka and Godlewska, 2015; Marlęga, 2022; Nazarko, 2018; Tchórzewski, 2013; Zimmer and Englot, 2005) and the Perceptron Artificial Neural Network (see Vargas, Pedrycz and Hemerly, 2019; Osowski, 2013; Ruciński, 2022; Tadeusiewicz, 1993). The results of the literature review in this area are included in the paper by Marlęga (2021), which shows the originality of the conducted research against the background of the available world literature in this field.

[1]TGE S.A.: Towarowa Giełda Energii S.A. – Polish Energy Exchange Co., company running the commodity exchange for electric energy and for a number of other goods and services (eds.).

2. Formulations of the problems

In order to obtain a model of the DAM system, the basics of systems engineering as well as of control and systems theory were referred to (see Kaczorek et al., 2021; Konieczny, 1983; Staniszewski, 1990; Tchórzewski, 1990, 1992, 2013) and the identification of the system was carried out with the use of data recorded on Day-Ahead Market.

In order to conduct the experiments, leading to the development of the DAM system model, numerical data recorded on the DAM in 2019-2020 were used regarding the volume of delivered and sold electricity in each hour of the day and the obtained average weighted with the volume of prices of delivered and sold electricity. Thus, in individual models, the input quantities were the volumes of delivered and sold ep in individual hours of the day (24 input quantities each), and the output quantities were volume-weighted average prices obtained for the sold ep in the above-mentioned period for the appropriate hours of the day. In this publication, the obtained results have been limited to four hours of the day, i.e. 6:00, 12:00, 18:00 and 24:00 hours[2] (see: TGE S.A., 2002-2022).

In a more formalized perspective, the identification of the DAM system is the problem of finding a representation of the matrix of measurement data recorded on the DAM in the form (see Söderström and Stoica, 1997; Tchórzewski, 2013; Zimmer and Englot, 2003) Z^N =[input, output] in the model parameters vector [θ] stored in the form of the **th** matrix of theta format, where N is the number of ordered observations of successive values of the input variables and successive values of the output variables.

Therefore, the problem of identification includes both the measurement data acquisition technique and the computational procedures that are used to process numerical data strings so as to obtain the estimators of model parameters. It should be emphasized here that there are no results of research on the identification of the Day-Ahead Market system at TGE S.A. carried out in order to obtain models understood as equivalent schemes of real systems in terms of control theory and systems, including state space, hence the proposed approach is innovative in this respect (see Marlęga, 2021; Tchórzewski, Marlęga, 2019a).

[2]The article discusses four MISO-type models, i.e. obtained as a result of identification for 24 input quantities (volume of electricity quoted in all separate hours of the day) and for single output quantities (average price volume-weighted in a specific one hour of the day).

3. TGE S.A. Day-Ahead Market system identification models

Based on the collected numerical data, as a result of identification modeling, corresponding classes of system models are obtained, depending on the method used, such as:

- one-dimensional models: Single Input Single Output (SISO),
- multidimensional models: Multi Input Multi Output (MIMO), Multi Input Single Output (MISO), Single Input Multi Output (SIMO),
- linear and non-linear models,
- parametric and non-parametric models,
- models constant in time and models changing in time,
- models in the time domain and models in the frequency domain,
- models with continuous time and models with discrete and even pulse time,
- models with lumped parameters and models with distributed parameters,
- deterministic models and stochastic models, as well as a number of other kinds of models (Box and Jenkins, 1983; Chodakowska, Halicka, Kononiuk and Nazarko, 2005, Ejdys, Halicka and Godlewska, 2015; Halicka, 2006; Marlęga, 2022; Popławski and Weżgowiec, 2015; Ruciński, 2022; Tchórzewski and Marlęga, 2019a).

Due to the purpose of the research, which is reported here, including the need to select the type of identification process with the use of an appropriate method, it is worth paying attention to several basic and important types of models in identification, which include, among others, dynamic linear model, which can be defined by a discrete output signal $y(t)$ of the form (see Fig. 2):

$$y\,(t) = G(z^{-1})u(t) + H(z^{-1})\varepsilon(t), \tag{1}$$

where:

$G(z^{-1})$ – control path characteristics,

$H(z^{-1})$– characteristics of the interference path,

$u(t)$ – input signal,

$\varepsilon(t)$ – disturbance (noise),

z – time shift operator,

t – short time as an independent variable [day].

Expressing $G(z^{-1})$ and $H(z^{-1})$ as quotients of the polynomials in z^{-1} leads to the models of the form:

$$A(z^{-1})y\,(t) = \frac{B(z^{-1})}{F(z^{-1})}u\,(t) + \frac{C(z^{-1})}{D(z^{-1})}\varepsilon\,(t) \tag{2}$$

where:

$$A(z^{-1}) = 1{+}a_1 z^{-1} + \ldots + a_{na} z^{-na},$$

$$B(z^{-1}) = 1{+}b_1 z^{-1} + \ldots + b_{na} z^{-nb},$$

$$C(z^{-1}) = 1{+}c_1 z^{-1} + \ldots + c_{nc} z^{-nc},$$

$$D(z^{-1}) = 1{+}d_1 z^{-1} + \ldots + d_{nd} z^{-nd}$$

$$F(z^{-1}) = 1{+}f_1 z^{-1} + \ldots + f_{nf} z^{-nf},$$

$z^{-\mathbf{1}}$ – time delay by one unit, e.g. $y(t)z^{-1} = y(t\text{-}1)$,
na – degree of the polynomial $A(z)$,
nb – degree of the polynomial $B(z)$,
nc – degree of polynomial $C(z)$,
nd – degree of polynomial $D(z)$,
nf – degree of polynomial $F(z)$.

So, if:

$$C(z^{-1}) = D(z^{-1}) = F(z^{-1}) = 1,$$

the model of the ARX form is obtained:

$$y\,(n) = \frac{B(z^{-1})}{A(z^{-1})} u\,(n) + \frac{1}{A(z^{-1})} \varepsilon\,(n) \tag{3}$$

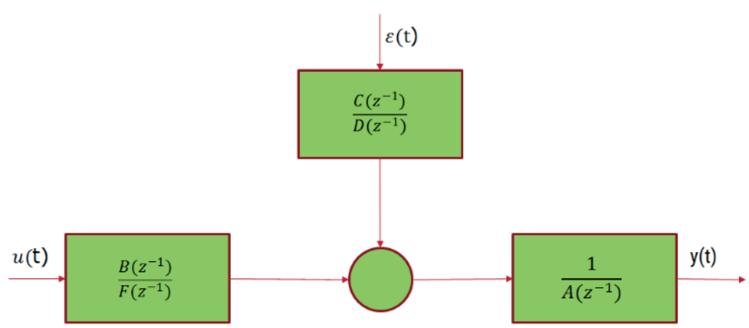

Figure 1. Block diagram of the parametric model in general form. Notations are explained in the text. Source: Söderström and Stoica (1997)

From the point of view of the capacities characterising the methods of identifying the TGE S.A. system, which are used, referring also to individual subsystems, including the DAM system (Marlęga, 2021), several groups of models can be listed in this context, including, in particular:

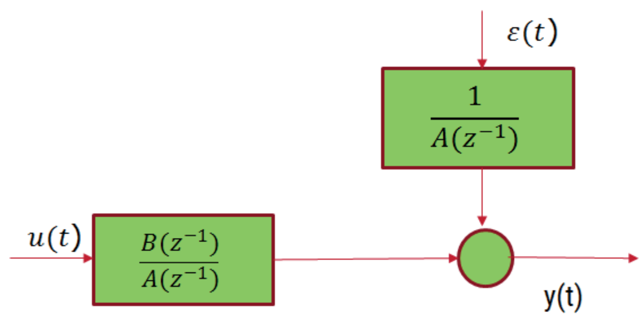

Figure 2. Block diagram of the parametric ARX model. Notations are explained in the text. Source: Söderström and Stoica (1997), Zimmer and Englot (2003)

1. Models of electricity price forecasting as a group of models, which are used to determine the price of electricity, taking into account the competitiveness of the Electricity Market, based on the simulation of the operation of the National Power System and the related information on the costs of generation, transmission and use, etc. The main obstacle, regarding the effective application of this approach, is the requirement to have a sufficiently long real time period for collecting the data used in the investigations. Such simulation methods work very well if used by market operators and regulatory authorities that are authorized to collect precise data, e.g. about equipment, losses, etc. (see Chodakowska et al., 2005; Ejdys, Halicka and Godlewska, 2015; Nazarko, 2018; Voronin, 2013; Ziel and Weron, 2018; Zimmer and Englot, 2005);
   (a) Electricity pricing models based on the principles of game theory that focus on the impact of the bidder on strategic changes in electricity prices. In this particular context, it was found, among others, that prices on the electricity market are closely related to the strategies of submitting bids and setting prices by market participants (see Wesołowski, 2011; Voronin, 2013);
   (b) Stochastic models of ee pricing based on the modified Brownian geometric motion method as a method of stochastic modeling of electricity prices. In this type of models, calculation difficulties are caused by the procedures, related to consideration of the physical properties of power systems, such as transmission losses, transmission congestion, etc., which may cause significant mismatches between the determined electricity prices and the actually recorded electricity prices on the market (see Söderström and Stoica, 1997; Voronin, 2013);

2. Time series models used in modeling electricity prices, covering two main groups of models: regressive models, such as, e.g., moving average autoregression (ARMA), transfer functions and dynamic regression, threshold autoregressive models of the TAR type, conditional autoregression of the ARCH type, Engle models, GARCH models and many other models, as well as increasingly often used artificial intelligence (AI) models, e.g. neural models, neural models with evolutionary weight tuning, models using swarm algorithms, including ant algorithms, models using fuzzy sets, artificial life models, machine learning models, hybrid models, etc. (see Box and Jenkins, 1983; Popławski and Weżgowiec, 2015; Söderström and Stoica, 1997; Tchórzewski, 2010; Trusz and Tserakh, 2017; Voronin, 2013, Zimmer and Englot, 2005).

Time series models also include parametric models, obtained as a result of identification, including ARX-type methods, used for many years, especially in technical and economic sciences, and in particular – for management and control of the DAM system. Various methods of obtaining parametric models are used, depending on the object of research, including: AR, ARX, ARMAX, etc.

The review of the literature on the subject shows, among others, that in the previous research on the Day-Ahead Market system, forecasting, planning or programming models for determining electricity prices were obtained in the effect of the use of time series modeling methods known in the literature, which were then used to determine new prices.

Various modeling methods (analytical, identification and neural) were also used, as a result of which models were obtained based on specific inputs such as economic, environmental and even social factors, and outputs such as the price of electricity. The latter models were then used to predict the price of electricity.

It should be noted here that the literature on the subject lacks models of the TGE S.A. Day-Ahead Market system. (and similar markets such as Nord Pool), understood as technical or technical and economic systems, obtained as a result of parametric identification carried out using methods of control theory and systems engineering, which are finally converted into state space models.

It is also worth adding, in particular, that the demand for such models of the TGE S.A. Day-Ahead Market system and its subsystems is increasing due to the increase in their share in electricity trading on the Electricity Market (see Mielczarski, 2000).

4. Selection of the method for identifying the Day-Ahead Market system

In order to identify the DAM system, preliminary research was carried out consisting in the selection of the identification method. Due to the properties of the DAM system, and, in particular, the fact that electricity volume quotations are used as input values, and electricity price volume-weighted quotations as output values, the following methods were taken into account: arx, nlarx, IV4, armax, arxqs, which were then used in the analysis performed with SIT MATLAB (see Guide for ..., 1992-2021b).

Then, the identification of the DAM system was carried out using the data listed on the stock exchange between 5:01 and 6.00 hours recorded for all 365 days of 2019. The above was adopted for comparison of five identification methods to choose the best identification method. MISO type models were used, i.e. 24 input quantities representing the volume recorded in each hour of the day were taken into account (meaning simply the consecutive hourly periods between 0:00 and 24:00 hours), and at the output only one output value was taken into account, i.e. the volume-weighted average price of electricity obtained in a given hour of the day for electricity supplied and sold (in the case under consideration, for the 6^{th} hour, that is, between 5:01-6:00). The obtained results are presented in Table 1.

Table 1. The degree of matching of the obtained models to real data using selected identification methods for the 6^{th} hour (between 5:01 – 6.00) in 2019. Source: Own elaboration using MATLAB and System Identification Toolbox (Guide for ..., 1992-2021b)

Identification method	arx	arxnl	iv4	armax	arxqs
Degree of matching	75.66%	54.67%	51.08%	50.75%	66.96%

The best method turned out to be the parametric identification method arx, using which the match of 75.66% was obtained, and the worst method was armax with the match of 50.57%. For these reasons, the method of parametric identification arx was used in further research.

5. Identification of the DAM system using data from 2019

The DAM system has been identified for hourly data downloaded from TGE S.A. for all days, first in 2019, and then verified on data from 2020. As a result of identification, MISO type models of the DAM system were obtained for 2019 and as a result of verification for 2020 for each distinguished hour, i.e. for hours of the day: 6, 12, 18 , 24. For each of these hours, a discrete parametric model arx was determined, which was converted into a continuous parametric model, and this into a state-space continuous model. In total, three models were obtained for each hour of the day, i.e. 12 models for 2019 and, as a result of verification, 3 models for 2020[3].

Therefore, for the detailed analysis, performed for the purposes of this publication, four discrete hourly arx parametric models were used, i.e. for the hours: 6, 12, 18 and 24, and the corresponding continuous hourly parametric models and continuous models in the state space. As an example, parametric arx discrete and continuous models as well as continuous models of state variables for the 6^{th} hour are presented here, with discrete full parametric models for all four hours used in the simulation model.

So, for the 6^{th} hour in 2019, the parametric arx discrete model was obtained in the following form:

$$A_6(z) \cdot y_6(t) = B_1(z) \cdot u_1(t) + B_2(z) \cdot u_2(t) + ... + B_{24}(z) \cdot u_{24}(t) + e(t), \quad (4)$$

where:

$$\begin{aligned}
A_6(z) &= 1 - 0.3777 \cdot z^{-1} + 0.1551 \cdot z^{-2} - 0.1395 \cdot z^{-3} - 0.1729 \cdot z^{-4} + \\
&\quad +0.07633 \cdot z^{-5} - 0.0725 \cdot z^{-6}, \\
B_1(z) &= -0.1233 \cdot z^{-1} - 0.021 \cdot z^{-2} + 0.002389 \cdot z^{-3} - 0.003882 \cdot z^{-4} + \\
&\quad +0.045 \cdot z^{-5} + -0.0008132 \cdot z^{-6} - 0.01498 \cdot z^{-7} - 0.01678 \cdot z^{-8} - \\
&\quad -0.002096 \cdot z^{-9} + 0.0196 \cdot z^{-10}, \\
B_2(z) &= -0.02169 \cdot z^{-1} + 0.02292 \cdot z^{-2} - 0.005696 \cdot z^{-3} + 0.01106 \cdot z^{-4} - \\
&\quad -0.08478 \cdot z^{-5} + -0.002471 \cdot z^{-6} + 0.02813 \cdot z^{-7} + 0.04245 \cdot z^{-8} + \\
&\quad +0.02165 \cdot z^{-9} - 0.03317 \cdot z^{-10},
\end{aligned}$$

....

$$\begin{aligned}
B_{24}(z) &= -0.01645 \cdot z^{-1} - 0.000098 \cdot z^{-2} + 0.01285 \cdot z^{-3} + \\
&\quad +0.009628 \cdot z^{-4} + 0.009773 \cdot z^{-5} + -0.000536 \cdot z^{-6} + 0.001467 \cdot z^{-7} + \\
&\quad +0.001157 \cdot z^{-8} + 0.003415 \cdot z^{-9} + 0.01136 \cdot z^{-10}.
\end{aligned}$$

[3]Due to the size of the article, the obtained test results are not included in this publication.

When converting the parametric discrete arx model to the continuous arx parametric model using the form function[4]:

th6101h62019=d2c(arx6101h62019,'tustin') (5)

the following model was obtained:

$$y_6(t) = \frac{B_1(s)}{F(s)} \cdot u_1(t) + \frac{B_2(s)}{F(s)} \cdot u_2(t) + ... + \frac{B_{24}(s)}{F(s)} \cdot u_{24}(t) + \frac{C_1(s)}{D_1(s)} \cdot e(t), \quad (6)$$

where:

$$\begin{aligned} B_1(s) = &-0.02731 \cdot s^{10} - 0.2861 \cdot s^9 + 2.816 \cdot s^8 - 9.212 \cdot s^7 + 49.11 \cdot s^6 + \\ &- 103.4 \cdot s^5 + 198.3 \cdot s^4 - 254.7 \cdot s^3 - 42.29 \cdot s^2 - 81.72 \cdot s - 3.196, \end{aligned}$$

$$\begin{aligned} B_2(s) = &\ 0.03977 \cdot s^{10} + 0.501 \cdot s^9 - 6.2 \cdot s^8 + 18.33 \cdot s^7 - 90.71 \cdot s^6 + \\ &+ 193.0 \cdot s^5 - 306.8 \cdot s^4 + 339.7 \cdot s^3 + 169.0 \cdot s^2 + 36.33 \cdot s + 14.83, \end{aligned}$$

....

$$\begin{aligned} B_{24}(s) &= 0.006079 \cdot s^{10} + 0.03525 \cdot s^9 + 2.183 \cdot s^8 - 2.081 \cdot s^7 + 32.8 \cdot s^6 + \\ &\quad -58.96 \cdot s^5 + 74.968 \cdot s^4 - 211.2 \cdot s^3 + 19.52 \cdot s^2 - 68.22 \cdot s + 19.93, \\ C_1(s) &= s^6 + 12 \cdot s^5 + 60 \cdot s^4 + 160 \cdot s^3 + 240 \cdot s^2 + 192 \cdot s + 64, \\ D_1(s) &= s^6 + 11.04 \cdot s^5 + 41.99 \cdot s^4 + 107.2 \cdot s^3 + 128.6 \cdot s^2 + 125.3 \cdot s + 13.46, \\ F(s) &= s^{10} + 19.04 \cdot s^9 + 154.3 \cdot s^8 + 740 \cdot s^7 + 2363 \cdot s^6 + 5246 \cdot s^5 + \\ &\quad 8203 \cdot s^4 + 8943 \cdot s^3 + 6389 \cdot s^2 + 2435 \cdot s + 215.4, \end{aligned} \quad (7)$$

Thus, as a result of converting the parametric model of the discrete arx into the parametric model of continuous *th*, and this into the continuous model in the state space by means of the function[5] (see: Guide for ..., 1992-2022b):

[A B C D KRDN X0RDN]= th2ss(th6101h62019). (8)

a model in the state space (ss) was obtained, consisting of 246 state variables (matrix **A** had dimensions of 246 x 246), however, due to the cyclically repeating groups of state variables (with identical 10 model coefficients), the model has

[4]The function d2c(arx6101h62019,'tustin') located in the Control System Toolbox library of the MATLAB environment converts the model named arx6101h62019 obtained as a result of identification in discrete form into a parametric model in continuous form named th6101h62019, the argument of this function, apart from the model name arx6101h62019, is the method tustin integration.

[5]The function th2ss(th6101h62019) located in the Control System Toolbox library of the MATLAB environment converts the parametric model in a continuous form named th6101h62019 into a continuous model in the state space (ss), i.e. into the appropriate equation matrices of state variables and the output equation, i.e. **A, B, C, D, KRDN, X0RDN**.

been simplified to the first 10 state variables, and, consequently, to the following 10 state equations:

$$\frac{dx_1(t)}{dt} = -19.0379 \cdot x_1(t) - 9.6436 \cdot x_2(t) - 5.7815 \cdot x_3(t) - 4.6152 \cdot x_4(t)$$
$$-2.5617 \cdot x_5(t) + -2.0026 \cdot x_6(t) - 1.0917 \cdot x_7(t) - 0.7799 \cdot x_8(t)$$
$$-0.5945 \cdot x_9(t) - 0.2103 \cdot x_{10}(t) + u_1(t), \tag{9}$$

$$\frac{dx_2(t)}{dt} = -16 \cdot x_1(t),$$
$$\frac{dx_3(t)}{dt} = -19 \cdot x_2(t),$$
$$\frac{dx_4(t)}{dt} = 4 \cdot x_3(t),$$
$$\frac{dx_5(t)}{dt} = 4 \cdot x_4(t), \tag{10}$$
$$\frac{dx_6(t)}{dt} = 2 \cdot x_5(t),$$
$$\frac{dx_7(t)}{dt} = 2 \cdot x_6(t),$$
$$\frac{dx_8(t)}{dt} = x_7(t),$$
$$\frac{dx_9(t)}{dt} = 0.5 \cdot x_8(t),$$
$$\frac{dx_{10}(t)}{dt} = 0.25 \cdot x_9(t),$$

and for 6 o'clock in 2019, the output equation takes the following form:

$$y_6(t) = 0.2338 \cdot x_1(t) + 0.4394 \cdot x_2(t) + 0.0859 \cdot x_3(t) + 0.2219 \cdot x_4(t)$$
$$+0.0195 \cdot x_5(t) + 0.1032 \cdot x_6(t) - 0.0013 \cdot x_7(t) + 0.0161 \cdot x_8(t) - 0.0037 \cdot x_9(t)$$
$$+0.026 \cdot x_{10}(t) + ... + -0.0273 \cdot u_1(t) + 0.0398 \cdot u_2(t) - 0.0053 \cdot u_3(t)$$
$$-0.0193 \cdot u_4(t) - 0.0036 \cdot u_5(t) + 0.0064 \cdot u_6(t) + 0.0215 \cdot u_7(t) + 0.0071 \cdot u_8(t)$$
$$-0.0108 \cdot u_9(t) - 0.0410 \cdot u_{10}(t) + 0.0470 \cdot u_{11}(t) - 0.0456 \cdot u_{12}(t) + 0.0643 \cdot u_{13}(t)$$
$$-0.0103 \cdot u_{14}(t) + 0.0055 \cdot u_{15}(t) - 0.00068 \cdot u_{16}(t) - 0.0226 \cdot u_{17}(t) - 0.0654 \cdot u_{18}(t)$$
$$+0.0720 \cdot u_{19}(t) + 0.0017 \cdot u_{20}(t) - 0.007 \cdot u_{21}(t) - 0.0037 \cdot u_{22}(t)$$
$$-0.0087 \cdot u_{23}(t) + 0.0061 \cdot u_{24}(t), \tag{11}$$

where: $x_1(t)$, $x_2(t)$,... , $x_{10}(t)$ – relevant state variables,
$u_1(t)$, $u_2(t)$,... , $u_{24}(t)$ – relevant input variables.

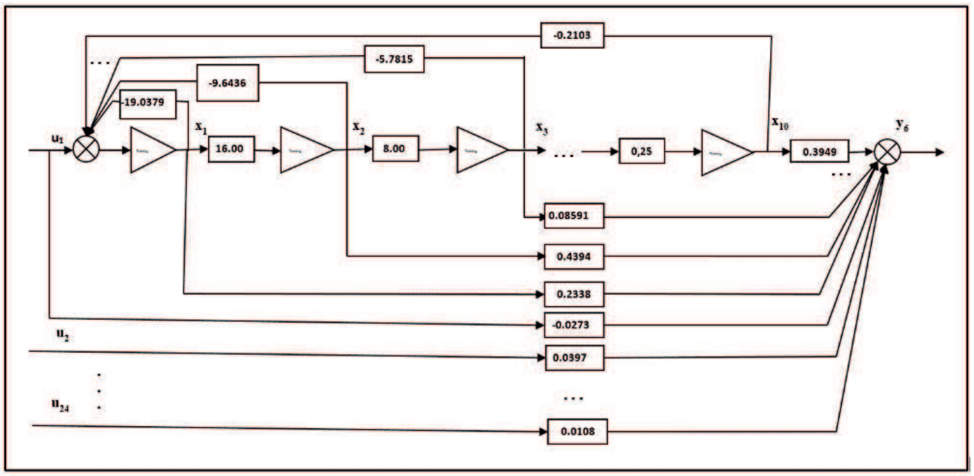

Figure 3. Block diagram of the state variables of the TGE S.A. Day-Ahead Market system Notations are provided in the text. Source: Own elaboration in terms of control and systems theory (see Kaczorek et al., 2021; Tchórzewski, 2013)

The model of the Day-Ahead Market system in the state space described by the state equations (6) and the output equation (7) is shown in the form of a block diagram in Fig. 3.

One can notice the specific role of the input (control) variable from the first hour (u_1) on the formation of the output (response) y_6 of the DAM system, as well as the presence of the negative feedback from all state variables to the system input, and also the direct influence of the input (control) signals on the output signal (the presence of a non-zero matrix **D**), with the elements of the matrix **D** being close to zero (these values are of the order of magnitude of $10^{-2} \div 10^{-3}$).

6. Full model and interpretation of DAM model parameters in the state space

The analysis of the structure and parameters of matrix **A** shows that the degree of internal organization of the state of the DAM system in 2019 and its changes were determined mainly by one derivative of the state variable, i.e. $dx_1(t)/dt$, which was influenced by all state variables adopted in the model, their number being established at 10, and the elements of the first row of matrix **A**, related to it. The derivatives of the remaining state variables were affected by the single state variables and the elements of matrix **A** associated with them, i.e. $dx_2(t)/dt$ was affected by the state variable $x_1(t)$, then, $dx_3(t)/dt$ was affected by the state variable $x_2(t), \ldots$, and, finally, $dx_{10}(t)/dt$ was affected by the state variable $x_9(t)$.

In order to examine the nature of the state variables, it was first established that matrix **B** shows that only the input variable $u_1(t)$ and the associated element b_{11} of matrix **B** affects the level of control, while all input quantities affect the output from the model, because there is a non-zero matrix **D**, but with values close to zero.

Thus, the first equation of state, included in equations (6), shows, among others, that due to the fact that the input variable $u_1(t)$ concerns the volume of ep delivered for sale at 0:01-1:00 [in MWh], hence the state variable $x_1(t)$ can also be interpreted as the volume of ee delivered for sale at 0-1, assuming that element b_{11} of matrix **B** is expressed in [1/h] and element a_{11} of matrix **A** is expressed in [1/h]. On this basis, subsequent state variables can also be interpreted as volumes of electricity delivered for sale at 0-1, assuming that the subsequent elements of matrix **A** (a_{12}, a_{13}, ..., $a_{1,10}$) are also expressed in [1/h].

Due to the fact that the subsequent equations of state variables, x_{11}, $x_{12,\ldots,}\, x_{20}$, are as follows:

$$\begin{aligned}\frac{dx_{11}(t)}{dt} &= -19.0379 \cdot x_{11}(t) - 9.6436 \cdot x_{12}(t) - 5.7815 \cdot x_{13}(t) \\ &-4.6152 \cdot x_{14}(t) - 2.5617 \cdot x_{15}(t) + -2.0026 \cdot x_{16}(t) - 1.0917 \cdot x_{17}(t) \\ &-0.7799 \cdot x_{18}(t) - 0.5945 \cdot x_{19}(t) - 0.2103 \cdot x_{20}(t) + u_2(t),\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{dx_{12}(t)}{dt} &= -16 \cdot x_{11}(t), \\ \frac{dx_{13}(t)}{dt} &= -19 \cdot x_{12}(t), \\ \frac{dx_{14}(t)}{dt} &= 4 \cdot x_{13}(t), \\ \frac{dx_{15}(t)}{dt} &= 4 \cdot x_{14}(t), \\ \frac{dx_{16}(t)}{dt} &= 2 \cdot x_{15}(t), \\ \frac{dx_{17}(t)}{dt} &= 2 \cdot x_{16}(t), \\ \frac{dx_{18}(t)}{dt} &= x_{17}(t),\end{aligned} \tag{13}$$

$$\frac{dx_{19}(t)}{dt} = 0.5 \cdot x_{18}(t),$$
$$\frac{dx_{20}(t)}{dt} = 0.25 \cdot x_{19}(t),$$

the output equation then has the following additional elements in terms of the elements of matrix **C**:

$$\begin{aligned} & y_6(t) = ... - 0.2562 \cdot x_{11}(t) - 0.771 \cdot x_{12}(t) - 0.0868 \cdot x_{13}(t) \\ & -0.3607 \cdot x_{14}(t) - 0.0076 \cdot x_{15}(t) + -0.1546 \cdot x_{16}(t) - 0.0020 \cdot x_{17}(t) \\ & -0.0104 \cdot x_{18}(t) - 0.0148 \cdot x_{19}(t) + 0.0061 \cdot x_{20}(t) + ... \end{aligned} \tag{14}$$

Hence, from the first equation of state, included in equations (13), it follows, among others, that due to the fact that the input variable $u_2(t)$ concerns the volume of ep [MWh] delivered for sale between 1:01 – 2.00 o'clock, the state variable $x_{11}(t)$ can also be interpreted as the volume of ee delivered for sale between 1:01-2:00 o'clock, assuming that the element $b_{1,11}$ of matrix **B** is expressed in [1/h], and element $a_{1,11}$ of matrix **A** is also expressed in [1/h].

On this basis, subsequent state variables can also be interpreted as volumes of electricity delivered for sale at 1-2 hours, assuming that subsequent elements of matrix **A** ($a_{1.12}$, $a_{1.13}$, ..., $a_{1.20}$) are also expressed in [1/h].

This situation is repeated until the equations of state:

$$\begin{aligned} & \frac{dx_{231}(t)}{dt} = -19.0379 \cdot x_{231}(t) - 9.6436 \cdot x_{232}(t) - 5.7815 \cdot x_{233}(t) \\ & -4.6152 \cdot x_{234}(t) - 2.5617 \cdot x_{235}(t) + -2.0026 \cdot x_{236}(t) - 1.0917 \cdot x_{237}(t) \\ & -0.7799 \cdot x_{238}(t) - 0.5945 \cdot x_{239}(t) - 0.2103 \cdot x_{240}(t) + 0.5 \cdot u_{24}(t), \end{aligned}$$

$$\begin{aligned} & \frac{dx_{232}(t)}{dt} = -16 \cdot x_{231}(t), \\ & \frac{dx_{233}(t)}{dt} = -19 \cdot x_{232}(t), \\ & \frac{dx_{234}(t)}{dt} = 4 \cdot x_{233}(t), \\ & \frac{dx_{235}(t)}{dt} = 4 \cdot x_{234}(t), \\ & \frac{dx_{236}(t)}{dt} = 2 \cdot x_{235}(t), \\ & \frac{dx_{237}(t)}{dt} = 2 \cdot x_{236}(t), \\ & \frac{dx_{238}(t)}{dt} = x_{237}(t), \end{aligned} \tag{15}$$

$$\frac{dx_{239}(t)}{dt} = 0.5 \cdot x_{238}(t),$$
$$\frac{dx_{240}(t)}{dt} = 0.25 \cdot x_{239}(t),$$

and then the output equation has the following additional elements in terms of elements of matrix **C**, respectively:

$$y_6(t) = ... - 0.2562 \cdot x_{231}(t) - 0.771 \cdot x_{232}(t) - 0.0868 \cdot x_{233}(t)$$
$$-0.3607 \cdot x_{234}(t) - 0.0076 \cdot x_{235}(t) + -0.1546 \cdot x_{236}(t) - 0.0020 \cdot x_{237}(t)$$
$$-0.0104 \cdot x_{238}(t) - 0.0148 \cdot x_{239}(t) + 0.0061 \cdot x_{240}(t) + ..., \quad (16)$$

which involves taking into account the impact of subsequent input quantities on the derivatives of successive state variables, of which the last input quantity, i.e. $u_{24}(t)$, has an impact on the derivative of the state variable $x_{231}(t)$, which also applies to the appropriate interpretation of the state variables based on the volume of electricity from the next hour until 23-24 hours, followed by the following equations of state:

$$\frac{dx_{241}(t)}{dt} = -19.0379 \cdot x_{241}(t) - 9.6436 \cdot x_{222}(t) - 5.7815 \cdot x_{243}(t)$$
$$-4.6152 \cdot x_{244}(t) - 2.5617 \cdot x_{245}(t) - 2.0026 \cdot x_{246}(t),$$

$$\frac{dx_{242}(t)}{dt} = -16 \cdot x_{241}(t),$$
$$\frac{dx_{243}(t)}{dt} = -19 \cdot x_{242}(t),$$
$$\frac{dx_{244}(t)}{dt} = 4 \cdot x_{243}(t), \quad (17)$$
$$\frac{dx_{245}(t)}{dt} = 4 \cdot x_{244}(t),$$
$$\frac{dx_{246}(t)}{dt} = 2 \cdot x_{245}(t),$$

which is no longer related to taking into account the influence of the input quantities on the derivatives of subsequent state variables, because the last input quantity has already been used in the first equation of state variables (17), and therefore the output equation has the following additional elements in the range of elements only included in the matrix **C**:

$$y_6(t) = ... - 11.0379 \cdot x_{241}(t) - 5.2493 \cdot x_{242}(t) - 3.3492 \cdot x_{243}(t)$$
$$-2.0087 \cdot x_{244}(t) - 0.9788 \cdot x_{245}(t) + -0.4206 \cdot x_{246}(t).$$

Therefore, in this case, due to the nature of the output variable $y_1(t)$, which is the volume weighted average ee at hours 5-6, the state variables $x_{241}(t)$, $x_{242}(t)$, ..., $x_{246}(t)$ can be taken as the volumes of ee delivered and sold at hours 5-6, assuming that the corresponding elements of the matrix **C** are expressed in [h], and the corresponding elements of the matrix **C** are expressed in [1/h].

Of course, an in-depth interpretation can be adopted due to the possibility of differentiating state variables by adopting a different interpretation of individual elements of matrix **A**, which goes beyond the scope of this publication. Examples of in-depth interpretation of state variables are presented on the example of the development of the power system in Tchórzewski (2013).

7. Simulation of the DAM system model using data from 2019

The block diagram of the DAM system model for 2019 is built with the use of appropriate Simulink blocks, which are in fact appropriate Matlab m-files, including: From Workspace, To Workspace, Demux, Mux, Math Function, Abs, Gain, Product, Constant, Divide, Transport Delay, Scope, Sum, Idpoly, NNET, Subsystem, Inport, Outport, Hitch link, etc. (see Appendix 1 and Appendix 2 to this paper).

The hourly model of the DAM system in the general form includes 17 subsystems of the following eight types:

- volume separator ep from hourly data of the DAM system (one subsystem),
- hourly efficiency generator of the DAM system (one subsystem),
- hourly efficiency mileage meter of the DAM system (one subsystem),
- a measure of the relative error of robustness between the model and the DAM system (four subsystems),
- price separator ep for hourly data of the DAM system (one subsystem),
- a measure of the relative efficiency error between the model and the DAM system (four subsystems),
- generator of model quality assessment indicators for the DAM system (efficiency, effectiveness, robustness, etc.) (four subsystems),
- hourly model of the DAM system (one subsystem),

Appendix 1 contains the scheme of the simulation models of individual subsystems, and their descriptions are provided in Appendix 2.

Each of the above-mentioned subsystems is built of an appropriate number of successive relevant subsystems and other accompanying elements, forming a hierarchical model of the DAM system. The block diagram of the DAM system was used to carry out, inter alia, simulation tests and sensitivity tests. The stimulation study was performed on data on the volume of electricity supplied and sold in 2019, including the data for hours: 6:00, 12:00, 18:00 and 24:00, and a sensitivity study was based on ep volume data for 2020.

8. Hourly model of the DAM system based on the 2019 data

The hourly model of the DAM system determines: DAM system models for each hour of the day (as already specified, the Subsystem includes four selected hours of the daily cycle, i.e. 6, 12, 18 and 24). This subsystem was built using the following blocks: In, Subsystem designating the hourly model of the DAM system, Subsystem designating the relative error of the model to the system, Scope, ToWorkspace, Out. The Subsystem that determines the model of the DAM system consists of the following blocks: In, Idmodel, Gain, Custom Neural Network, Scope, ToWorkspace and Out (Fig. 2).

The basic block of this subsystem is the Idmodel described by the function:

$$\text{yidmodelh62019=idpoly(arx6101h62019)}, \tag{18}$$

whose argument is a parametric model with an input containing 24 input quantities related to the volume of delivered and sold ee in particular hours of the day, in the here considered example – in 2019.

9. Simulation tests of the accuracy of the DAM system model

The simulation tests of the accuracy of the DAM system model were carried out for each hour of the day, and the present paper includes selected test results for the four indicated hours, i.e. for the hours 6:00, 12:00, 18:00 and 24:00 of the day. In order to compare the accuracy of the model with respect to the system, the simulation model was used, including the hourly model of Fig. 4 of the subsystem to measure the absolute and relative error of the DAM system model, shown in Fig. 5. In each of the individual cases considered, the absolute error or discrepancy between the output from the model and the output from the system was determined first, and then the relative error. The time histories of the relative errors for the above-mentioned four hours of the day are shown in Figs. 7 through 10.

The obtained annual results, as well as the results for a randomly selected month (December 2019) and for a randomly selected week, accounting for the exclusion of working days (the first full week of December) are presented in Table 2.

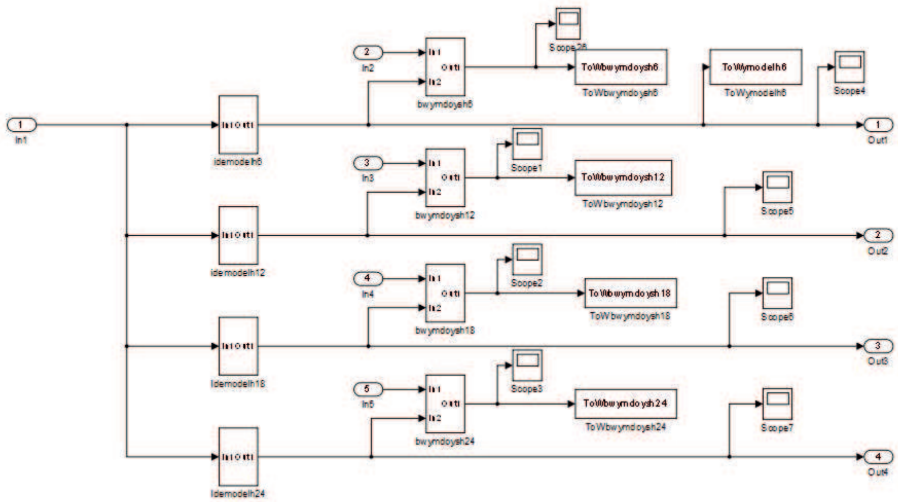

Figure 4. Hourly model of the DAM system. Symbols are explained in the text. Source: own study using Simulink Environment (see Guide for . . . , 1992-2021b)

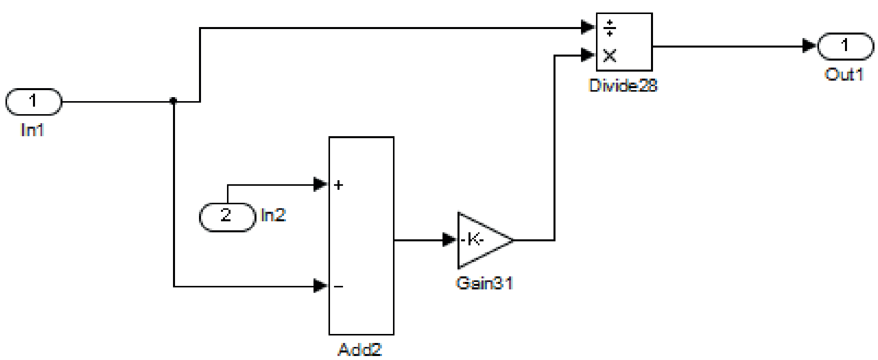

Figure 5. Subsystem designating the relative error of the model with respect to the system. Notations are explained in the text. Source: own study using the Simulink environment (see Guide . . . , 1992-2021b)

The determined relative errors for the individual four hours for 2019 for the arx parametric model of the discrete DAM system (corrected with ANN) ranged from 5.39% to 10.54%. In addition, the MAPE errors of the DAM system model and the model corrected with the ANN were determined for the hours

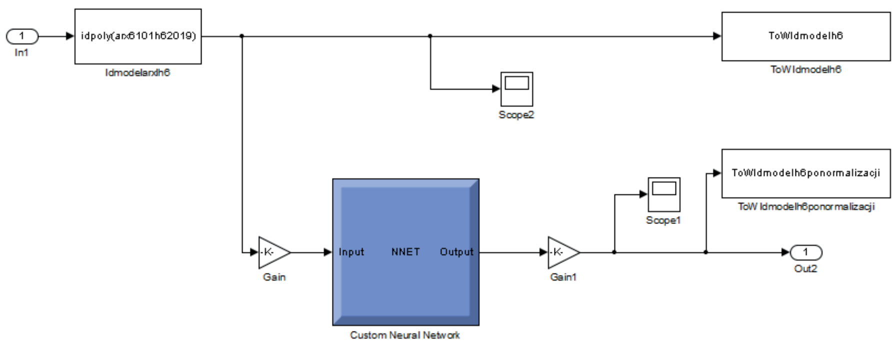

Figure 6. Subsystem designating the model of the DAM system. Notations are explained in the text. Source: own study using the Simulink environment (see Guide . . . , 1992-2021b)

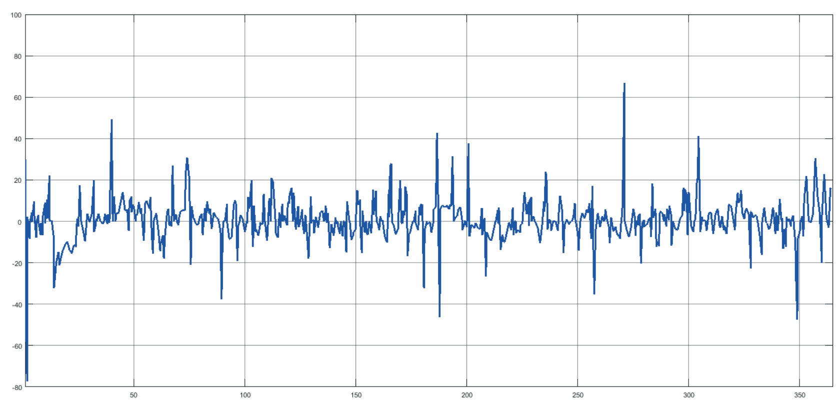

Figure 7. Curves of the relative error of the model in relation to the DAM system for 6 o'clock in 2019. Notation: X axis - long time [year]. Source: Own study using the Simulink environment (see Guide for . . . , 1992-2021b)

6:00, 12:00, 18:00, 24:00 of 2019. The values of the MAPE error for selected periods of 2019 are presented in Table 3.

In terms of summarizing the results, provided in Tables 2 and 3, we shall now list the ranges of the corresponding errors for various criteria, models and time periods modelled.

Relative errors ranged from the value of 5.39% for 6 hours of 2019 to 10.54% for 12 hours of 2019.

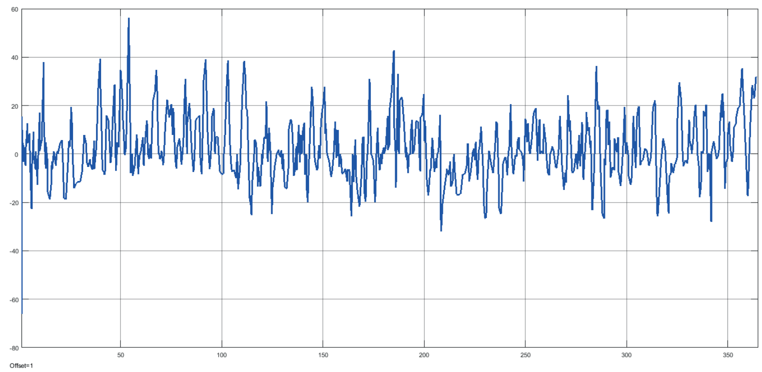

Figure 8. Curves of the relative error of the model in relation to the DAM system for 12 o'clock in 2019. Notation: X axis - long time [year]. Source: Own study using the Simulink environment (see Guide for . . . , 19922-2021b)

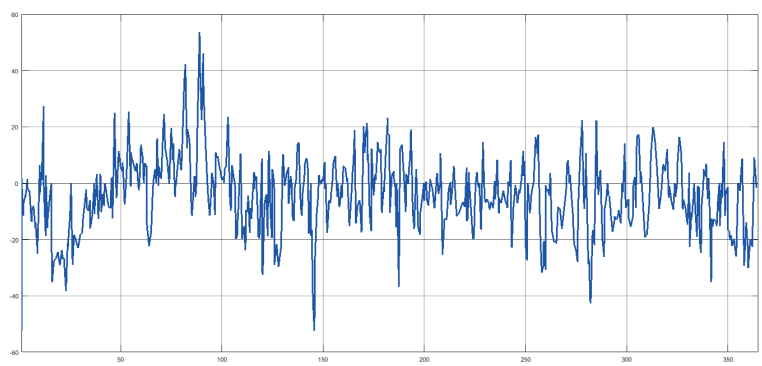

Figure 9. Runs of the relative error of the model in relation to the DAM system for 6 pm in 2019. Notation: X axis - long time [year]. Source: Own study using the Simulink environment (see Guide for . . . , 1992-2021b)

Moreover, errors were determined for selected months of 2019, and these errors ranged from 3.46% for 6 hours in December 2019 for the adjusted DAM system model up to the value of 13.69% for 12 o'clock in December 2019 for the DAM system model.

Errors were also determined for selected weeks (the first full week of December) of 2019, and these errors ranged from the value of 0.71% for 6 o'clock in December 2019 for the corrected DAM system model up to the value of 14.91% for 12 o'clock in December 2019 for the uncorrected model of the DAM system.

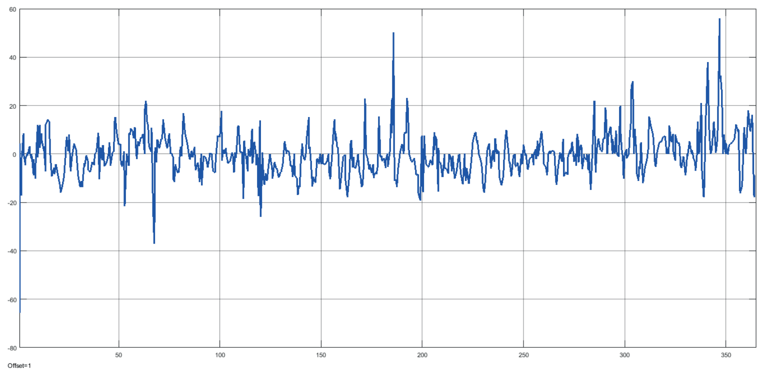

Figure 10. Curves of the relative error of the model in relation to the DAM system for 24 o'clock in 2019. Notation: X axis - long time [year]. Source: Own study using the Simulink environment (see Guide for . . . , 1992-2021b)

Table 2. Summary of relative errors [in %] for selected periods (2019, December, 1^{st} full week). Source: own elaboration using MATLAB and Simulink (see Guide for . . . , 1992-2021b)

Model	**year**	**month**	**month (working days)**	**week**	**week (working days)**
Idmodel h6	5.46	4.86	4.81	3.01	2.85
Idmodel h6 with ANN correction	5.30	3.46	2.52	0.71	1.26
Idmodel h12	11.29	13.69	13.92	11.55	11.43
Idmodel h12 with ANN correction	10.54	18.45	20.63	20.85	23.20
Idmodel h18	11.77	12.40	12.44	14.91	18.85
Idmodel h18 with ANN correction	9.68	8.24	7.04	9.61	10.27
Idmodel h24	6.18	11.02	11.70	11.61	12.10
Idmodel h24 with ANN correction	6.39	11.15	10.83	8.99	10.18

Taking into account only working days in a month, the errors ranged from the value of 2.52% to the value of 13.92%, and during the week they ranged from the value of 1.26% to the value of 11.42%.

Table 3. Summary of MAPE errors [%] for selected periods of 2019 (2019, December, 1^{st} full week). Source: own elaboration using MATLAB and Simulink (see Guide for . . . , 1992-2021b)

Model	**2019**	**December 2019**	**only working days Dec. 2019**	**1-st full week Dec. 2019**	**only working days 1-st full week Dec. 2019**
Idmodel h6	5.46	6.57	6.15	3.01	2.85
Idmodel h6 with ANN correction	5.39	6.81	6.51	3.77	4.58
Idmodel h12	11.50	13.07	14.33	11.24	13.91
Idmodel h12 with ANN correction	9.55	12.21	12.49	12.81	14.48
Idmodel h18	11.85	8.30	8.60	3.65	3.53
Idmodel h18 with ANN correction	9.92	11.86	11.99	5.98	5.77
Idmodel h24	6.94	7.89	7.93	8.80	8.89
Idmodel h24 with ANN correction	6.31	11.07	11.55	5.99	5.57

The MAPE errors ranged from the value of 5.39% for 6 o'clock for 2019 to 9.55% for 12 o'clock in 2019.

Moreover, MAPE errors were determined for selected months of 2019, with values from 6.57% for 6 o'clock in December 2019 for the DAM system model to 12.21% for 12 o'clock in December 2019 for the adjusted DAM system model.

The relative MAPE errors were determined for selected weeks (the first full week of December) of 2019, and they ranged, respectively, from the value of 3.01% for 6 o'clock in December 2019 to the value of 11.24% for 12 o'clock in December 2019 for the DAM system model.

Taking into account only the working days in the month the relative MAPE errors ranged from the value of 6.15% to the value of 12.49% and during the week they ranged from the value of 2.85% to the value of 13.91%.

At this point, Table 4 shows a comparison of the results, obtained by the present authors, with a selection of those available in the literature, focusing on the MAPE error values.

Thus, when analyzing the content of Table 4, one can notice, in particular, that the arx model, which was obtained by the present authors on the basis of the data for the year 2019, has the MAPE error values, which are compared as follows with the other results, for the following lengths of the forecast:

year - 5.39%, but when corrected with ANN - 5.03%, so it is definitely a smaller error than in the case of S. Voronin's (2013) model: ARMA - 13.74%, GARCH - 16.17%, ARMA + GARCH - 12.84%,

month - 3.46%, meaning that it is an error comparable with that of K. Halicka's (2006) neural model: ANN I - 3.69%, ANN II - 3.35%, and month (working days only) – 2.52%,

week - 0.71% and for the working days week - 1.26%, so it is a lower error than that obtained by T. Popławski and M. Weżgowiec (2015) for the additive Winters model - 4.51%, the multiplicative Winters model - 7.01%, and the Holt model - 2.98 %.

It is worth noting, however, that the main goal of the research, reported in this paper, was not to achieve the results characterized by the lower MAPE errors than those characterising the existing models, but to obtain, on the one hand, a database for meta-identification in order to build a meta-model of the Day-Ahead Market system, and, on the other hand, to obtain models of the Day-Ahead Market system in the space of states, which have very interesting interpretations, e.g. in the case of the currently developed multi-source system of the Electricity Market in the world.

10. The testing study of the DAM system model

The testing study experiments of the DAM system model were carried out on the DAM system model for 6 o'clock in 2019 for contractual figures regarding the volume of delivered and sold ep in 2020.

The performance of the volume-weighted average price of ep, generated by the model in relation to the price actually generated by the DAM system was examined, with the assumption that the first fifteen input quantities relate to the initial conditions of the model (due to the delays occurring in both polynomials, $\boldsymbol{A}(z)$ as well as $\boldsymbol{B}(z)$). The resulting MAPE error was obtained and summarized in Table 5 for the selected periods of 2020.

Table 4. Selected results of comparative studies of the Day-Ahead Market system models, including the Nord Pool Spot models (see Marlęga, 2021)

Name of system	**Data for identification**	**Model type**	**Data for forecasting**	**MAPE [%]**	**Environment**
Day-Ahead Market (Ejdys, Halicka and Godlewska, 2015)	16 months	Holt-Winters ANN	day	37.32 4.71	STATISTICA
Day-Ahead Market (Popławski and Weżgowiec, 2015)	quarter	Winters Additive Winters Multiplicative Holt Model	week (working days)	4.51 7.01 2.98	-
Day Ahead Market (Halicka, 2006)	year	ANN I ANN II	month	3.69 3.35	STATISTICA
Nord Pool Spot (Voronin, 2013)	11 years	ARMA Model GARCH Model ARMA+GARCH	year	13.74 16.17 12.84	-
Day Ahead Market (Marlęga, 2021)	year	ARX Model ANN corrected	year	5.39 5.03	MATLAB
			month	3.46	
			month (working days)	2.52	
			week	0.71	
			week (working days)	1.26	

Table 5. List of MAPE errors for selected periods of 2020 (year, December, week two) [%]. Source: Own elaboration using MATLAB and Simulink (see Guide for . . . , 1992-2021b)

Model used for 6 o'clock	**2019**	**December 2019**	**Only working days December 2019**	**1st full week December 2019**	**Only working days 1st full week December 2019**
Idmodel	32.00	7.00	6.70	10.05	4.58
Idmodel with ANN correction	31.33	7.24	5.99	6.96	3.83

11. Summary

As a result of the parametric identification using the arx model for the data recorded on the DAM in the period from January 1, 2019, to December 31, 2019, four DAM discrete-type system models were obtained for the hours 6:00, 12:00, 18:00 and 24:00 of the day, respectively, in 2019. In a detailed analysis of these models, in particular, absolute errors, relative errors and MAPE errors for the whole year were determined for randomly selected months and weeks.

Simulation tests on the Day-Ahead system model carried out for the hours of the day in 2019 made it possible to determine the absolute error value and thus the discrepancy between the output from the model and the output from the system, and then the values of relative errors. The determined relative errors for individual four hours for the entire year 2019 for the arx parametric model of the DAM adjusted by ANN ranged from 5.39% to 10.54%, and in December 2019 from 3.46% to 18.45%.

The smallest relative error was obtained at 06:00 in the first full week of December 2019, amounting to 0.71%, and the highest at 12:00 in the first full week of December 2019 (counting only working days) amounting to 30.2%, which shows that one can model the RDN system much more accurately at 6:00 a.m. than at 12:00 p.m. These significantly different results show, among others, that the DAM system can be modeled much more accurately at 6:00 hours than at 12:00 hours, this fact being strongly associated with similar behavior of people during peak hours and quite different behavior during off-peak hours. Nevertheless, investigations, related to these differences have not been conducted, and they may constitute a new direction of research.

In addition, the DAM system models obtained on the basis of data from 2019 were tested on the basis of data from 2020. In particular, it was checked what was the accuracy of the outputs obtained from the DAM system for the inputs from 2020 in relation to the outputs for the data from 2019, which is very important from the point of view of the possibility of using the parameters of the DAM system models in meta-identification (see Tchórzewski and Marlęga, 2019b), in the search of metamodels of the DAM system.

The relative error for the first full week of December 2019 (only working days) for the parametric model was 4.58%, and after correcting the model with the Perceptron ANN, 3.83%, so it was relatively low. On the other hand, the relative error obtained using the arx parametric model obtained for 6 o'clock in 2019 and for the 2020 data was 32.00%, and after correcting with the Perceptron ANN it was still at 31.33%.

The relative error was also relatively small for the month of December, as it amounted to 7.00%, and for the working days of December, 6.77% (after correcting the model with ANN - 5.99%), which indicates that this type of modeling can be used for the purposes of meta-identification in order to obtain models of the DAM system using metamodels.

Research in this direction continues. An important and still topical issue is also to study the effectiveness, efficiency and robustness of the models and respective systems (see Sienkiewicz, 1987; Staniszewski, 1990; Toczyłowski, 2008; Tchórzewski and Marlęga, 2021).

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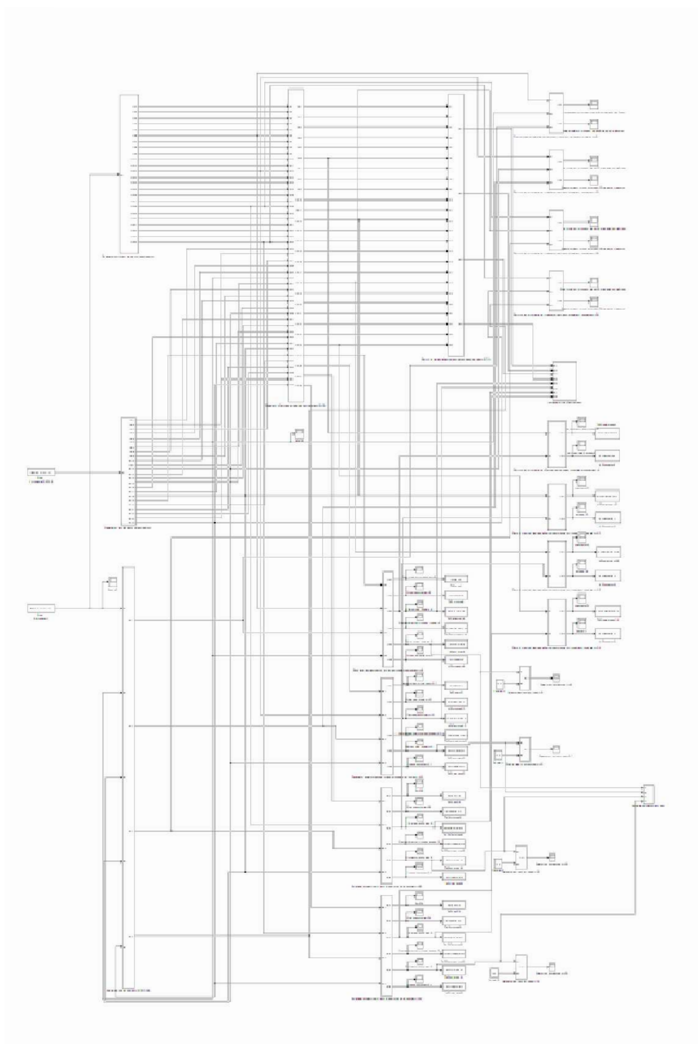
Appendix 1

Simulation model of the DAM system for hours 6, 12, 18 and 24 in 2019. Notations explained in Appendix 2. Source: Own elaboration using Simulink environment (see: Guide . . . , 2002-2021b)



Appendix 2

Selected Simulink blocks used to build a simulation model of the Day-Ahead Market system[6]

Abs, which generates the absolute value of the output. In the modeled cases, this block was used to obtain the absolute value of the discrepancy between two signals;

Constant, which generates a real or composite constant value. The output can be scalar, vector or matrix signals. The block output has the same dimensions and elements as the input signals, in the case under consideration they were constant values;

Demux, which extracts the components of the input vector signal and outputs separate signals. The output ports are ordered top to bottom. In the considered example, the input signal to the block is a matrix containing 24 quantities related to the input signal u (volume of sold ee), and the output signals to individual volumes ee for each hour of the day;

Divide, which divides the signal values entered into the numerator (marked x) by the signal values entered into the denominator (marked: ÷). The block exit is the signal that is the result of the division. The numerator and denominator must have the same dimensions. In the case under consideration, this block was used to determine the effectiveness of the DAM system;

From Workspace, which reads data values specified in time series, matrices or other forms from the MATLAB workspace, e.g. [tpp365 u242019], where: tpp365 - data retrieval control vector, u242019 - matrix of input values for the volume sold ee in individual hours of the day in 2019;

Gain, which is the proportional term used to multiply a signal by a number or matrix of numbers. In the case under consideration, this block is used as a proportional element, i.e. describing the numerical values present in the model (e.g. data used to scale the input signal according to its parameter value), here: constant value 100. Input signal and block parameter (gain) it can be a scalar [???], vector, or matrix;

Hitch link, which splits the incoming scalar or vector signal into two identical signals (so-called tap);

Idpoly, which is described by the function idpoly (arx6101h62019) and thus is an implementation of the arx6101h62019 discrete parametric model in Simulink. It requires that its argument be stored in Workspace. The argument of the

[6]When designing the simulation model, appropriate Simulink blocks were used (which are in fact m-files of Matlab) in accordance with the adopted algorithm leading to obtaining a computer program in the Simulink environment, which could then be used in simulation and comparative studies and in testing the sensitivity of the system model to relevant input signals.

function, which is in fact a model of the DAM system, must be, inter alia, an idpoly object, as in the case under consideration for 6 o'clock 2019. This function is performed in Simulink by means of a block that generally supports models with or without continuous time input-output delays. The initial states must be a vector with a length equal to the model row. In the case of the considered idpoly model, the initial conditions are zero;

Inport, which is an input link to the subsystem from the system environment (most often from the master system). They are automatically numbered in the order from the highest level, starting with input 1. You can assign the port a sampling time as the rate at which the ambient signal is downloaded to the system. A value of -1 causes the block to inherit its sampling time from the controlling block;

Mux, which combines a plurality of scalar inputs into a single vector output, the input signal being a vector signal. In the discussed example, this block was used to compile the robustness waveforms of the signal from the DAM system model and from the DAM system measured for 6:00 a.m. 2019, so that both signals could run on one graph;

Math Function, which, as a block of mathematical functions, has typical mathematical functions assigned by means of parameters. In the case under consideration, this block was used in the corrector of the output signal from the neural model as a mathematical function of lifting to the power with the given parameter v;

NNET, which simulates the operation of an Artificial Neural Network learned to correct the output signal from a parametric arx discrete model. In the considered example, this model is arx6101h62019, that is the DAM system model for 6 am in 2019. This block adopts the parameters of ANN designed and implemented using NNT;

Outport, which is the output link from the subsystem to the system environment (most often to the master system). The outports are automatically numbered in order from the highest level, starting with input 1. You can assign a sampling time to the port as the rate at which the signal is sent from the system to the environment;

Product, which is used for the multiplication and division of scalar signals and non-scalar signals with the same dimensions or for matrix multiplication and inversion. In the discussed example, this block was used to compile the robustness waveforms of the signal from the DAM system model and from the DAM system measured for 6:00 a.m. 2019, so that both signals could run on one graph;

Scope, which displays the waveforms generated during the simulation. In the case under consideration, the displayed signal is the effectiveness of the DAM system model for 6 o'clock in 2019;

Sum, which enables the operation of adding and / or subtracting signals introduced into the block. The number and type of inputs are the parameters of the block as well as the type of the block shape. In the case under consideration, the block was used to determine the absolute error between the system efficiency and the model efficiency;

Subsystem, which is a subsystem consisting of several integrated Simulink blocks, e.g. Bwymdoysh6 is a subsystem containing a model for determining the absolute error and difference error between the arx parametric model and the DAM system for 6 o'clock in 2019;

To Workspace, which inputs the signal and writes the signal data to the MATLAB workspace. During simulation, the block writes data to an internal buffer. Block description e.g. ToWbwemdosh6 as the name of the output is passed to the MATLAB workspace, in this case the relative error between the model and the DAM system for 6 o'clock in 2019;

Transport Delay, which delays input by a predetermined time unit, e.g., a delay unit resulting from the $z - i$ time shift operator present in the arx parametric model. At the start of simulation, the block outputs a start output parameter until the simulation time exceeds the $z - i$ time delay parameter, then input delay is generated. In the case of discrete signals, the appropriate value is present at the output at the required time. In the models considered in the study, this block was used in the idpoly model as a subsystem.