Technical Sciences 16(4), 2013, 265–280

SINGLE-EPOCH PRECISE POSITIONING USING MODIFIED AMBIGUITY FUNCTION APPROACH

Sławomir Cellmer

Institute of Geodesy
Univeristy of Warmia and Mazury in Olsztyn

K e y w o r d s: GNSS data processing, Ambiguity Function, MAFA method.

Received 27 June 2013; accepted 29 October 2013; available on line 31 October 2013

A b s t r a c t

Single-epoch positioning is a great challenge in recent research related to GNSS data processing. The Modified Ambiguity Function Approach (MAFA) method can be applied to perform this task. This method does not contain a stage of ambiguity resolution. However the final results take into account their integer nature. The functional model of the adjustment problem contains the conditions ensuring the integer nature of the ambiguities. A prerequisite for obtaining the correct solution is a mechanism ensuring appropriate convergence of the computational process. One of such mechanisms is a cascade adjustment, applying the linear combinations of the L1 and L2 signals with the integer coefficients and various wavelengths. Another method of increasing the efficiency of the MAFA method is based on the application of the integer de-correlation matrix to transform observation equations into equivalent, but better conditioned, observation equations. The next technique of improving the MAFA method is search procedure. This technique together with the de-correlation procedure allows to reduce the number of stages of the cascade adjustment and to obtain correct solution even in the case when *a priori* position is a few meters away from the actual position. This paper presents some problems related to search procedure. The results of single-epoch positioning using improved MAFA method are presented.

Introduction

The MAFA method is based on the least squares adjustment (LSA) with condition equations in the functional model of the adjustment problem (CELLMER et al. 2010, CELLMER 2012a). This ensures that the condition of the ambiguity 'integerness' is satisfied in the final results. The functional model for the carrier phase adjustment is relatively weak. Therefore different linear combinations (LC) of L1 and L2 GPS carrier phase observations are applied in

* Corespondence: Sławomir Cellmer, Instytut Geodezji, Uniwersytet Warmińsko-Mazurski, ul. Oczapowskiego 1, 10-719 Olsztyn, phone: 48 89 524 52 83, e-mail: slawomir.cellmer@uwm.edu.pl

the cascade adjustment so that the appropriate convergence of the computational process can be assured (HAN, RIZOS 1996, JUNG, ENGE 2000). Another method of improving the efficiency of the MAFA method was proposed by CELLMER (2011b). This technique exploits an integer de-correlation procedure. After transformation of the observation equations with an integer de-correlation matrix, a model of an adjustment problem turns into an equivalent model, but a better conditioned one. There are some limitations in applying the MAFA method. CELLMER (2012a) derived the necessary condition for obtaining correct solution with MAFA method. This condition describes the relationship between the accuracy of the *a priori* position and the wavelength of LC forming an observation set. The *a priori* position must be placed inside a certain region around the actual position. Therefore the approximate position in carrier phase process should be as good as possible. The accuracy of the approximate position can be increased using Network Code DGPS Positioning (BAKUŁA 2010). However this accuracy can be still insufficient for MAFA method, even if the de-correlation procedure and the cascade adjustment are applied. Therefore, the search procedure is proposed, as the technique of overcoming this problem. This procedure is based on testing the objective function values for different vectors of misclosures in the functional model of the adjustment problem. This procedure allows obtaining correct solution, even if the *a priori* position is a few meters away from the actual position. The next section presents the theoretical basis of the MAFA method followed by the description of the techniques improving its efficiency. In the third section the search procedure is presented. In the last part of the paper a numerical example, the results of the tests and some conclusions are given.

Theoretical basis of the MAFA method

The following simplified form of the observation equation for double differenced (DD) carrier phase observable is assumed (HOFMANN-WELLENHOF et al 2008, LEICK 2004, TEUNISSEN 1998):

$$\Phi + v = \frac{1}{\lambda}\,\rho(\mathbf{X_c}) + N \tag{1}$$

where:

Φ – DD carrier phase observable (in cycles)

λ – length of the carrier wave

v – residual (measurement noise)

$\mathbf{X_c}$ – receiver coordinate vector
$\rho(\mathbf{X_c})$ – DD geometrical range
N – integer number of cycles (DD initial ambiguity)

Then taking into account the integer nature of the ambiguity parameter N and assuming that the residual values are much lower than half a cycle (HOFMANN-WELLENHOF et al. 2008), the Eq. (1) can be rewritten in the following form:

$$\Phi + v - \frac{1}{\lambda}\rho = \text{round}\left(\Phi - \frac{1}{\lambda}\rho\right) \tag{2}$$

or

$$v = \text{round}\left(\Phi - \frac{1}{\lambda}\rho\right) - \left(\Phi - \frac{1}{\lambda}\rho\right) \tag{3}$$

where round is a function of rounding to the nearest integer value. The residual (3) takes into account the integer nature of ambiguities. The right side of the Eq. (3) can be expressed in the form of the following, differentiable and continuous function (CELLMER 2011b):

$$\Psi = \text{round}(s) - s = \begin{cases} -\frac{1}{\lambda}\arcsin\left[\sin(\pi s)\right] \text{ for } s \in \{s : \cos(\pi s) \geq 0\} \\ \frac{1}{\lambda}\arcsin\left[\sin(\pi s)\right] \text{ for } s \in \{s : \cos(\pi s) < 0\} \end{cases} \tag{4}$$

where s is an auxiliary variable:

$$s = \Phi - \frac{1}{\lambda}\rho \tag{5}$$

Each of the nonlinear observation equation for double differenced carrier phase observables is linearized. After linearization, the general formula of the residual equations can be shown in the following form (CELLMER et al. 2010):

$$\mathbf{V} = \frac{1}{\lambda}\mathbf{AX} + \Delta \tag{6}$$

with:

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \rho_1}{\partial x} & \frac{\partial \rho_1}{\partial y} & \frac{\partial \rho_1}{\partial z} \\ \frac{\partial \rho_2}{\partial x} & \frac{\partial \rho_2}{\partial y} & \frac{\partial \rho_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial \rho_n}{\partial x} & \frac{\partial \rho_n}{\partial y} & \frac{\partial \rho_n}{\partial z} \end{bmatrix} \tag{7}$$

$$\Delta = round\ (\Phi - \frac{1}{\lambda}\rho) - (\ \Phi - \frac{1}{\lambda}\rho) \tag{8}$$

where:

V – residual vector ($n \times 1$),
X – parameter vector (increments to a priori coordinates vector X_0),
A – design matrix ($n \times 3$),
Δ – misclosure vector ($n \times 1$),
ρ_0 – DD geometric distance vector computed using *a priori* position and satellite coordinates.

The LS solution of the formula (6) is:

$$\mathbf{X} = -\lambda(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}\Delta \tag{9}$$

with **P** standing for the weight matrix.

In order to assure the convergence of the computational process to the correct solution, in classic form of MAFA method, three linear combinations (LC) of L1 and L2 GPS carrier phase observables, preserving integer nature of ambiguities and long wavelengths are applied in the cascade adjustment (CELLMER et al. 2010). The efficiency of the MAFA method can be also improved using the de-correlation procedure. The ambiguities (N) are usually strongly correlated. Hence, fixing one value of ambiguity through rounding value *s* in (5) to the nearest integer as in (2), has an impact on the rest of the ambiguities. Therefore, the correlation between ambiguities should be taken into account at rounding the right side of the equation (2) or alternatively the observation equations should be transformed into the equivalent form with de-correlated ambiguities.

Let us assume that **Z** is the integer de-correlation matrix (LIU et al. 1999, TEUNISSEN 1995):

$$\mathbf{Q}_{\mathrm{Nz}} = \mathbf{Z}\mathbf{Q}_{\mathrm{N}}\mathbf{Z}^{\mathrm{T}} \tag{10}$$

where:
Z – integer de-correlation matrix
$\mathbf{Q}_{\mathrm{N}}$ – ambiguity covariance matrix
$\mathbf{Q}_{\mathrm{Nz}}$ – diagonal transformed ambiguity covariance matrix.

By multiplying Eq. (1) with **Z**, one can obtain a new equation with a new integer ambiguity vector $\mathbf{N}_{\mathrm{z}}$:

$$\Phi_{\mathrm{z}} + \mathbf{V}_{\mathrm{z}} = \frac{1}{\lambda}\rho_{\mathrm{z}}(\mathbf{X}_{\mathrm{c}}) + \mathbf{N}_{\mathrm{z}} \tag{11}$$

The above formula can replace equation (1). Further considerations are the same but de-correlated observation equation (11) in the place of equation (1) increases the probability of obtaining the correct solution. The subsequent part of the computation process results from this equation. There are many various methods of finding the **Z** decorrelation matrix (HASSIBI, BOYD 1998, JONGE, TIBERIUS 1996, LIU et al. 1999, XU 2001). In order to find the **Z** matrix, the ambiguity covariance matrix ($\mathbf{Q}_{\mathrm{N}}$) is required. This matrix can be evaluated on the basis of the system of observation equations (1) after linearization:

$$\mathbf{V} = \mathbf{AX} + \mathbf{BN} - \mathbf{L} \tag{12}$$

where:
L – misclosures (observed minus computed) vector
B – ambiguity functional model matrix

The covariance matrix of the unknown vector $\mathbf{X}_{\Omega} = [\mathbf{X}, \mathbf{N}]^{\mathrm{T}}$ can be presented as:

$$\mathbf{C}_{\mathrm{X}_{\Omega}} = \begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{PA} & \mathbf{A}^{\mathrm{T}}\mathbf{PB} \\ \mathbf{B}^{\mathrm{T}}\mathbf{PA} & \mathbf{B}^{\mathrm{T}}\mathbf{PB} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{Q}_{\mathrm{X}} & \mathbf{Q}_{\mathrm{XN}} \\ \mathbf{Q}_{\mathrm{NX}} & \mathbf{Q}_{\mathrm{N}} \end{bmatrix} \tag{13}$$

where:

$$\mathbf{Q}_{\mathrm{N}} = [\mathbf{B}^{\mathrm{T}}\mathbf{PB} - \mathbf{B}^{\mathrm{T}}\mathbf{PA}(\mathbf{A}^{\mathrm{T}}\mathbf{PA})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{PB}]^{-1} \tag{14}$$

In the case of the single epoch data, matrix **B** is an identity matrix and $\mathbf{Q}_N$ computed according to formula (17), is not positive definite. It causes difficulties with the de-correlation procedure and leads to incorrect solutions. Therefore, an additional coefficient k is inserted (CELLMER 2011a, 2011b, 2012b):

$$\mathbf{Q}_{Nk} = [\mathbf{P} - k\mathbf{PA}(\mathbf{A}^T\mathbf{PA})^{-1}\mathbf{A}^T\mathbf{P}]^{-1} \tag{15}$$

The use of the coefficient k is equivalent to the simulation of additional observations, e.g. code observations, as in the generalized least squares model presented in WIELGOSZ (2011). The matrix $\mathbf{Q}_{Nk}$ can be applied to the de-correlation procedure as an approximation of the ambiguity covariance matrix.

Search procedure in MAFA method

In (CELLMER 2012a) the necessary condition for applying MAFA method was described. This condition can be formulated as follows:

$$\mathrm{N} = \mathrm{round}(\Phi - \frac{1}{\lambda}\rho_0) \tag{16}$$

where:
N – true ambiguity
ρ_0 – DD geometric distance computed using *a priori* position coordinates.

If the above condition is not satisfied then the observation equation (2) takes the following form:

$$\Phi + v - \frac{1}{\lambda}\rho = \mathrm{round}(\Phi - \frac{1}{\lambda}\rho) + \mathrm{N_e} \tag{17}$$

with integer $\mathrm{N_e}$.

Hence in place of Eq. (3) and (4) are adequately:

$$\mathrm{v_e} = \mathrm{round}(\Phi - \frac{1}{\lambda}\rho) - (\Phi - \frac{1}{\lambda}\rho) + \mathrm{N_e} \tag{18}$$

and

$$\Psi_e = \text{round}(s) - s = \begin{cases} -\dfrac{1}{\lambda}\arcsin\,[\sin(\pi s)] + N_e \text{ for } s \in \{s : \cos\,(\pi s) \geq 0\} \\ \\ \dfrac{1}{\lambda}\arcsin\,[\sin(\pi s)] + N_e \text{ for } s \in \{s : \cos\,(\pi s) < 0\} \end{cases} \tag{19}$$

Based on linearization of the function Ψ_e, the observation equation (6) is rewritten as follows:

$$\mathbf{V} = \frac{1}{\lambda}\mathbf{AX} + \Delta_e \tag{20}$$

with the new misclosures vector:

$$\Delta_e = \text{round}(\Phi - \frac{1}{\lambda}\rho_0) - (\Phi - \frac{1}{\lambda}\rho_0) + \mathbf{N}_e \tag{21}$$

Due to the integer values of the vector $\mathbf{N}_e$ the search procedure is necessary. The search procedure will consist of testing the values of the objective function $\mathbf{V}^T\mathbf{PV}$ for different vectors $\mathbf{N}_e$. It is proposed that the vector $\mathbf{N}_e$ will consist only of the following values -1, 0 and 1. This assumption significantly reduces of the search region. All possible vectors $\mathbf{N}_e$ can be represented by column vectors **e**i forming matrix **E**. The vectors $\mathbf{e}_i$ consists of the elements -1, 0 or 1 in all possible combinations. Generally the matrix **E** can be formed using the following recursive formula:

$$\mathbf{E}_1 = [-1 \quad 0 \quad 1]$$

$$\mathbf{E}_n = \begin{bmatrix} \mathbf{E}_1 \otimes \underset{1\times 3^{n-1}}{\mathbf{1}} \\ \\ \underset{1\times 3}{\mathbf{1}} \otimes \mathbf{E}_{n-1} \end{bmatrix} \tag{22}$$

where:

$\underset{1\times k}{\mathbf{1}}$ – k-element row vector of ones

$\otimes$ – Kronecker product symbol

n – number of ambiguities.

The dimension of matrix **E** is $n \times m$ with the number of columns:

$$m = 3^n \tag{23}$$

The example of the matrix **E** for $n = 3$ is:

$$\mathbf{E}_3 = \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \tag{24}$$

Each column of the **E** matrix is substituted into (21) for $\mathbf{N}_e$ and the criteria $\mathbf{V}^T\mathbf{PV}$ = min is tested. The value of $\mathbf{e}_i$ minimizing $\mathbf{V}^T\mathbf{PV}$ is chosen to final positioning according to formula (9). Delta expression in this equation is calculated by (21) and contains the optimal value of $\mathbf{N}_e$ from the search procedure. Summarizing, the search procedure is based on the misclosure vector modifications followed by the test of the objective function value. The misclosures vector consists of the actual measurements minus predicted measurements. In consecutive tests, the different values of the predicted measurements are substituted taking into account the integer nature of the ambiguities. As it was mentioned in section 2, in classic form of the MAFA method, the three stage cascade adjustment is applied. However due to the search procedure the efficiency is increased and therefore the cascade process can be limited to two stages: only widelane L1-L2 and single L1 observations are employed in the cascade adjustment (definitely both observation sets consist of double differenced observations). The MAFA method together with the search procedure can be used for processing of the observations obtained from a single-epoch.

The processing algorithm in this case will consist of the following stages:

- *a priori* position determination (e.g. using code observations)
- forming double difference of L1, L2, geometric ranges, model matrix **A** and weight matrix **P**
- de-correlation procedure
- search procedure
- final position determination (vector of the coordinates)

When using cascade adjustment two last stages are repeated for each LC. The solution obtained from L1 is assumed as the final position. However as it is shown in the next section, sometimes in the last step of cascade adjustment the criterion of $\mathbf{V}^T\mathbf{PV}$ minimization can indicate on wrong solution. Therefore, it is proposed to apply search procedure only in the first step of the two-steps cascade adjustment (for L1-L2). This optimal approach was determined on the basis of tests presented in section 5.

Case study – numerical example

The test of the presented algorithm was based on the real data. This example relates to a special case – when a search procedure does not give a correct solution in the final step of cascade adjustment. Taking into account this special case, a general algorithm was modified. The input data are listed in Tables 1 and 2. In the first row of Table 1, the coordinates of an *a priori* position are placed. In the second row, there are coordinates obtained from an 8-hour static session processing using Bernese software (DACH et al. 2007). These values are presented for the purposes of comparison with the single epoch processing results. The first column of the Table 2 contains the design matrix. The second and third columns contain double differenced carrier phase observations of the signals L1 and L2. In the fourth column there are double-differenced geometric distances computed from *a priori* position.

Table 1

A priori and 'true' coordinates

	X [m]	Y [m]	Z [m]
a priori (DGPS)	3,717,669.061	1,254,116.079	5,011,896.056
True	3,717,669.167	1,254,115.775	5,011,894.647

Table 2

Input data

A	DD_L1[cycles]	DD_L2[cycles]	DD_dist [m]
0.039 0.233 -0.048	1,818,996.301	1,096,262.797	-483.100
-0.271 0.102 0.878	-7,995,014.281	-6,203,167.191	-37.579
0.494 -0.524 -0.020	7,721,422.793	5,631,992.707	1,226.360
0.368 0.746 -0.037	8,635,207.515	6,393,878.608	-1,439.328
0.814 -0.680 0.138	-10,839,038.439	-8,774,623.852	1,693.053
-0.055 -0.365 0.300	17,445,922.208	13,280,625.091	825.281

The weight matrix was obtained as an inverse of the LC double differenced carried phase covariance matrix:

$$\mathbf{P} = \mathbf{C}^{-1} \tag{25}$$

with the following structure of matrix **C**:

$$\mathbf{C} = m^2\sigma^2 \begin{bmatrix} 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 4 & 2 & 2 & 2 \\ 2 & 2 & 2 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 \end{bmatrix} \tag{26}$$

where:

σ – instrumental accuracy of the carrier phase observation (σ = 0.01 cycle)
m – LC noise, dependent on LC coefficients ($m^2 = i^2 + j^2$, where i = 1, j = -1 for wide lane and i =1, j = 0 for L1 only).

The processing was carried out using MAFA method. A two-step cascade adjustment preceded by de-correlation process was carried out. A search procedure was also applied. The ambiguity covariance matrix was obtained using formula (15) with the coefficient k = 0.9. The decorrelation procedure was performed using algorithm of united ambiguity decorrelation (LIU et al, 1999) implemented by the author in Matlab for the purpose of this contribution. As a result of this procedure the following integer transformation matrix was obtained:

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & -1 & 1 \\ 1 & 1 & 0 & - & 0 & -2 \end{bmatrix} \tag{27}$$

The results of the search procedure on each stage of the cascade processing are listed in the Table 3.

Table 3

The search procedure results

LC#	Without search		With search		With search on $LC_{1,-1}$ and without search on L1	
	$\mathbf{e}_i$	$\mathbf{V}^T\mathbf{PV}$	$\mathbf{e}_i$	$\mathbf{V}^T\mathbf{PV}$	$\mathbf{e}_i$	$\mathbf{V}^T\mathbf{PV}$
$LC_{1,-1}$	$[0\ 0\ 0\ 0\ 0\ 0]^T$	208.65	$[0\ 0\ 0\ 0\ 0\ -1]^T$	2.18	$[0\ 0\ 0\ 0\ 0\ -1]^T$	2.18
$LC_{1,0}$ = L1	$[0\ 0\ 0\ 0\ 0\ 0]^T$	7.11	$[0\ 1\ 1\ 0\ 1\ 1]^T$	7.11	$[0\ 0\ 0\ 0\ 0\ 0]^T$	7.93

The first column contains the name of linear combinations at each stage of cascade processing. The next two columns contain the results of computations without using search procedure. The fourth and fifth columns concern the scenario of implementing the search procedure in every step of cascade adjustment. The results in the last two columns concern the case of using search procedure only in the first step of cascade adjustment. The columns labeled '$\mathbf{e}_i$' contain the constant vector that must be added to the vector Δ in order to obtain minimum value of $\mathbf{V}^T\mathbf{PV}$. Minimum values of $\mathbf{V}^T\mathbf{PV}$ relating to $\mathbf{e}_i$ are listed in the columns labeled '$\mathbf{V}^T\mathbf{PV}$'.

Table 4 contains the residuals, referenced to the 'true' coordinates at each stage of the cascade processing for three scenarios: without search procedure, with search procedure and with search procedure only on the first stage of cascade adjustment. The graphical representations of the ΔX, ΔY, ΔZ values are depicted in Fig 1. The horizontal axis on the zero level depicts the 'true' value of the coordinates. The blue, red and green lines show the residuals of the coordinates referenced to their 'true' values. In the first scenario (without search procedure) in the first step of cascade adjustment (for $LC_{1,-1}$), the V^TPV value is 208.65, whereas in the second and third scenarios (with search procedure) this value equals 2.18 (for e_i = [0 0 0 0 0 –1]).

Table 4

The results of elaboration

LC#	Without search			With search			With search on $LC_{1,-1}$ and without search on L1		
	ΔX	ΔY	ΔZ	ΔX	ΔY	ΔZ	ΔX	ΔY	ΔZ
DGPS	-0.107	0.304	1.410	-0.107	0.304	1.410	-0.107	0.304	1.410
$LC_{1,-1}$	0.070	0.150	0.979	0.007	-0.009	0.026	0.007	-0.009	0.026
$LC_{1,0}$ = L1	0.231	0.182	0.925	0.231	0.182	0.925	-0.008	-0.005	0.002

The residual values (Tab. 4 and Fig. 1) are much lower if using search procedure in the first step of cascade adjustment (for $LC_{1,-1}$). There was a significant improvement as a result of applying the search procedure on this stage of processing. However on final stage of cascade adjustment (for L1) the search procedure gave wrong solution. The correct solution is obtained without search procedure ($\mathbf{e}_i$ = [0 0 0 0 0 0], V^TPV = 7.93) although minimum of V^TPV equals 7.11 (for $\mathbf{e}_i$ = [0 1 1 0 1 1]). The above example shows that criterion: $\mathbf{V}^T\mathbf{PV}$ = min can sometimes lead to incorrect solution. Especially when dealing with L1 data. Therefore it is proposed to modify the general algorithm by using search procedure only on the first stage (for $LC_{1,-1}$) of cascade adjustment. Tests presented in the next section confirm this finding.

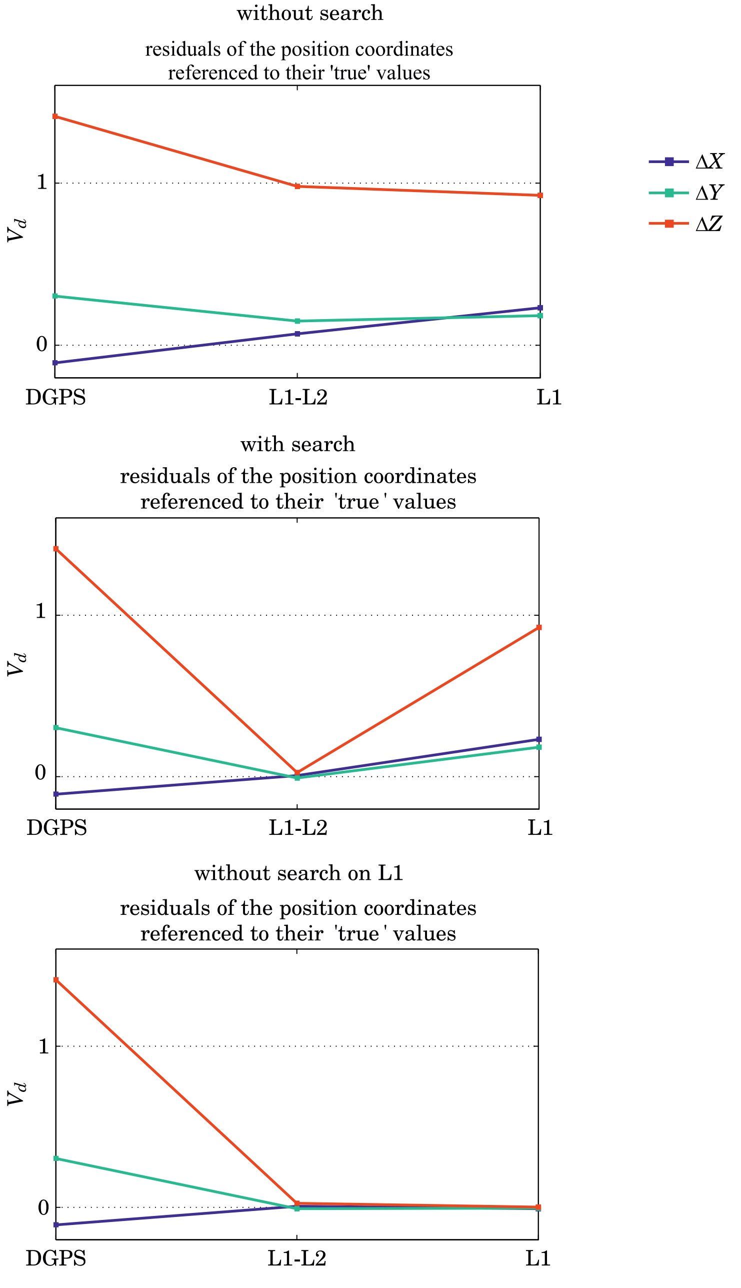

Fig. 1. Residuals of the position coordinates referenced to their 'true' values

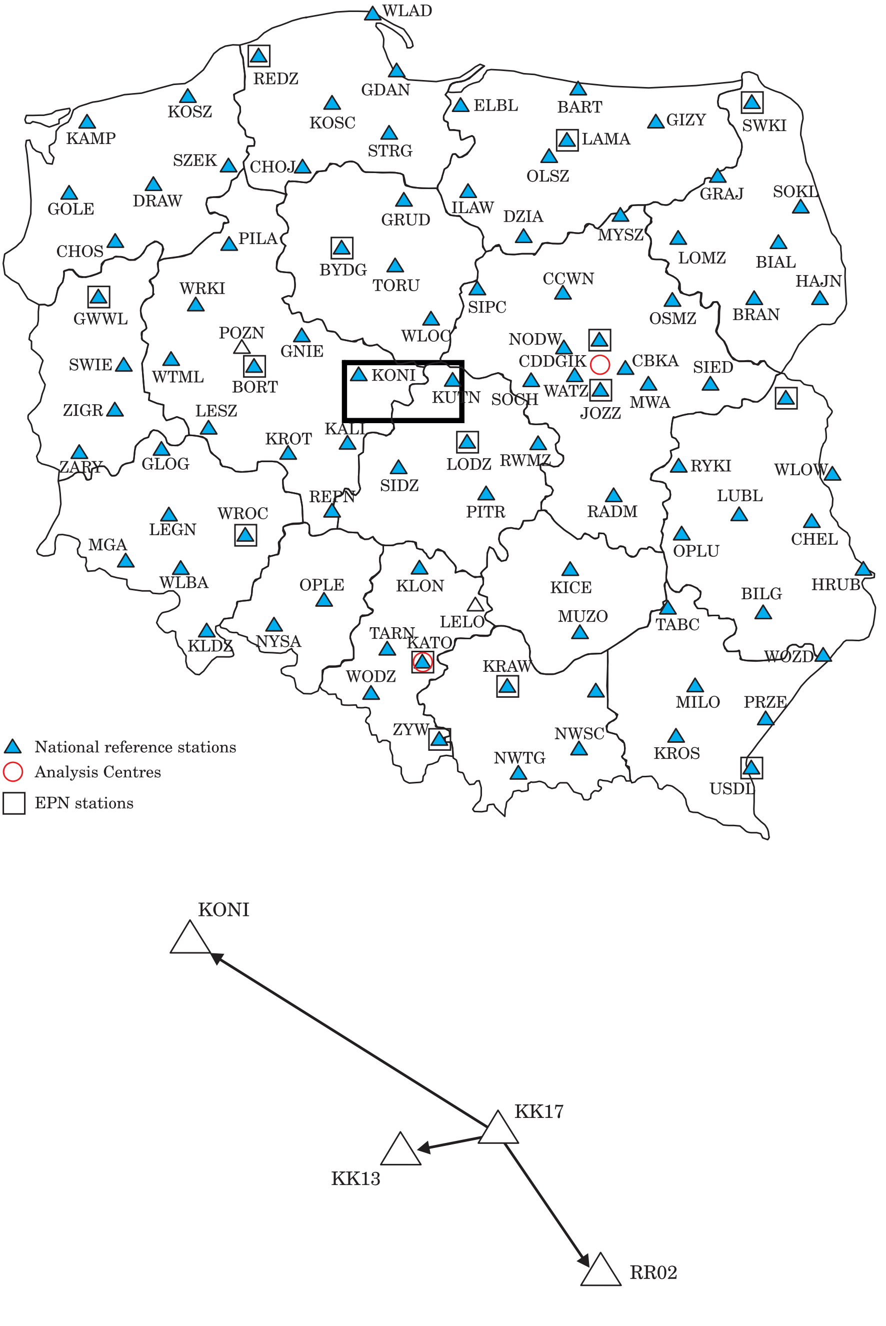

Fig. 2. The location of the test surveys http://www.asgeupos.pl/webpg/graph/dwnld/map_pl_EN.jpg

Experiment design

In order to test the efficiency of the proposed algorithm, the real GPS data of three baselines were used. Tha data come from campaign performed in order to monitor local deformation in open-pit mine 'Adamów' in Central Poland. This project is managed by Dr. Radoslaw Baryła from the Chair of Satellite Geodesy and Navigation of University of Warmia and Mazury in Olsztyn. Figure 2 depicts the location of the surveys. One GPS station of ASG-EUPOS (Polish part of European Positioning System active geodetic network) was used in test surveys ('KONI'). The surveys were performed on December 9th, 2008, on 30.7 km, 10.2 km and 2.1 km baselines, with a 30-second sampling rate. Data sets of each baseline consisted of 120 epochs. The data were processed according to the proposed approach independently for each epoch. The ambiguity covariance matrix was formed according to formula (15), as a basis for the de-correlation procedure. The 'true' coordinates were derived using Bernese software based on an 8-hour data set.

Test results

Figure 3 presents the comparison of the results of 120 single epoch data processing for different scenarios: without search procedure, with search procedure on each stage of cascade adjustment and for scenario with search procedure only on the first stage of cascade adjustment.

The horizontal lines on Fig. 3 depict the linear residuals of the position obtained independently in each epoch using MAFA method, with respect to the 'true' position. The residuals were computed as: $\sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$, where ΔX, ΔY and ΔZ are components of the residuals with respect to the 'true' position. The blue lines relate to the first scenario (without search procedure), the red lines relate to second scenario (with search procedure on each stage of cascade adjustment) and the green lines relate to third scenario (search procedure only on the first stage of cascade adjustment). In most cases, *a priori* position was farther than 1m from the 'true' position. For each case the percentage of correct solutions is shown in text box. There were 20%–40% correct solutions in first scenario (without search procedure) depending on length of baseline. In second scenario the percentage of correct solutions ranged from 76% to 82%. There has been a significant improvement of the results. Further improvement can be obtained using third scenario of data processing- through applying search procedure only in the first step of cascade adjustment. In this case the percentage of correct solutions reaches even 92%. Summarizing, the number of correct single-epoch solutions using optimal scenario of data processing varied from 85% to 92%.

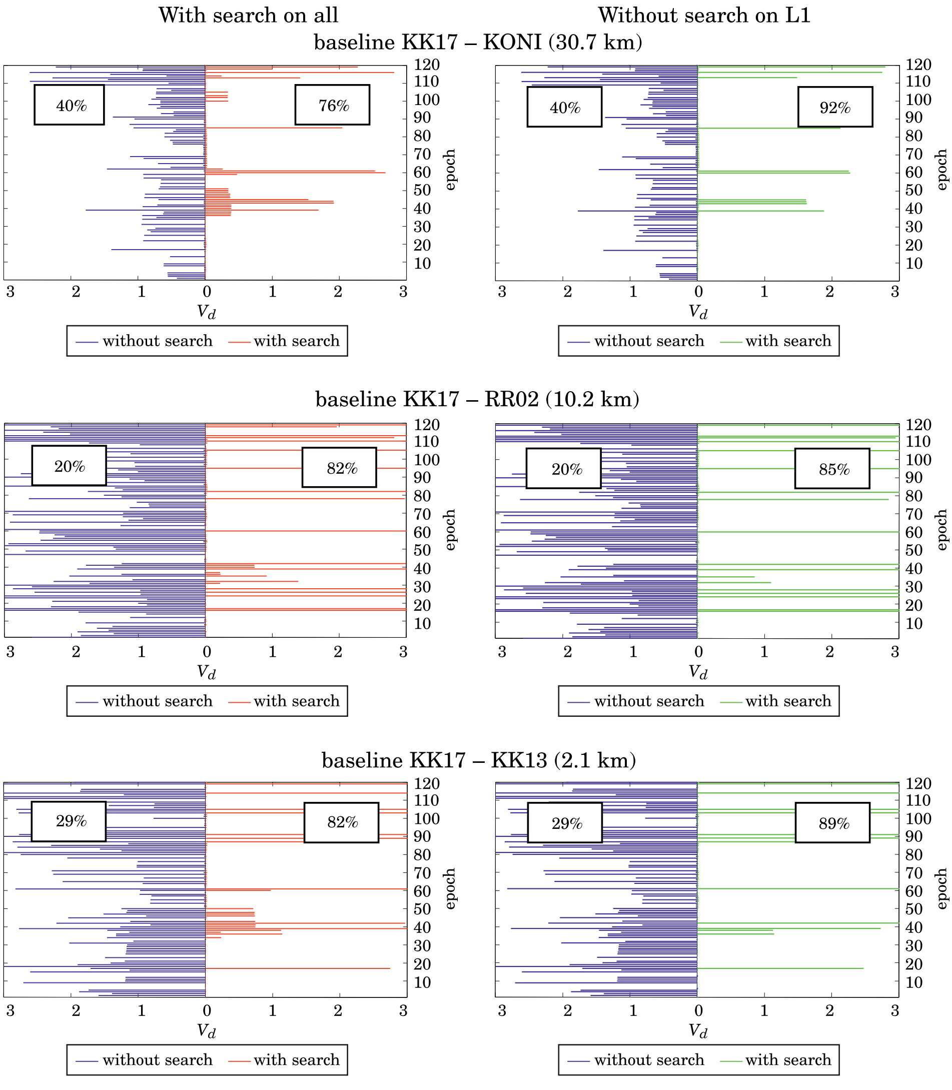

Fig. 3. Linear residuals of the final position for three baselines expressed in meters

Conclusions

The MAFA method was improved with implementing a search procedure. The detailed algorithm of such procedure was elaborated and presented in this paper. The computational process allows obtaining precise position even on the basis of only single observational epoch. The results of the tests show the usefulness of the proposed solutions. The high efficiency of the proposed algorithm was confirmed by tests performed for short and medium baselines (shorter than 30 km).

References

BAKUŁA M. 2010. *Network Code DGPS Positioning and Reliable Estimation of Position Accuracy*, Survey Review, 42(315): 82–91.

CELLMER S., WIELGOSZ P., RZEPECKA Z. 2010. *Modified ambiguity function approach for GPS carrier phase positioning.* Journal of Geodesy, 84: 267–275.

CELLMER S. 2011a. *The real time precise positioning using MAFA method.* The 8th International Conference ENVIRONMENTAL ENGINEERING, selected papers, III, Vilnius, 1310–1314.

CELLMER S. 2011b. *Using the Integer Decorrelation Procedure to increase of the efficiency of the MAFA Method.* Artificial Satellites, 46(3): 103–110.

CELLMER S. 2012a. *A Graphic Representation of the Necessary Condition for the MAFA Method.* Transactions on Geoscience and Remote Sensing, 50(2): 482–488.

CELLMER S. 2012b. *On-the-fly ambiguity resolution using an estimator of the modified ambiguity covariance matrix for the GNSS positioning model based on phase data.* Artificial Satellites, 47(3): 81–90.

DACH R., HUGENTOBLER U., FRIDEZ P., MEINDL M 2007. *BERNESE GPS Software Version 5.0.* Astronomical Institute, University of Berne.

GLENN J., SVEDENSEN G. 2006. *Some properties of decorrelation techniques in the ambiguity space.* GPS Solut, 10: 40–44.

HASSIBI A., BOYD S. 1998. *Integer parameter estimation in linear models with application to GPS*. IEEE Trans SignallProc, 46: 2938–2952.

HAN S., RIZOS C. 1996. *Improving the computational efficiency of the ambiguity function algorithm*. Journal of Geodesy, 70(6): 330–341.

HOFMANN-WELLENHOF B., LICHTENEGGER H., WASLE E. 2008. *GNSS-Global Navigation Satellite Systems – GPS, GLONASS, Galileo & more*, Springer-Verlag Wien.

JUNG J., ENGE P. 2000. *Optimization of Cascade Integer Resolution with Three Civil GPS Frequencies* Proc. ION GPS'2000, Salt Lake City, September.

JONGE P. DE., TIBERIUS CH. 1996. *The LAMBDA method for integer ambiguity estimation: implementation aspects* Delft Geodetic Computing Centre LGR Series.

LEICK A. 2004. *GPS Satellite Surveying.* 3rd edition, John Wiley and Sons, Inc.

LIU L.T., HSU H.T., ZHU Y.Z., OU J.K. 1999. *A new approach to GPS ambiguity decorrelation.* Journal of Geodesy, 73: 478–490.

TEUNISSEN P. J.G. 1995. *The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation*, Journal of Geodesy, 70: 65–82.

TEUNISSEN P.J.G., KLEUSBERG A. 1998. *GPS for Geodesy*, (selected papers of Intenational School lectures) Springer – Verlag, Berlin Heidelberg New York.

WIELGOSZ P. 2011. *Quality assessment of GPS rapid static positioning with weighted ionospheric parameters in generalized least squares.* GPS Solutions, 15(2): 89–99.

XU P. 2001. *Random simulation and GPS decorrelation*. Journal of Geodesy, 75: 408–423.