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COMPARISON OF TIME-STEPPING METHODS FOR TRANSIENT MAGNETIC FIELD COMPUTATIONS IN COMSOL MULTIPHYSICS

The computations of a transient magnetic field problem are presented. The problem has been constructed and solved in COMSOL Multiphysics which is one of the most popular and versatile commercial programs that allow to deal with solving electromagnetic field problems by means of the finite element method. A simple 2D problem of a coaxial cable has been chosen so that an analytical solution can be used in order to easily compute an error. The main topic of the paper concerns the comparison of the time-stepping methods that are available in COMSOL Multiphysics i.e. the BDF and generalized alpha methods. It is also possible to select various options for these methods, which have an influence on the solution accuracy. These have been also considered in the analysis.

KEYWORDS: time-stepping, COMSOL, transient magnetic field.

1. INTRODUCTION

The paper is a part of a project that concerns electromagnetic field computations and efficiency analyses of the finite element method.

The authors' efforts are mostly directed at the improvement of the efficiency of transient magnetic field computations (including nonlinear problems). The efficiency improvement is sought as either:

– the reduction of the computation time while keeping a relatively similar accuracy,
– an improvement of solution accuracy while the computation time remains in a relatively similar scale.

While the quality of a transient problem solution depends on many factors, the current stage of the analysis concerns only the inspection of the most specific feature of time-dependent solvers i.e. the applied time-stepping methods.

In previous studies [1, 2] it has been shown that it is possible to achieve efficient transient magnetic field computations when applying higher order BDF

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(**B**ackward **D**ifferentiation **F**ormula [3]) methods with time-step adaptation. These computations have been performed with the help of an original Matlab script.

In contrast, this paper concerns the application of one of the most popular programs for electromagnetic field computations – COMSOL Multiphysics [4].

COMSOL Multiphysics is a powerful tool for simulating electromagnetic field phenomena by means of the finite element method. In the program it is possible to solve time-dependent FEM problems by either the BDF or the so called generalized-alpha method [5]. Both these methods (with various options) are compared in the paper. A simple 2D problem has been chosen so that it is also possible to obtain an analytical solution, which will allow to ascertain the accuracy of the numerical results.

2. MAGNETIC FIELD EQUATIONS

The study has been performed on the standard package of COMSOL Multiphysics where transient computations in the Magnetic Field module are not supported [6]. However, it is possible to perform a time-dependent magnetic field study by properly implementing the equations in the General Form PDE (**P**artial **D**ifferential **E**quation) feature. As it is common in magnetic field computations, the magnetic vector potential formulation is used, which in a 2D analysis is simplified in terms of the vector's components:

$$\vec{A} = A\vec{1}_z. \tag{1}$$

The general form PDE has the form:

$$e_\mathrm{a}\frac{\partial^2 A}{\partial t^2} + d_\mathrm{a}\frac{\partial A}{\partial t} + \vec{\nabla}\cdot\vec{\Gamma} = f. \tag{2}$$

For the above to assume the form of Amperé's law one must define the vector $\vec{\Gamma}$ as:

$$\vec{\Gamma} = H_y\vec{1}_x - H_x\vec{1}_y, \tag{3}$$

so that it leads to:

$$\vec{\nabla}\cdot\vec{\Gamma} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = (\vec{\nabla}\times\vec{H})\cdot\vec{1}_z. \tag{4}$$

With an assumption of a low frequency field (or generally – a slowly varying field) one can omit displacement currents hence:

$$e_\mathrm{a} = 0, \quad d_\mathrm{a} = \gamma. \tag{5}$$

Finally the parameter f represents the externally applied current density:

$$f = J_\mathrm{ext}. \tag{6}$$

In addition to the general form PDE one must define the magnetic field strength as an auxiliary variable in terms of the magnetic vector potential component A.

3. TRANSIENT MAGNETIC FIELD PROBLEM

A simple transient magnetic field problem is considered (Fig. 1), where a coaxial cable leads a current of two different frequencies:

$$i = i(t) = \sin(\omega_1 t) + 0.1\sin(9\omega_1 t), \tag{7}$$

where $\omega_1 = 2\pi f_1$, $f_1 = 50$ Hz.

By imposing several harmonics one can preview how the solver would deal with a nonlinear problem (however a linear problem has been chosen as it has an easily obtainable analytical solution).

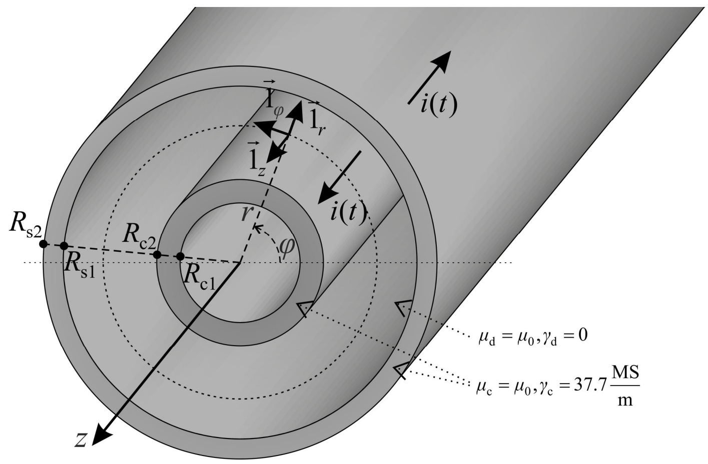

Fig. 1. Coaxial cable leading a non-sinusoidal current

When assuming that no corona effects are present in the insulation one can assume the following boundary conditions for the magnetic field strength:

$$\begin{cases} H_\varphi\big|_{r=R_{c1}} = 0, \\ H_\varphi\big|_{r=R_{c2}} = \dfrac{i}{2\pi R_{c2}}, \\ H_\varphi\big|_{r=R_{s1}} = \dfrac{i}{2\pi R_{s1}}, \\ H_\varphi\big|_{r=R_{s2}} = 0, \end{cases} \tag{8}$$

which are actually the Neumann boundary conditions:

$$\begin{cases} \dfrac{1}{\mu}\dfrac{\partial A}{\partial n}\bigg|_{r=R_{c1}} = 0, \\ \dfrac{1}{\mu}\dfrac{\partial A}{\partial n}\bigg|_{r=R_{c2}} = \dfrac{\pm i}{2\pi R_{c2}}, \\ \dfrac{1}{\mu}\dfrac{\partial A}{\partial n}\bigg|_{r=R_{s1}} = \dfrac{\pm i}{2\pi R_{s1}}, \\ \dfrac{1}{\mu}\dfrac{\partial A}{\partial n}\bigg|_{r=R_{s2}} = 0, \end{cases} \tag{9}$$

Note that whether the second and third boundary condition is imposed with a plus or minus depends on the boundary normal vector's direction with respect to the radial axis.

4. AC SOLUTION ACCURACY

The time-dependent solution of the considered problem can be at most as accurate as the superposition of AC solutions obtained for the frequencies f_1 and $9f_1$. An AC magnetic field analysis can be performed by means of the Helmholtz Equation interface in COMSOL Multiphysics. In here the general equation is:

$$\vec{\nabla} \cdot (-c\vec{\nabla}\underline{A}) + \underline{a}\,\underline{A} = \underline{f}. \tag{10}$$

Again the scalar value f is the externally applied current density, while:

$$c = \frac{1}{\mu}, \quad \underline{a} = \mathrm{j}h\omega_1\gamma, \tag{11}$$

with h being either 1 or 9 (in reference to the respective frequencies).

The solution has been obtained for the mesh presented in Fig. 2, which yields 12925 degrees of freedom. The solution has been obtained in approximately 5 seconds so the mesh could be a lot finer – however, the same mesh is to be used for the time-dependent study, which is estimated to take at least several times longer to finish.

The magnetic field strength solution at the line $x \in [R_{c1}, R_{s2}]$, $y = 0$ is compared with the analytical solution [7]:

– inside the conductors:

$$\underline{H}_{\varphi}\Big|_{r\in[R_{c1},R_{c2}]} = \underline{c}_1\mathrm{I}_1(\underline{\Gamma}r) + \underline{c}_2\mathrm{K}_1(\underline{\Gamma}r), \tag{12}$$

$$\underline{H}_{\varphi}\Big|_{r\in[R_{s1},R_{s2}]} = \underline{c}_3\mathrm{I}_1(\underline{\Gamma}r) + \underline{c}_4\mathrm{K}_1(\underline{\Gamma}r), \tag{13}$$

– in the insulation:

$$\underline{H}_{\varphi}\Big|_{r\in[R_{c2},R_{s1}]} = \frac{-\mathrm{j}}{2\pi r}. \tag{14}$$

$\underline{\mathrm{I}}_1$ and $\underline{\mathrm{K}}_1$ are the modified Bessel functions of the first and second kind respectively. $\underline{c}_1$, $\underline{c}_2$, $\underline{c}_3$, $\underline{c}_4$ are complex coefficients that are dependent on the imposed boundary conditions. $\underline{\Gamma}$ is the skin effect parameter which equals $\mathrm{j}h\omega_1\mu\gamma$.

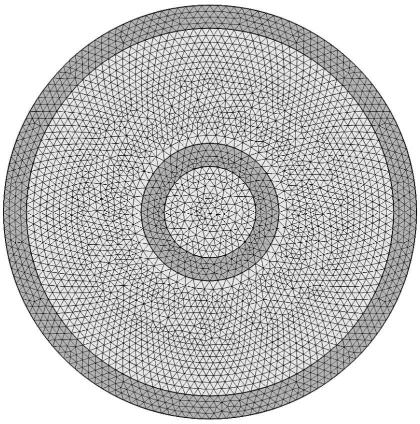

Fig. 2. Magnetic field problem mesh

After obtaining the numerical solution in COMSOL, the magnetic field strength $\underline{H}_y$ at the selected line (representing $\underline{H}_\varphi$) has been exported for automatically selected points. These values are then placed inside the vector $\underline{\boldsymbol{H}}_{\mathrm{num}}$. Further on, for the selected points the analytical solution is determined. These values in turn are placed in the vector $\underline{\boldsymbol{H}}_{\mathrm{an}}$. Basing on a relative average difference of the numerical and analytical solutions, the following error formula is defined:

$$e_{\mathrm{r}} = \left(\overline{\left|\underline{\boldsymbol{H}}_{\mathrm{num}} - \underline{\boldsymbol{H}}_{\mathrm{an}}\right|} / \overline{\left|\underline{\boldsymbol{H}}_{\mathrm{an}}\right|}\right)\cdot 100\%, \tag{15}$$

The computed error values for the frequencies f_1 and $9f_1$ are respectively:

$$e_{\mathrm{r1}} = 0.2087\,\%, \quad e_{\mathrm{r9}} = 1.1056\,\%. \tag{16}$$

5. TRANSIENT SOLUTION ACCURACY

In the standard package of COMSOL Multiphysics it is possible to solve time-dependent problems with either one of two general methods:

- the BDF method with an adaptively changing order within a fixed range [n_{min}, n_{max}], where n_{min} can be at most 2 and the maximum value of n_{max} is 5,
- the generalized alpha method, with a high frequency dampener (expressed further on by the symbol $\zeta \in [0, 1)$) and a linear or constant predictor.

In order to compare the efficiency of both methods with various options, the transient magnetic field problem presented in Section 2 has been solved. The solution has been obtained for a total time of $5/f_1$ in order to obtain a steady-state. The complex valued solutions for f_1 and $9f_1$ have been obtained through the application of the Vaniček method [8] on the final computed time period.

A comparison has been performed with respect to the computation time and the relative error value (15) for the frequencies f_1 and $9f_1$ (denoted further on as e_{r1} and e_{r9} respectively). The results of this comparison are presented in Figure 3.

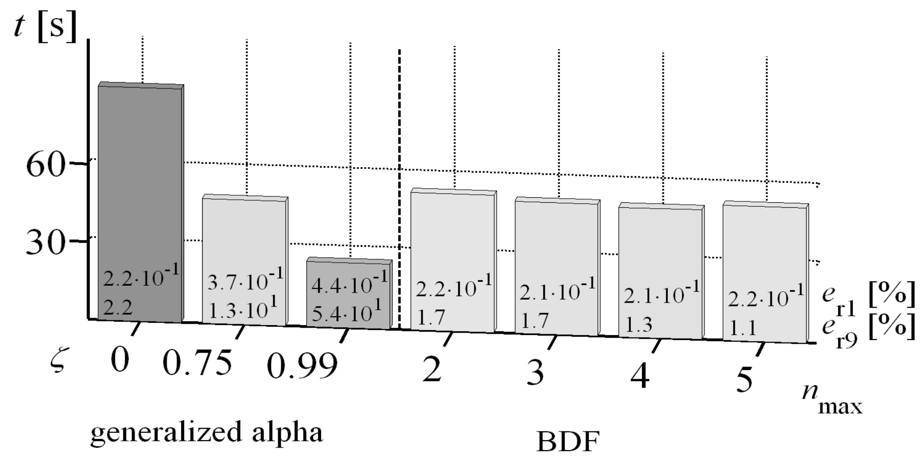

Fig. 3. Comparison of computation times and relative errors for the transient magnetic field solutions for the BDF and generalized alpha methods

The results depict that the e_{r1} error is close to the AC solution when choosing either of the BDF methods. The e_{r9} error tends to the AC solution error when a BDF method of higher order is chosen.

The generalized alpha method with $\zeta = 0$ seems to perform worse than all of the BDF methods. In the case of $\zeta = 0.75$ the computation time isn't much smaller in comparison to the BDF methods but when choosing $\zeta = 0.99$ the computation time equals to about half the typical time it takes when applying a BDF method. However a major drawback of the method is that it should be only used if fast changes of the variable are insignificant – which can be noticed by the large e_{r9} error values.

Taking into account the above observations and the fact that the computation times are similar in all of the BDF methods then unless one deals with a very different problem – it is advised for the BDF method of $n_{max} = 5$ to be chosen. This should guarantee the biggest chances of obtaining an optimal result when taking into account both computation time and solution accuracy.

6. CONCLUSIONS

A comparison has been performed for the various methods that can be chosen in COMSOL Multiphysics for a time-dependent study. A transient magnetic field problem of a coaxial cable has been considered, where the conductors lead a current consisting of two different frequencies. An AC solution has been obtained in order to check the possible accuracy that can be achieved in the 2D problem with a selected mesh. For a general criterion of checking the solution accuracy – the analytical solution of the problem has been used. COMSOL Multiphysics allows to solve time-dependent problems with the use of either the BDF methods or the generalized alpha method. Both methods (with various options) have been compared in terms of the solution accuracy and the computation time. After performing the computations it has been observed that the greatest chances of an optimal result can be achieved when choosing the BDF method of $n_{\max} = 5$.

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