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COLLISION-FREE TRAJECTORY PLANNING FOR MOBILE MANIPULATORS SUBJECT TO CONTROL CONSTRAINTS

A method of planning collision-free trajectory for a mobile manipulator tracking a line section path is presented. The reference trajectory of a mobile platform is not needed, mechanical and control constraints are taken into account. The method is based on a penalty function approach and a redundancy resolution at the acceleration level. Nonholonomic constraints in a Pfaffian form are explicitly incorporated to the control algorithm. The problem is shown to be equivalent to some point-to-point control problem whose solution may be easier determined. The motion of the mobile manipulator is planned in order to maximise the manipulability measure, thus to avoid manipulator singularities. A computer example involving a mobile manipulator consisting of a nonholonomic platform (2,0) class and a 3 DOF RPR type holonomic manipulator operating in a three-dimensional task space is also presented.

1. Introduction

The main task performed by a robot is to move the end-effector from a given initial position to a given final position in a workspace. The shape of the path, the end-effector moves along, is important in some tasks, such as handling and stacking operations, parts assembly (pick and place) or inspection tasks. In such cases, the path is specified by a higher level module, and the end-effector of the robot has to follow it.

In order to extend performance capabilities of the manipulator, its arm is mounted on a mobile wheeled platform. By combining the mobility of the platform with the dexterous capability of the manipulator, such a system gains kinematic redundancy. The redundant degrees of freedom render it possible

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to accomplish complex tasks in complicated workspaces with obstacles, but redundancy also causes the solution of the mobile manipulator task is difficult because of its ambiguity. Moreover, the task becomes more complicated when the mobile manipulator, in addition to holonomic constraints, also has nonholonomic ones.

In the literature, different approaches to solving such problems have been developed. They can be classified according to different criteria, e.g. optimality of the solution, treating the mobile manipulator as a single system or two interconnected subsystems, definition of the task.

Much of the existing research addresses the problem using only kinematic equations of the mobile manipulator, and the dynamics of the robot is not considered at all. Bayle [2], [3] has proposed a pure kinematic solution based on the pseudo-inverse of the Jacobian matrix. Galicki in [13] and [14] has presented a solution at the kinematic level to the inverse kinematic problem to solve point-to-point problems in a workspace with obstacles. In order to determine a unique solution, an instantaneous performance index describing an energy lost has been used. A kinematic control method based on the transverse function approach has been proposed by Fruchard et al. [9]. The realisation of the manipulation task has been set as the prime objective, the control objective for the platform has been expressed in the form of a secondary cost function, whose exact minimisation has been not a strict requirement. In [26] Seraji has proposed approach which treats nonholonomic constraints of the mobile platform and the kinematic redundancy of the manipulator arm in a unified manner to obtain the mobile manipulator motion at the kinematic level. The inverse kinematic problem for a mobile manipulator has been solved in [28] by applying an endogenous configurations that drive the end-effector to desirable positions and orientations in the task space.

Some of existing research addresses the problem using the dynamics of the mobile robot. Desai and Kumar [5] have presented an approach based on the calculus of variations to obtain optimal trajectories for multiple mobile manipulators, taking their dynamics and obstacles in the workspace into consideration. This approach requires knowledge of the final mobile manipulator configuration and the shapes of obstacles. In [29] Yamamoto and Yun have considered the end-effector trajectory tracking task and they have studied dynamic interactions between the manipulator and the mobile platform. The solution at the torque/force level has been presented by Galicki in [15], [16]. The class of controllers, fulfilling state equality and inequality constraints, and generating collision free mobile manipulator trajectory with instantaneous minimal energy has been proposed. Nevertheless in these solutions control constraints have not been taken into consideration. In [27] Tan et al. have proposed integrated task planning and control approach for manipulating

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a nonholonomic cart by mobile robot consisting of a holonomic platform and on-board manipulator. The kinematic redundancy and dynamic properties of the platform and the manipulator arm are considered, the manipulator dexterity is preserved, nevertheless the mobile platform is holonomic. Mohri et al. [19] have presented the sub-optimal trajectory planning method of a mobile manipulator. The planning problem has been formulated as an optimal control problem and solved using an iterative algorithm based on the gradient function synthesised in a hierarchical manner considering the order of priority. In [18] Mazur has developed an input-output feedback linearization technique for different types of nonholonomic mobile robots. The author has proposed a form of output functions which makes it possible to move simultaneously, the mobile platform and the manipulator mounted on it.

Some methods require knowledge of the mobile platform and end-effector reference trajectories that should be traced by the mobile manipulator. In the work [4] Chung et al. have considered the mobile robot as two subsystems, and they have designed two independent controllers for the mobile platform and the manipulator. These controllers are coordinated by a nonlinear interaction-control algorithm. The trajectory found in this approach is not optimal in any sense. Egerstedt and Hu in the paper [6] have proposed error-feedback control algorithms in which the trajectory for the mobile platform is planned in such a way that the end-effector trajectory is reachable for the manipulator arm. Mazur [17] has presented the control algorithm for nonholonomic mobile manipulators following along the desired path. This problem has been decomposed into two separated tasks defined for the end-effector of the manipulator and the nonholonomic platform. This solution can be applied to mobile manipulators with fully known dynamics.

This paper presents a sub-optimal motion planning method for applications, where only the knowledge of the end-effector path is needed. The task of the mobile manipulator is to move the end-effector along a prescribed geometric path being a line section. The robot's trajectory is planned in a manner to maximise the manipulability measure to avoid manipulator singularities. In addition, the constraints imposed on mechanical limits and mobile manipulator controls are taken into account. Boundary conditions resulting from the task to be performed are also considered.

In the proposed solution, the path following problem is transformed into an optimisation problem with holonomic and nonholonomic equality constrains, and inequality constraints resulting from mechanical limitations and collision avoidance conditions. This task is shown to be equivalent to some point-to-point control problem whose solution may be easier determined. The resulting trajectory is scaled to fulfil the constraints imposed on the controls. The task was solved by using penalty functions and a redundancy resolution

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at the acceleration level. The asymptotic stability of the solution implies fulfilment of all the constraints imposed. The presented solution assumes that kinematic and dynamic parameters are fully known, even if they cannot be measured directly they can be estimated by an identification technique [1], [25]. The most important advantage of this solution is simplification of the problem which results in reduction of the computational burden. The use of this method is limited to movement of the end-effector along the line section path, but such a task is common in practical applications. The proposed approach can be extended to the case of trajectory planning for the mobile manipulator following the end-effector path given as a general curve, however, in such a case the numerical complexity increases significantly.

To the best of the authors' knowledge, no research has considered the problem formulated in the above manner. Opposite to similar works, the proposed solution does not require the reference path for the platform, which makes it possible to apply our method in complicated workspaces including many obstacles. All research known to the authors, which take into account the kinematic and dynamic model, require transformation of the nonholonomic constraints in a Pfaffian form to a driftless control system. This transformation is not unique which makes difficult to choose a suitable driftless dynamic system. Opposite to these approaches, our method incorporates nonholonomic constraints in a Pfaffian form explicitly to the control algorithm. The presented solution is distinguished by the method of determining the controls which fulfil the assumed constraints. Moreover, the knowledge of the robot dynamic is not needed to find the mobile manipulator trajectory. The proposed method, using scaling techniques [22]-[24], produces the mobile manipulator trajectory parameterised by gain coefficients in such away as to make the corresponding controls fulfil the imposed constraints, so the dynamic model of the mechanism is necessary to determine the values of gain coefficients only. Other research considering the dynamics of the robot produces controls parameterised by gain coefficients of the controller directly. In such cases, it is very difficult (or impossible) to find coefficients which fulfil control limits. Moreover, our solution maximises the manipulability measure of whole manipulator. In consequence, a robot is far from its singular configurations during execution the task (much of existing literature does not deal with any optimality at all).

The paper is organised as follows. Section 2 formulates the problem of optimal control. A solution to this problem is presented in Section 3. The proposed method is demonstrated numerically in Section 4, for a mobile manipulator consisting of a nonholonomic (2,0) class platform and a 3DOF RPR type holonomic manipulator operating in a three-dimensional task space.

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2. Problem formulation

The mobile manipulator task is to move the end-effector in the m-dimensional task space along the given line section between initial point $\mathbf{P}_0$ and final point $\mathbf{P}_f$:

$$\mathbf{P}(\mathbf{q}) = \mathbf{P}_0 + s\left(\mathbf{P}_f - \mathbf{P}_0\right). \tag{1}$$

Right-hand side of the above equation describes the end-effector path in the parametric form, $s \in [0,\ 1]$ is a scaling parameter which depends on time t, i.e. $s = s(t)$, $t \in [0, T]$, T stands for an unknown time of task accomplishment. Function $\mathbf{P} : \Re^n \to \Re^m$ denotes m-dimensional mapping, which describes the position and orientation of the end-effector in the workspace. $\mathbf{q} \in \Re^n$ is a vector of generalised coordinates describing a whole mobile manipulator composed of a nonholonomic platform and holonomic manipulator with kinematic pairs of the 5$^{\text{th}}$ class:

$$\mathbf{q} = \begin{pmatrix} \mathbf{q}^p & \mathbf{q}^r \end{pmatrix}^T. \tag{2}$$

Vector $\mathbf{q}$ depends on time t, i.e. $\mathbf{q} = \mathbf{q}(t)$, $\mathbf{q}^p \in \Re^p$ means the vector of the coordinates of the nonholonomic platform, $\mathbf{q}^r \in \Re^r$ is the vector of joints coordinates of the holonomic manipulator. p and r determine the numbers of coordinates describing the nonholonomic platform and the holonomic manipulator, respectively, $n = p + r$.

The platform motion is subject to nonholonomic constraints described in the Pfaffian form:

$$\tilde{\mathbf{A}}(\mathbf{q}^p)\ \dot{\mathbf{q}}^p = \mathbf{0}, \tag{3}$$

where $\tilde{\mathbf{A}}(\mathbf{q}^p)$ is $(k \times p)$ Pfaffian full rank matrix and k is the number of nonholonomic constraints.

The dynamics of the mobile manipulator is given in a general form, as:

$$\mathbf{M}(\mathbf{q})\ \ddot{\mathbf{q}} + \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{A}(\mathbf{q})^T \lambda = \mathbf{B}\mathbf{u}, \tag{4}$$

where $\mathbf{M}$ denotes $(n \times n)$ positive definite inertia matrix, $\mathbf{F}$ is n-dimensional vector representing Coriolis, centrifugal, viscous, Coulomb friction and gravity forces. $\mathbf{A}(\mathbf{q}) = \begin{bmatrix} \tilde{\mathbf{A}}(\mathbf{q}^p) & \mathbf{0}_{k\times r} \end{bmatrix}$ is the Pfaffian matrix augmented by $(k \times r)$ zero matrix $\mathbf{0}_{k\times r}$, λ stands for the k-dimensional vector of the Lagrange multipliers corresponding to nonholonomic constraints (3). $\mathbf{u}$ denotes $(n - k)$-dimensional vector of controls (torques/forces) and $\mathbf{B}$ is $(n \times (n - k))$ full rank matrix (by definition) describing which state variables of the mobile manipulator are directly driven by the actuators.

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The assumption that the robot at the initial moment of motion, i.e. for $t=0$, is in the acceptable configuration is taken into account. Additionally, for this configuration the end-effector of the mobile manipulator should be at the initial location of the path:

$$\mathbf{P}(\mathbf{q}(0)) - \mathbf{P}_0 = \mathbf{0}. \tag{5}$$

The practical processes that are accomplished by the industrial robots impose some conditions at the beginning and end of a trajectory. It is natural to assume that at the initial and final moments of the task performance, the velocities of the mobile manipulator equal zero:

$$\dot{\mathbf{q}}(0) = \mathbf{0}, \quad \dot{\mathbf{q}}(T) = \mathbf{0}. \tag{6}$$

The constraints connected with the existence of mechanical limits for the mobile manipulator configuration $\mathbf{q}$ and the fact that the robot should not collide with the obstacles will be considered. They are given, in general form, as a set of L inequalities:

$$\{c_i(\mathbf{q}) \geq 0\}, \quad i = 1 : L, \tag{7}$$

where $c_i(\mathbf{q})$ are scalar functions, which involve the fulfilment of the constraints.

For collision avoidance purposes, the method of obstacles enlargement with simultaneous reducing of the manipulator size [12] is used. In this case, the collision test leads to checking a finite number of inequalities, so scalar functions $c_i(\mathbf{q})$ are expressed as:

$$c_i(\mathbf{q}) = S_j(\mathbf{p}) - \delta;$$

where $S_j(\cdot)$ denotes the equation of the j-th obstacle surface, $\mathbf{p}$ is a point from the discrete set of points which approximate the mobile manipulator and δ is small positive scalar, safety margin.

Additionally, the constant constraints of control are also assumed:

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}, \tag{8}$$

where $\mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$ are $(n-k)$-dimensional vectors, lower and upper limits on $\mathbf{u}$, respectively.

In practice, it is important for the configuration of mobile manipulator's joints to be far away from singular configurations. This assumption corresponds to a search for the trajectory for which the instantaneous performance

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index is minimized (maximising the manipulability measure [30]) at each time instant $t \in [0, T]$.

$$\tilde{I}(\mathbf{q}) = -\det\left(\mathbf{j}\,\mathbf{j}^T\right)^{1/2}, \tag{9}$$

where $\mathbf{j} = \partial\mathbf{P}(\mathbf{q})/\partial\mathbf{q}$ is $(m \times n)$ dimensional Jacobian matrix of the mobile robot.

The dependencies above formulate the robotic task as an optimal control problem expressed in rather general terms. The fact that there are inequality constraints imposed on the vector $\mathbf{q}$ makes its solution difficult. The next section will present an approach that renders it possible to solve the above optimisation problem.

3. Trajectory generation

To solve the problem defined in the above section, the approach which is an extension of previous works [20], [21], [24] on trajectory planning for stationary holonomic redundant manipulators has been used. The method uses penalty functions (interior or exterior) [7], [8], [11] which cause the inequality constraints to be satisfied, but the value of the performance index (9) to be somewhat increased. In this case the performance index takes a new form:

$$I(\mathbf{q}) = \tilde{I}(\mathbf{q}) + \sum_{i=0}^{L} \kappa_i\left(c_i(\mathbf{q})\right), \tag{10}$$

where $\kappa_i()$ is the penalty function which associates a penalty with a violation of a constraint.

In order to find mobile manipulator motion along the path (1), at first we give necessary conditions for minimum of function (10) at the pose $\mathbf{P}_f$. Let us note that, in fact, we are now solving a point-to-point control problem. In this case, the task of searching for an optimal configuration for the given final point can be described as follows:

$$\mathbf{P}(\mathbf{q}) - \mathbf{P}_f = \mathbf{0}, \tag{11}$$

$$\mathbf{A}(\mathbf{q})\ \dot{\mathbf{q}} = \mathbf{0}, \tag{12}$$

$$\min_{\mathbf{q}} I(\mathbf{q}). \tag{13}$$

Following derivation method presented in the work [10], the necessary conditions for minimum of function (13) with equality constraints (11)-(12) take the form, as follows:

$$\left[\ \left(\left(\mathbf{J}^R(\mathbf{q})\right)^{-1}\mathbf{J}^F(\mathbf{q})\right)^T \quad -\mathbf{1}_{n-m-k}\ \right] \mathbf{I}_q(\mathbf{q}) = \mathbf{0}, \tag{14}$$

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where $\mathbf{J}(\mathbf{q}) = \left[\begin{array}{cc} (\mathbf{j}(\mathbf{q}))^T & (\mathbf{A}(\mathbf{q}))^T \end{array}\right]^T$ is $((m+k)\times n)$ dimensional full rank matrix (implied by maximisation the manipulability measure), $(m+k) < n$, $\mathbf{J}^R(\mathbf{q})$ is $((m+k)\times(m+k))$ dimensional matrix constructed from $(m+k)$ linear independent columns of $\mathbf{J}$, $\mathbf{J}^F(\mathbf{q})$ is $((m+k)\times(n-m-k))$ dimensional matrix obtained by excluding $\mathbf{J}^R$ from $\mathbf{J}$, $\mathbf{1}_{n-m-k}$ denotes $(n-m-k)\times(n-m-k)$ identity matrix, $\mathbf{I}_q(\mathbf{q}) = \partial I/\partial\mathbf{q}$ is n-dimensional vector.

Equation (14) introduces $(n-m-k)$ dependencies which, in combination with the conditions (11) and (12), allow finding an optimal configuration for a given final point. At last, a solution to the system of equations given below yields the robot optimal configuration.

$$\mathbf{E}(\mathbf{q},\dot{\mathbf{q}}) = \left(\begin{array}{c} \mathbf{P}(\mathbf{q}) - \mathbf{P}_f \\ \left[\begin{array}{cc} \left(\left(\mathbf{J}^R(\mathbf{q})\right)^{-1}\mathbf{J}^F(\mathbf{q})\right)^T & -\mathbf{1}_{n-m-k} \end{array}\right]\mathbf{I}_q(\mathbf{q}) \\ \mathbf{A}(\mathbf{q})\ \dot{\mathbf{q}} \end{array}\right) = \mathbf{0}. \tag{15}$$

The mapping $\mathbf{E}$ may be interpreted as some measure of error between a current configuration $\mathbf{q}(t)$ and an acceptable nonsingular final configuration $\mathbf{q}(T)$. The m-first components of $\mathbf{E}$ is responsible for reaching the given final point, the next $(n-m-k)$ dependencies are responsible for the fulfilment of inequality constraints (7) and for maximising the manipulability measure (9), the k-last components are responsible for the fulfilment of nonholonomic constraints (3).

Introducing the following substitutions:

$$\mathbf{E}^I(\mathbf{q}) = \left(\begin{array}{c} P(\mathbf{q}) - P_f \\ \left[\begin{array}{cc} \left(\left(\mathbf{J}^R(\mathbf{q})\right)^{-1}\mathbf{J}^F(\mathbf{q})\right)^T & -\mathbf{1}_{n-m-k} \end{array}\right]\mathbf{I}_q(\mathbf{q}) \end{array}\right), \quad \mathbf{E}^{II}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{A}(\mathbf{q})\ \dot{\mathbf{q}}$$

dependency (15) may be rewritten as:

$$\mathbf{E}(\mathbf{q},\dot{\mathbf{q}}) = \left(\begin{array}{c} \mathbf{E}^I(\mathbf{q}) \\ \mathbf{E}^{II}(\mathbf{q},\dot{\mathbf{q}}) \end{array}\right). \tag{16}$$

The solution of equation (16) is the final nonsingular configuration $\mathbf{q}(T)$. To find the trajectory of the mobile manipulator $\mathbf{q}(t)$ from the initial point $\mathbf{P}_0$ to the final $\mathbf{q}(T)$, the following dependencies are proposed:

$$\begin{array}{r} \ddot{\mathbf{E}}^I + \mathbf{\Lambda}_V^I\,\dot{\mathbf{E}}^I + \mathbf{\Lambda}_L^I\,\mathbf{E}^I = \mathbf{0} \\ \dot{\mathbf{E}}^{II} + \mathbf{\Lambda}_L^{II}\,\mathbf{E}^{II} = \mathbf{0} \end{array}, \tag{17}$$

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where $\mathbf{\Lambda}_V^I = diag\left(\Lambda_{V,1}^I, \ldots, \Lambda_{V,n-k}^I\right)$ and $\mathbf{\Lambda}_L^I = diag\left(\Lambda_{L,1}^I, \ldots, \Lambda_{L,n-k}^I\right)$ are $((n-k) \times (n-k))$ diagonal matrices with positive coefficients $\Lambda_{V,i}^I$, $\Lambda_{L,i}^I$ ensuring the stability of the first equation, $\mathbf{\Lambda}_L^{II} = diag\left(\Lambda_{L,1}^{II}, \ldots, \Lambda_{L,k}^{II}\right)$ is $(k \times k)$ diagonal matrix with positive coefficients $\Lambda_{L,i}^{II}$ ensuring the stability of the second equation.

Eq. (17) is a system of homogeneous differential equations with constant coefficients. In order to solve it and find the trajectory of the mobile manipulator, $(2n-k)$ consistent dependencies should be given. These dependencies are given by the initial conditions, obtained from the mapping $\mathbf{E}$ for $t = 0$ taking into account dependencies (5) and (6):

$$\mathbf{E}_{t=0}^I = \left(E_{0,1}^I, \ldots E_{0,n-k}^I\right)^T, \quad \dot{\mathbf{E}}_{t=0}^I = \mathbf{0}_{1\times(n-k)}, \quad \dot{\mathbf{E}}_{t=0}^{II} = \mathbf{0}_{1\times k} \tag{18}$$

As is easy to see, the form of differential equation (17) ensures that its solution is asymptotically stable for positive coefficients $\Lambda_{V,i}^I$, $\Lambda_{L,i}^I$, $\Lambda_{L,i}^{II}$. Additionally, if these coefficients satisfy the inequalities $\Lambda_{V,i}^I > 2\sqrt{\Lambda_{L,i}^I}$ function $\mathbf{E}(\mathbf{q}, \dot{\mathbf{q}})$ is also a strictly monotonic function. In [21] it has been proved that the properties of the solution imply fulfillment of the conditions (11) and (6), i.e. the mobile manipulator reaches the final point $\mathbf{P}_f$ with zero velocity. Moreover, for the initial nonsingular configuration, fulfilling constraint (7), i.e. satisfying the condition (14), robotic motion is free of singularities and fulfils constraints (7) during the movement to the final point $\mathbf{P}_f$.

Let us note that, in the presented method, the Pfaffian constraints are not involved by the penalty function approach. To incorporate them, the mapping $\mathbf{E}^{II}$ has been introduced and used to formulate the error dynamic equation (lower dependency (17)). Taking into account initial condition (6) the mapping $\mathbf{E}^{II} = \mathbf{0}$ for $t = 0$. Additionally, the solution of the differential equation (17) is asymptotically stable, so mapping $\mathbf{E}^{II}$ is equal to zero in each time instant. Hence, in the presented method, the Pfaffian constraints are satisfied exactly for the whole time interval.

The trajectory of the mobile manipulator determined from equation (17) depends of the choice of the parameters $\mathbf{\Lambda}_V^I$, $\mathbf{\Lambda}_L^I$ and $\mathbf{\Lambda}_L^{II}$. It can be seen, from dependency (16), that m-first elements of matrices $\mathbf{\Lambda}_V^I$ and $\mathbf{\Lambda}_L^I$ specifies the end-effector motion. Particular solution of differential equation (17) for these components can be written as:

$$E_i^I = E_{0,i}^I \frac{r_{i2}}{r_{i2} - r_{i1}} \left(e^{r_{i1}t} - \frac{r_{i1}}{r_{i2}} e^{r_{i2}t}\right), \quad i = 1 : m, \tag{19}$$

where r_{i1} and r_{i2} are roots of the characteristic equation of the equation (17).

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If $\Lambda^I_{V,i} = \Lambda^I_{V,j}$ and $\Lambda^I_{L,i} = \Lambda^I_{L,j}$ for $i, j = 1 : m$ then m-first components of differential equation (17) have the same characteristic equation, so $r_{i1} = r_{j1}$ and $r_{i2} = r_{j2}$ for $i, j = 1 : m$. Hence, a linear dependence between two different elements of mapping $\mathbf{E}$ can be seen:

$$E^I_i = \frac{E^I_{0,i}}{E^I_{0,j}} E^I_j, \quad i \neq j, \quad i, j = 1 : m. \tag{20}$$

Dependency (20) forces the end-effector to move along the line section between initial and final points. Therefore, constraint (1) is not considered further on, because it is automatically satisfied for the trajectory obtained according to the proposed approach. In this solution, the end-effector path is not given explicitly, so it is not possible to set the orientation of the end-effector during its movement, however, it is possible to set the orientation of the end-effector at the end of the motion.

Finally, the trajectory of the mobile manipulator tracing the line section path can be determined by simple transformations from the dependency (17) as:

$$\begin{bmatrix} \mathbf{E}^I_q \\ \mathbf{E}^{II}_{\dot{q}} \end{bmatrix} \ddot{\mathbf{q}} = - \begin{bmatrix} \frac{d}{dt}\left(\mathbf{E}^I_q\right) \dot{\mathbf{q}} + \mathbf{\Lambda}^I_V \mathbf{E}^I_q \dot{\mathbf{q}} + \mathbf{\Lambda}^I_L \mathbf{E}^I \\ \mathbf{E}^{II}_q \dot{\mathbf{q}} + \mathbf{\Lambda}^{II}_L \mathbf{E}^{II} \end{bmatrix}, \tag{21}$$

where $\mathbf{E}^I_q = \partial \mathbf{E}^I(\mathbf{q}) \big/ \partial \mathbf{q}$, $\mathbf{E}^{II}_{\dot{q}} = \partial \mathbf{E}^{II}(\mathbf{q}, \dot{\mathbf{q}}) \big/ \partial \dot{\mathbf{q}}$ and $\mathbf{E}^{II}_q = \partial \mathbf{E}^{II}(\mathbf{q}, \dot{\mathbf{q}}) \big/ \partial \mathbf{q}$.

Equation (21) specifies the system of differential equations of the second order. The trajectory of a mobile manipulator is the solution of this system. To determine values of controls which are required to realise the trajectory it is necessary to transform the dynamic equation (4). For this purpose, the nonholonomic constraints are expressed by an analytic driftless dynamic system:

$$\dot{\mathbf{q}}^p = \tilde{\mathbf{N}}(\mathbf{q}^p)\ \nu, \tag{22}$$

where $\tilde{\mathbf{N}}(\mathbf{q}^p)$ denotes $(p \times (p-k))$ dimensional matrix and ν is $(p-k)$ dimensional vector including scaled angular velocities of the platform driving wheels.

Introducing the full rank matrix:

$$\mathbf{N}(\mathbf{q}) = \begin{bmatrix} \tilde{\mathbf{N}}(\mathbf{q}^p) & \mathbf{0}_{p\times r} \\ \mathbf{0}_{r\times(p-k)} & 1_{r\times r} \end{bmatrix} \tag{23}$$

and multiplying the dynamic equation (4) by $\mathbf{N}^T(\mathbf{q})$, noting that $\tilde{\mathbf{A}}(\mathbf{q}^p)\ \tilde{\mathbf{N}}(\mathbf{q}^p) = \mathbf{0}$, we obtain:

$$\mathbf{N}^T(\mathbf{q})\ \mathbf{M}(\mathbf{q})\ \ddot{\mathbf{q}} + \mathbf{N}^T(\mathbf{q})\ \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{N}^T(\mathbf{q})\ \mathbf{B}\,\mathbf{u}. \tag{24}$$

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The above equation allows determination of controls for a current trajectory.

In order to ensure fulfilment of constraints (4) an idea of trajectory scaling, presented in [22]-[24], is used. In this case it is suggested to modify values of gain coefficients $\mathbf{\Lambda}_V^I$ and $\mathbf{\Lambda}_L^I$. Using dependency (24), control constraints (8) may be written as follows:

$$\mathbf{u}_{\min} \leq \left(\mathbf{N}^T(\mathbf{q})\,\mathbf{B}\right)^{-1}\mathbf{N}^T(\mathbf{q})\,\mathbf{M}(\mathbf{q})\;\ddot{\mathbf{q}} + \left(\mathbf{N}^T(\mathbf{q})\,\mathbf{B}\right)^{-1}\mathbf{N}^T(\mathbf{q})\,\mathbf{F}(\mathbf{q},\dot{\mathbf{q}}) \leq \mathbf{u}_{\max} \tag{25}$$

Let us note that the matrix $\mathbf{N}^T\mathbf{B}$ is nonsingular. Writing matrix $\mathbf{B}$ in the form of:

$$\mathbf{B} = \begin{bmatrix} \tilde{\mathbf{B}} & \mathbf{0}_{p\times r} \\ \mathbf{0}_{r\times(p-k)} & \mathbf{1}_{r\times r} \end{bmatrix},$$

where $\tilde{\mathbf{B}}$ is $p\times(p-k)$ matrix describing which state variables of the mobile platform are directly driven by actuators, $\mathbf{N}^T\mathbf{B}$ can be determined as:

$$\mathbf{N}^T(\mathbf{q})\,\mathbf{B} = \begin{bmatrix} \tilde{\mathbf{N}}^T(\mathbf{q}^p)\,\tilde{\mathbf{B}} & \mathbf{0}_{(p-k)\times r} \\ \mathbf{0}_{r\times(p-k)} & \mathbf{1}_{r\times r} \end{bmatrix}.$$

$\mathbf{N}^T\mathbf{B}$ is full rank matrix if $\tilde{\mathbf{N}}^T\tilde{\mathbf{B}}$ is full rank matrix. For the mobile platform of $(2,0)$ class, considered in the Numerical example section, matrices $\tilde{\mathbf{N}}$ and $\tilde{\mathbf{B}}$ take the following form:

$$\tilde{\mathbf{N}}^T = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1/a & 2/r & 0 \\ \cos(\theta) & \sin(\theta) & -1/a & 0 & 2/r \end{bmatrix}, \quad \tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{0}_{3\times 2} \\ \mathbf{1}_{2\times 2} \end{bmatrix},$$

where a is a one half of the distance between platform wheels and r denotes the wheel radius.

Hence, $\tilde{\mathbf{N}}^T\tilde{\mathbf{B}}$ is a nonsingular diagonal matrix and $\mathbf{N}^T\mathbf{B}$ is nonsingular too.

After simple calculations, using (21), inequalities (25) can be rewritten in a compact form:

$$\mathbf{u}_{\min} \leq \;\mathbf{a}(\mathbf{q},\,\dot{\mathbf{q}})\,\mathbf{\Lambda}(t) + \mathbf{b}(\mathbf{q},\,\dot{\mathbf{q}}) \leq \mathbf{u}_{\max} \tag{26}$$

where $\mathbf{a}(\mathbf{q},\,\dot{\mathbf{q}},) = -\mathbf{N}^{\#}\mathbf{M}\begin{bmatrix} \mathbf{E}_q^I \\ \mathbf{E}_{\dot{q}}^{II} \end{bmatrix}^{-1}\begin{bmatrix} diag\left(\mathbf{E}_q^I\dot{\mathbf{q}}\right), & diag\left(\mathbf{E}^I\right) \\ \multicolumn{2}{c}{\mathbf{0}_{k\times 2(n-k)}} \end{bmatrix}$, $\mathbf{b}(\mathbf{q},\,\dot{\mathbf{q}}) = -\mathbf{N}^{\#}\mathbf{M}\begin{bmatrix} \mathbf{E}_q^I \\ \mathbf{E}_{\dot{q}}^{II} \end{bmatrix}^{-1}\begin{bmatrix} \frac{d}{dt}\left(\mathbf{E}_q^I\right)\dot{\mathbf{q}} \\ \mathbf{E}_q^{II}\dot{\mathbf{q}} + \mathbf{\Lambda}_L^{II}\mathbf{E}^{II} \end{bmatrix} + \mathbf{N}^{\#}\mathbf{F}$, $\mathbf{N}^{\#} = \left(\mathbf{N}^T\mathbf{B}\right)^{-1}\mathbf{N}^T$ and $\mathbf{\Lambda}(t) = \left[\Lambda_{V,1}^I,\ldots,\Lambda_{V,n-k}^I,\;\Lambda_{L,1}^I,\ldots,\Lambda_{L,n-k}^I\right]^T$.

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Dependency (26) introduces $2(n-k)$ inequalities, whereas $\dim(\mathbf{\Lambda}) = 2(n-k)$, hence, assuming the full rank of the matrix $\mathbf{a}(\mathbf{q}, \dot{\mathbf{q}})$ it is possible to determine $2(n-k)$ gain coefficients $\mathbf{\Lambda}(t)$ ensuring fulfilment of constraints (8) at each time instant. Using gain coefficients $\mathbf{\Lambda}_V^I$ and $\mathbf{\Lambda}_L^I$. to scaling the trajectory of the mobile robot can affect the solution of the equation (17). However, due to the fact that rapid changes of gain coefficients lead to large values of controls it is practically reasonable to assume that $\mathbf{\Lambda}_V^I$, $\mathbf{\Lambda}_L^I$ are slowly varying functions of time ($\dot{\mathbf{\Lambda}}_V^I \cong \mathbf{0}$, $\dot{\mathbf{\Lambda}}_L^I \cong \mathbf{0}$), and in such a case the analysis of the solution of equation (17) holds true. Nevertheless, in particular cases if the controls are close to given constraints, small deviations from straight-line path are possible. Finally, the solution of equation (21) with suitable parameters $\mathbf{\Lambda}(t)$ gives an sub-optimal trajectory satisfying path constraints (1), inequality constraints (7), control constraints (8) and boundary conditions (5) and (6).

For practical reasons, it is interesting to know the computational complexity of the equation (21). The dimension of the robot task space is assumed to be constant, estimations are carried out at any time instant of the robot task accomplishment. On the basis of equation (16), it can be shown that the complexity of $\mathbf{E}^{II}$ is of the order $O(n)$. Assuming that $\mathbf{J}(\mathbf{q})$, $I_q(\mathbf{q})$ are given analytically, the complexity of $\mathbf{E}^I$ is also $O(n)$. It follows that the computation of $\mathbf{E}_q^I$, $\mathbf{E}_q^{II}$, $\mathbf{E}_{\dot{q}}^{II}$ involves $O\left(n^2\right)$ operations. The computational complexity of $(d/dt)\,\mathbf{E}_q^I$ is of the order $O\left(n^3\right)$ and it is the most complex element of the right-hand side of the equation (21). Other calculations involve at most $O\left(n^2\right)$ operations. Determination of the value $\ddot{\mathbf{q}}$ from (21) requires calculating the inverse of the matrix $\left[\begin{array}{cc} \left(\mathbf{E}_q^I\right)^T & \left(\mathbf{E}_{\dot{q}}^{II}\right)^T \end{array}\right]^T$. The computational complexity of this operation is of the order $O\left(n^3\right)$. Finally, the computational complexity of the whole equation (21) is of the order $O\left(n^3\right)$. Although the computational complexity of the trajectory generator (21) seems to be relatively large, it takes into account all the control/state dependent constraints (mechanical and control constraints, collision avoidance conditions, maximising the manipulability measure), which is very important from the practical point of view.

4. Numerical example

In the numerical example, a mobile manipulator, shown in Fig. 1, consisting of a nonholonomic platform of $(\delta_m,\ \delta_s) = (2,\ 0)$ class and a 3DOF RPR type holonomic manipulator working in a three-dimensional task space is considered. In order to increase the degree of its redundancy, the orientation of its end-effector is not taken into consideration.

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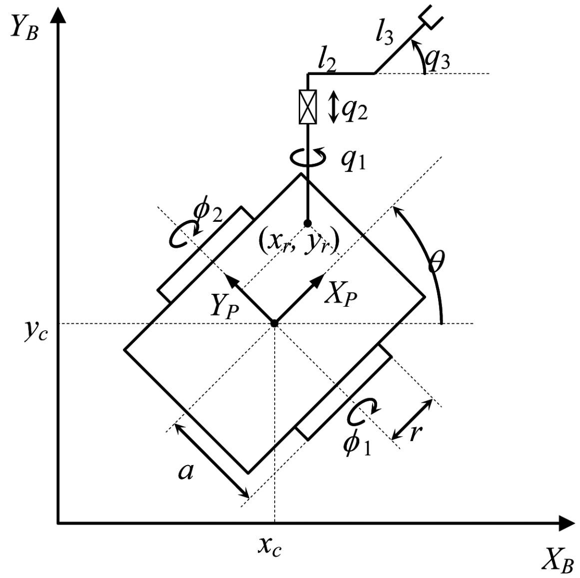

Fig. 1. Kinematic scheme of the mobile manipulator

The mobile manipulator is described by the vectors of generalised coordinates:

$$\mathbf{q}^p = (x_c,\, y_c,\, \theta,\, \phi_1,\, \phi_2)^T\,, \quad \mathbf{q}^r = (q_1,\, q_2,\, q_3)^T\,,$$

where (x_c, y_c) denotes the platform centre location and θ is the platform orientation, ϕ_1, ϕ_2 are angles of driving wheels, q_1, q_2, q_3 stand for angles and offset of the manipulator joints.

The platform works in $X_B Y_B$ plane of the base coordinate system $O_B X_B Y_B Z_B$. The coordinate system $O_P X_P Y_P Z_P$ is attached to the mobile platform at the midpoint of the line segment connecting the two driving-wheels. The holonomic manipulator is connected to the platform at the point $[x_r,\, y_r,\, 0]^T$ of $O_P X_P Y_P Z_P$ system. The kinematic equation of mobile manipulator is given as:

$$\mathbf{P}(\mathbf{q}) = \begin{pmatrix} 0.5 l_3 \left(\cos(\theta + q_1 - q_3) + \cos(\theta + q_1 + q_3)\right) + l_2 \cos(\theta + q_1) + \\ + x_r \cos(\theta) - y_r \sin(\theta) + x_c \\ 0.5 l_3 \left(\sin(\theta + q_1 - q_3) + \sin(\theta + q_1 + q_3)\right) + l_2 \sin(\theta + q_1) + \\ + x_r \sin(\theta) + y_r \cos(\theta) + y_c \\ q_2 - l_3 \sin(q_3) \end{pmatrix},$$

where l_2 and l_3 are the length of the second and the last arm of manipulator.

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The motion of the platform is subject to one holonomic and two non-holonomic constraints, so constraints (2) in this case take the following form:

$$\begin{bmatrix} 0 & 0 & 1 & -\frac{r}{2a} & \frac{r}{2a} \\ 1 & 0 & 0 & -\frac{r}{2}\cos(\theta) & -\frac{r}{2}\cos(\theta) \\ 0 & 1 & 0 & -\frac{r}{2}\sin(\theta) & -\frac{r}{2}\sin(\theta) \end{bmatrix} \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \mathbf{0},$$

where r is the radius of driving wheels and a stands for half-distance between the wheels.

The kinematic parameters of the mobile manipulator are given as (all physical values are expressed in the SI system): $l_2 = 0.3$, $l_3 = 0.2$, $a = 0.3$, $r = 0.075$, $x_r = 0.2$, $y_r = 0.0$. The masses of the mobile manipulator's elements amount to: $m_p = 94$, $m_2 = 20$, $m_3 = 20$, where m_p is the total mass of the platform and m_2, m_3 are the masses of the manipulator's arm.

The task of the manipulator is to trace a line section path between points: $\mathbf{P}_0 = (0.5,\ 0.0,\ 1.0)^T$ and $\mathbf{P}_f = (3.0,\ 1.5,\ 1.5)^T$.

Constraints (5), (7), (8) amount to:

$$\mathbf{q}_0 = (0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 1.2,\ \pi/2)^T$$

$$\mathbf{q}^r_{\min} = (-\pi,\ 0.5,\ -\pi/2)^T \qquad \mathbf{q}^r_{\max} = (\pi,\ 2.0,\ 3\pi/2)^T$$

$$\mathbf{u}_{\min} = (-5.0,\ -5.0,\ -2.0,\ 0.0,\ -2.0)^T \quad \mathbf{u}_{\max} = (5.0,\ 5.0,\ 2.0,\ 400.0,\ 10.0)^T$$

The penalty function introduced in order to take into account constraints (7) is taken as follows:

$$\kappa_i\left(c_i(\mathbf{q})\right) = \begin{cases} \rho\left(c_i(\mathbf{q}) - \varepsilon_i\right)^2 & for\ c_i(\mathbf{q}) \le \varepsilon_i \\ 0 & \text{otherwise} \end{cases},$$

where ρ denotes the constant positive coefficient determining the strength of penalty, ε_i is the constant positive coefficient determining the threshold value which activates the i-th constraint.

Three cases of performing this task are considered. The first one is an end-effector motion along the line section path without control constraints (8) and collision avoidance constraints. In the second case the mobile manipulator is used to solve the same task as in the first experiment, but control constraints (8) are taken into account. In the third simulation, there is an addition of obstacles in the workspace.

In the first case, the values of gain coefficients are taken as: $\Lambda^I_{L,i} = 1.0$, $\Lambda^I_{V,i} = 2.1$, $\Lambda^{II}_{L,i} = 1.0$. The mobile manipulator controls (u_1, u_2 – wheel

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torques, u_3, u_5 – joint torques and u_4 – joint force) obtained in the numerical simulations are shown in Fig. 2. For this solution, the final time T is 13.58 $[s]$, and it can be seen that the determined controls exceed the assumed constraints.

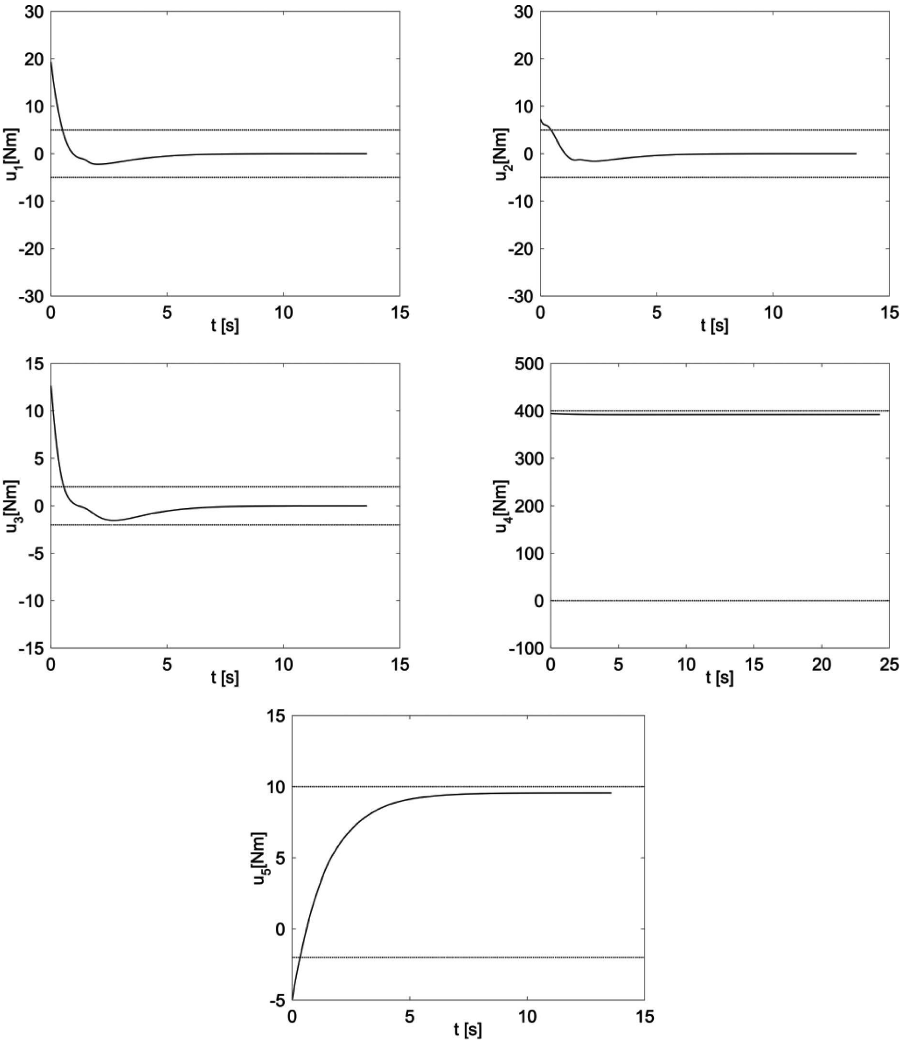

Fig. 2. Controls corresponding to the motion for the first case

The second simulation presents the solution of the same task as the first one, but control constraints (8) are considered. To satisfy these constraints, the values of gain coefficients $\mathbf{\Lambda}$ are determined from inequalities (26). For simplicity of numerical simulations, $\mathbf{\Lambda}$ are assumed to be constant: $\Lambda^I_{L,i} = 0.1$, $\Lambda^I_{V,i} = 0.66$. For this solution, the final time T is increased and it equals

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24.28 [s], but the determined controls do not exceed the assumed constraints. Fig. 3 presents the controls obtained in this simulation.

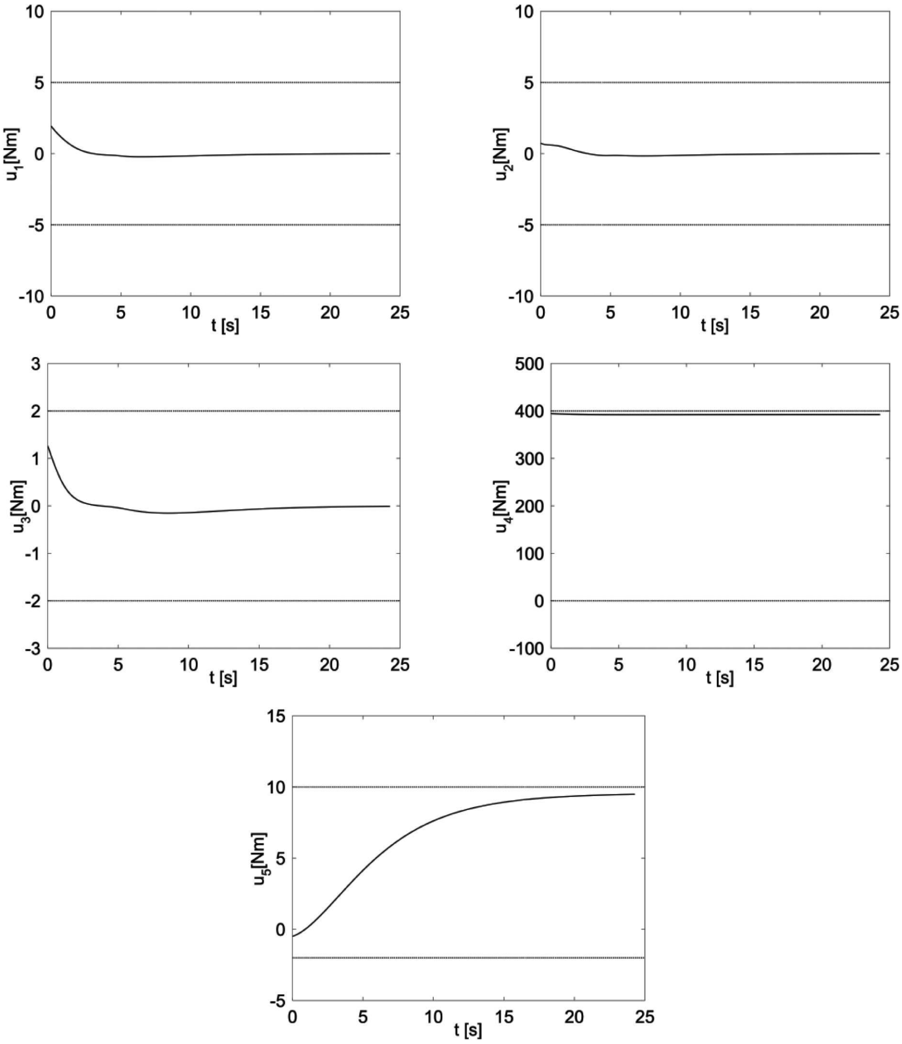

Fig. 3. Controls corresponding to the motion for the second case

In the third simulation, there are two obstacles which may collide with the mobile manipulator. The first one is represented by a cylinder with radius 0.25 and height 2.0. The centre point of its base is placed at $(0.7, 0.6, 0.0)^T$. The second obstacle is a sphere with radius 0.25 and centre point placed at $(2.0, 0.3, 0.5)^T$. If collision avoidance are not taken into account, the platform collides with the cylinder and the holonomic manipulator collides with the sphere (the platform doesn't collide with the sphere because it is above X_BY_B plane). To show these potential collisions, distances between the mobile ma-

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nipulator from the second simulation and the centres of obstacles introduced in the third simulation are shown in Fig. 4. As can be seen, the mobile robot collides with the first obstacle for $t \in [2.3, 5.6]$ and collides with the second obstacle for $t \in [6.6, 8.7]$.

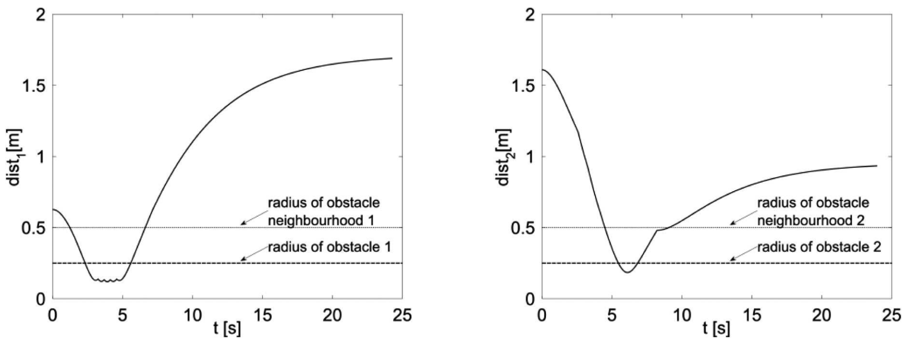

Fig. 4. Distances between the mobile manipulator and the centres of the obstacles if collision avoidance conditions are not taken into account

Parameters for the third simulation are the same as in the previous one. For this experiment, the final time T increased to 24.44 [s], but both control constraints and collision avoidance constraints are satisfied. Due to the obstacles acting on the mobile manipulator, the controls are slightly sharp. Figures 5, 6 and 7 present the manipulator motion, distances between the mobile manipulator and the centres of the obstacles and controls obtained in this simulation.

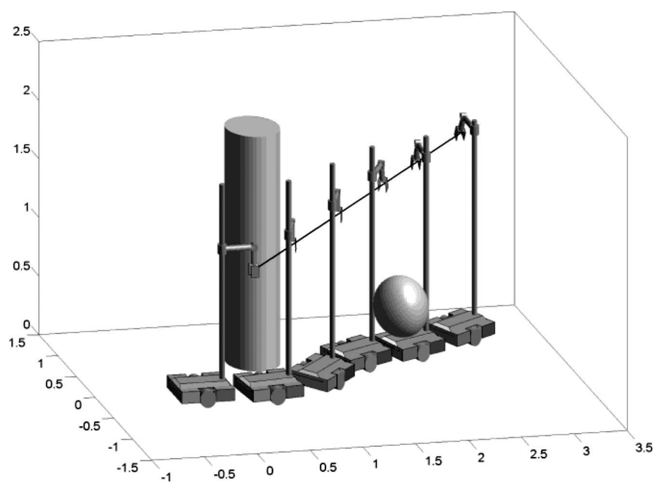

Fig. 5. Collision-free mobile manipulator motion for the third case

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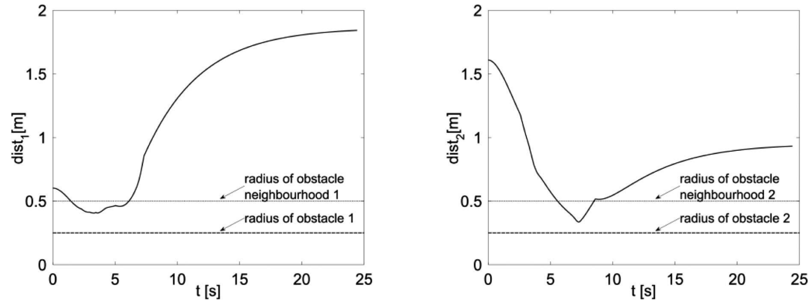

Fig. 6. Distances between the mobile manipulator and the centres of the obstacles for the third case

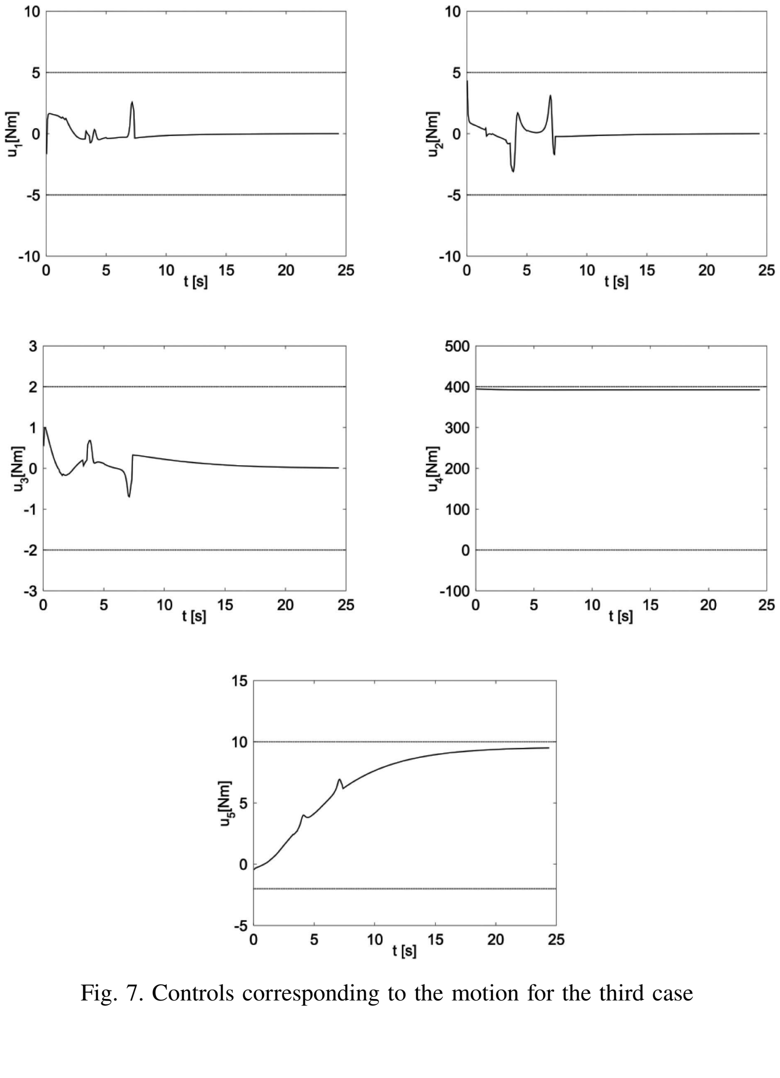

Fig. 7. Controls corresponding to the motion for the third case

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As can be seen from Fig. 6, the mobile manipulator penetrates the safety zone of the first obstacle from the time 1.5 to the time equal to 6.1. At the time instant 5.6 robot enters the safety zone of the second obstacle and it remains there to the time equal to 8.5.

5. Conclusions

In the paper, a sub-optimal motion of the mobile manipulators has been determined in the presence of the obstacles in the workspace. The task of the robot has been to move the end-effector along a prescribed geometric path being a line section, the reference trajectory of a mobile platform has not been needed. This task has been shown to be equivalent to some point-to-point problem whose solution may be easier determined. Constraints connected with the existence of mechanical limitations for manipulator configuration, collision avoidance conditions and control constraints have been considered. Additionally, boundary conditions resulting from the task to be perform have been also taken into account. Moreover, the manipulator motion has been planned in a manner to maximise the manipulability measure in order to avoid manipulator singularities.

The problem has been solved by using penalty functions and a redundancy resolution at the acceleration level. The resulting trajectory has been scaled in a manner to fulfil the constraints imposed on the controls. The property of asymptotic stability of the proposed solution implies fulfilment of all the constraints imposed. The proposed approach to trajectory planning is a computationally efficient method. The effectiveness of the solution is confirmed by the results of computer simulations.

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Planowanie bezkolizyjnej trajektorii manipulatorów mobilnych przy ograniczeniach na sterowania

S t r e s z c z e n i e

W pracy przedstawiono metodę planowania bezkolizyjnej trajektorii manipulatora mobilnego śledzącego odcinek prostoliniowy. Metoda nie wymaga określenia trajektorii dla mobilnej platformy, uwzględnia ograniczenia konstrukcyjne oraz ograniczenia na sterowania. W rozwiązaniu wykorzystano rozkład redundancji na poziomie przyspieszeń oraz metodę funkcji kary. Ograniczenia nieholonomiczne w formie Pfaffa zostały wprowadzone wprost do algorytmu sterowania. Pokazano, że zadanie robota jest równoważne pewnemu problemowi point-to-point, którego rozwiązanie może być łatwiej wyznaczone. Ruch mobilnego manipulatora jest planowany w taki sposób, aby maksymalizować miarę manipulowalności dzięki czemu przebiega on z dala od konfiguracji osobliwych. Zastosowanie metody zostało zilustrowane symulacjami komputerowymi, w których rozpatrywano manipulator mobilny, składający się z nieholonomicznej platformy klasy (2,0) oraz holonomicznego manipulatora typu RPR, operujący w trójwymiarowej przestrzeni roboczej.

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