Scientific Journals
of the Maritime University of Szczecin

Zeszyty Naukowe
Akademii Morskiej w Szczecinie

2016, 45 (117), 159–167
ISSN 1733-8670 (Printed)
ISSN 2392-0378 (Online)
DOI: 10.17402/100

Received: *31.08.2015*
Accepted: *18.12.2015*
Published: *25.03.2016*

On the mathematical theory of evidence in navigation

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Key words: nautical evidence, uncertainty, belief structures, position fixing, fix accuracy, systematic errors evaluation

Abstract

In most problems encountered in navigation, imprecision and uncertainty dominate. Methods of their processing rely on rather obsolete formalisms based on probability and statistics. Available solutions exploit a limited amount of available data, and knowledge is necessary to interpret the achieved results. Profound a posteriori analysis is rather limited; thus, the informative context of solutions is rather poor. Including knowledge in a nautical data processing scheme is impossible. Remaining stuck with the traditional formal apparatus, based on probability theory, one cannot improve the informative context of obtained results. Traditional approaches toward solving problems require assumptions imposed by the probabilistic model that exclude possibility of modelling uncertainty. It should be noticed that the flexibility of exploited formalism decide the quality of upgrading models and, subsequently, on the universality of the final results. Therefore, extension of the available formalisms is a challenge to be met. Many publications devoted to the mathematical theory of evidence (MTE) and its adaptation for nautical science in order to support decision making in navigational processes have enabled one to submit and defend the following proposition. Many practical problems related to navigational ship conducting and to feature uncertainty can be solved with MTE; the informative context of the obtained results is richer when compared to those acquired by traditional methods. Additionally, a posteriori analysis is an inherent feature of the new foundations. The brief characteristics of a series of publications devoted to the new methodology are the main topics of this paper.

Introduction

The mathematical theory of evidence (MTE), also known as belief theory or Dempster-Shafer theory (Dempster, 1968; Shafer, 1976), exploits belief and plausibility measures and operates on belief assignments also known as belief functions. The theory also offers combination schemes in order to increase the informative context of the initial evidence. The evidence is meant as a collection of facts and knowledge. In navigation, facts are the results of observations such as bearings, distances or horizontal angles. Given pieces of evidence, a combination scheme is expected to enable the position fixing of a ship and its final uncertainty analysis as well as systematic errors identification. Extension of the theory for possibilistic platforms (Yen, 1990) created new opportunities for modelling initial uncertainty. In the presented applications, uncertainty is due to erroneous observations. It is widely known that all measurements are distorted by various errors.

Possibilistic extension enables the drawing of conclusions based on the results of fuzzy evidence combination, provided adequate formulas are at hand. Appropriate expressions are derived from the general scheme of possibilistic reasoning available in fuzzy systems. Formal descriptions of the problems encountered in navigation involve models that accept imprecise, erroneous and, therefore, uncertain data. In particular, position fixing and its accuracy evaluation along with systematic errors handling are important nautical issues. In addition, the concept is expected to be followed regarding quite numerous problems encountered in many related and different disciplines.

Practical navigation is based upon probability theory. The basis is enough to define distributions of random variables that are assumed to be of measured values. It also enables a priori evaluation of fixes taken according to certain schemata because accuracy is calculated with formulas designated for selected schedules of observations taking into account the constellation of landmarks and approximate and crisp valued measurements error.

Discrete models of calculations have not been exploited in navigation. Therefore, the Bayesian approach is not popular among seafarers even though it enables the exploration of the area in the vicinity of the fix. The output of exploration might deliver important information regarding the quality of the ship's fixed position. Discrete models require high computation power, which modern personal computers provide rather satisfactorily. The lack of popularity of the approach may also have resulted from the underestimated attractiveness of the Bayesian evidence reasoning scheme.

The Bayesian approach enables reasoning on the probability of the fix being located in each point within a search area, an important issue in navigation. Unfortunately, it does not allow for including uncertainty into the upgraded models. This appears to be the main disadvantage of the concept. Discrete models that include uncertainty can be created with MTE. The theory can be perceived as an extension of the Bayesian concept. It also offers a combination mechanism, enabling the enrichment of the informative context of initial evidence. Despite its broad ability, the theory still remains unpopular in the presented scope of interest.

Expectations regarding the flexibility of the upgraded models are high. All items that affect fixed position should be included in the computations. One can mention the kind of random distributions of measurements taken with a particular navigational aid and discrepancies in the parameters of such features. It is popular to state that the mean error of a bearing taken with radar is interval valued within the range of [±1°, ±2.5°]. The presented evaluation, a piece of knowledge regarding mean error appears as a fuzzy figure; thus, fuzziness should be accepted and taken into account during computations. Subjective assessment, also in form of linguistic terms, of each observation should be accepted and processed. Empirical distributions are also supposed to be recognized and included in the calculations. The most important requirement is the ability for objective evaluation of the obtained fix based on measures indicating the probability of its location within the explored areas. Meeting the above stated expectations is impossible with traditional formal apparatus. Its ability is almost exhausted in the considered applications. Research and published works devoted to new platforms and modern environments have put attention on evidence theory, which delivers a wide range of new opportunities.

A comparison of the traditional way of position fixing and an approach based on MTE has been presented in recent publications delivered by the author. The main feature of the proposed scheme of reasoning is that it utilizes the possibilistic approach. This approach is justified whenever insufficient data samples are available and when dealing with interval valued estimations of measurements distributions. Thanks to fuzziness, the methodology facilitates upgrading models that enable the introduction of knowledge into the processing scheme. In making a fix, one should consider observations data, nautical knowledge and other factors such as dead reckoning data. The last item is rather difficult to consider in the traditional approach.

First, the most important preliminary issues discussed in publications out of the series delivered by the author are discussed. Expectations regarding the normalization scheme are presented next. Popular ways of belief assignments conversions were proved to be inadequate for nautical applications. Thus, hints to their adjustment were proposed. The last part of the paper is devoted to observations errors handling.

Characteristic of selected preliminary publications

The first paper of the series (Filipowicz, 2009) referred to discussions on the practicality and functionality of the Bayesian and Dempster-Shafer concepts of evidence representation and reasoning and the possibility of the application of belief theory in geodetic positioning and navigational position fixing.

Many authors have pointed to numerous applications involving the first approach, whereas examples employing other concepts are rather scarce. At the time of publication, it was widely said that there are only a few meaningless practical problems solved with MTE (Burrus & Lesage, 2004). Meaningful applications are related to risk analyses (Sun, Srivastava & Mock, 2006) and expert system inference engine implementation (Srivastava, Dutta & Johns, 1996). It should be noted that maritime application of the theory was successful while solving multi-target detection problems (Ayoun & Smets, 2001).

In the paper, practical nautical problems were briefly presented and the potential of the Dempster-Shafer theory exploitation was depicted. One of the presented problems was establishing the imprecise distance from a navigational obstacle. The simple but representative problem appears to be of a data integration type, which is met within data fusion. The scheme of reasoning engaging inaccurate measurements delivered by aids of various credibility levels was presented and discussed based on fuzzy inference schemes available in MTE. The solution obtained was a set of supports for each considered fuzzy hypothesis on representation of the true distance. Support is expressed by belief and plausibility, measures exploited in the Dempster-Shafer theory.

Another problem considered in the paper was related to position fixing based on imprecise measurement data. It was assumed that available data are two dimensional random variables governed by Gaussian distributions. The assumption is often made in navigation. Hypothesis and evidence universes (frames) were defined for position fixing. Next, relations between hypothesis and evidence frames were considered as binary. Degrees of hypothesis point inclusions within measurements related sets were grades of so called location vectors. In the preliminary approach, considered vectors consisted of zero-one elements. Each vector was assigned a credibility value calculated based on the confidence interval probability calculated for assumed distributions. The results of vectors associations were explored with intuitive formulas in order to obtain the fixed position. The simplified approach was further developed.

The next paper (Filipowicz, 2009a), published in Polish, is solid and thoroughly devoted to the fuzzy approach to position fixing. The main idea that remained behind the research and publication was introducing a more flexible approach towards position fixing. The first attempt to engage binary locations seemed inadequate because many publications devoted to nautical science emphasized that the results of observations are random variables governed by various dispersions. However, their substitution with Gaussian distributions is common, and this is, in many cases, a justified assumption. Their parameters should be considered as interval valued rather than crisp ones. It is usually said that the mean error of the distance taken with medium class radar is within range of [±1%, ±1.5%] of the distance taken. Thus, binary representation of nautical knowledge is not adequate. A platform that accepts fuzziness along with multiple random distributions should be introduced. This new approach was presented in the paper. At first, membership functions were discussed and expectations regarding their properties were specified in the context of their nautical usage. Different functions were presented and compared from the point of view of the proposed application. Membership functions are used in order to upgrade belief assignments, which are then converted to belief structures (Denoeux, 2000) and combined in order to make a fix.

The results of belief structures combination are a kind of encoded knowledge base that should be explored in order to seek support for various hypotheses. Hypothesis fuzzy representation and appropriate formulas deliver measures to support the proposition on representing the fix with respect to facts related to imprecise data at hand as well as to nautical knowledge. Considering position fixing, one can simplify the hypothesis representations that take the form of a singleton. Provided with this type of referential, fuzzy set formulas describing belief and plausibility supports were derived and used in numerical examples included in the paper. Strong dependence of the belief support measure on the allocation of hypotheses points was depicted in the publication. Therefore, plausibility support was strongly recommended as the most important factor when a fixed position is selected (Filipowicz, 2009a).

A preliminary version of the algorithm for selecting the fixed position based on navigational aids indications was presented and discussed in detail. Indications were considered as two dimensional random variables governed by various and approximate distribution characteristics. Inconsistency was removed using the Yager concept of normalization (Yager, 1996). At the last stage of the publication, the algorithm was used for solution sensitivity analysis. Measures indicating the selected position versus the degree of uncertainty featured by initial data were compared.

The next paper (Filipowicz, 2010) contains discussions on algorithms implementing MTE and which are intended for position fixing based on various terrestrial observations. Two algorithms were presented. The primary one is designated for an iterative search for the fixed position, whereas the secondary one is intended for hypothesis frame location adjustment. The idea lying behind the supplementary procedure enabled avoiding missing local maxima of the calculated support measures. The concept of random reshuffle of the search space locations exploited in the algorithm is like that encountered in an evolutionary approach towards optimization.

The iterative search for the fixed position explores an area of decreasing size in order to achieve required accuracy. In each loop, for a given search area, new belief assignments are created, normalized and combined. In the final stage, the search area should be small enough to guarantee a satisfying quality of the solution. A regular mesh is spanned over the search area. Thus, the quality of the solution depends on the size of the mesh. It should be noted that the quality is also determined by other, widely known factors. Number and quality of observations as well as the constellation of observed landmarks are main factors deciding the quality of a fix.

In the paper, stopping conditions of the iterating process were also examined. It was suggested that quitting should occur once multiple adjacent points featuring the same maximum support plausibility value are discovered. Under these circumstances, further decrement of the explored area leads to ambiguity increment. It was also noticed that distance between hypothesis frame points should be comparable to mean error of the best observation.

Imprecise estimations of standard deviations result in fuzzy location vector grades. Grades are calculated with membership functions designated for selected confidence intervals with imprecise borders. Location vectors are assigned credibility masses, which refer to the cumulated probability calculated for a respective confidence interval. Crisp valued cumulative probabilities are not justified, because confidence intervals have imprecise limits. Thus, credibility masses should be interval or fuzzy valued. The kind of involved masses determines the types of belief structures. Consequently, their combination engages a more sophisticated formal apparatus to process the interactive variables (Denoeux, 2000). Coping with fuzzy belief assignment degrades the effectiveness of the position fixing algorithm. The necessity of solving multiple numerical constrained problems requires more computation power compared to obtaining the fix based on crisp valued assignments. Thus, position fixing calculations involving fuzzy belief structures are proposed to be split into two stages. At first, fuzzy masses are defuzzyfied to obtain crisp valued assignments that are used by an iterative algorithm until a reasonable estimation of the ship's position is achieved. At the very last stage, fuzzy masses are restored and processed in order to get a broader informative context of the solution.

The fourth paper (Filipowicz, 2011), in its introductory part, contains a compilation of nautical knowledge regarding observations and their isolines (i.e. functions that are measurement projections on a chart). Application of MTE in terrestrial or celestial navigation involves dealing with isolines and their gradients. Confidence intervals are established along gradient directions. The most frequently used are isolines of bearings, distances and horizontal angles, and these functions were discussed in detail. For each case, an example isoline, its gradient's module and direction were presented. Proposed observation evidence encoding was discussed for each considered isoline type.

A significant part of the paper was devoted to empirical type of the random variables distribution. This type of distribution is encountered very often in navigation. They are usually converted to Gaussian ones although it so happens that conversions are not theoretically justified. Thus, empirical distribution inclusion into evidence representation seems natural and necessary. In this case, confidence intervals are substituted by histogram bins, and cumulative probabilities are replaced by relative frequencies of observations falling within the bin. Because available histograms differ, calculated frequencies are rather ranges of values than single figures. Thus, belief assignments upgraded with empirical distributions are interval valued. It remains that a combination scheme involving interval valued structures engages different procedures.

The relation between observations accuracy and mass of combination inconsistency was depicted in the paper. The less accurate the initial data, the greater the inconsistency mass. The disadvantages of two popular normalization schemes, known as Yager and Dempster methods, were emphasized in context of the considered applications. In the Yager method, inconsistency mass increases the uncertainty, but the approach impairs the detection of inconsistency cases. Consequently, the quality of evidence at hand is usually overestimated. In the Dempster concept, all masses assigned to non-empty sets, including those representing uncertainty, are increased by a factor that is a function of the total inconsistency mass. Final masses calculated based on initial assignment is increased during normalization with the modification factor. The confusing behaviour of the approach while low quality or contradictory evidence is being handled was also pointed out.

One must pay attention to the data sets presented in the paper. It is seen that Dempster normalization reduces the number of elements in the final structure. In some cases a 50% reduction in the number of result items was achieved. In view of the exponential complexity of the combination process, the approach seems to dominate over the Yager method. Despite its obvious disadvantages, the last method should not be rejected, because it features an effectiveness that appears to be a serious advantage in coping with robust cases. The approach can be implemented for processing in flow association without recording the complete result structures, as is the case in Dempster normalization. The specificity of the discussed field of application stipulates the modified transformation of the evidence assignments. It should feature the advantages of both mentioned methods. These expectations are hinted in the publication. Details of the new proposal are presented and discussed in the paper that follows.

In the fifth paper (Filipowicz, 2011a), more problems met in maritime applications that feature imprecision and uncertainty are presented. Apart from position fixing and its accuracy evaluation, the scope also embraces the collective assessment afforded in floating object detection. This can be further exploited in solving monitoring area coverage problems and planning search and rescue operations. Analysing and solving the mentioned problems with MTE was the main inspiration for the publication. In the first part of the paper, binary evidential mapping was presented. Representations of uncertain facts and rules were considered. A modus ponens inference pattern was used for conjecture on the consequent given uncertain rule and its antecedent. The obtained result was the same as the outcome of the solution utilizing so-called complete evidential mapping.

Mappings involving fuzzy sets were considered in the second part of the paper. Measurements taken in navigation deliver pieces of evidence with fuzzy location vectors. Each measurement enables the creation of a single belief structure. Belief structures can be used for position fixing. Their combination results create a sort of knowledge base that should be explored in order to make a fix. Formulas enabling the exploration of the base were presented. The point within a hypothesis space with the highest plausibility and belief measures is assumed as the ship's position.

In the sixth publication (Filipowicz, 2012), Dempster-Shafer versus Bayesian approaches were confronted. Belief structures in nautical applications contain encoded evidence related to taken measurements. The result of structures combination is a two-dimensional table that embraces enriched data enabling reasoning on the fix. From a possibilistic viewpoint, this result is a belief assignment that is the distribution of possibilities regarding each hypothesis point's location within evidence related sets. Mechanisms and methods available in MTE can be exploited in order to derive formulas for calculating the interval valued probability of representing fixed positions by each of the hypothesis points.

Alternatively, from a probabilistic standpoint, the result of combination can be perceived as a Bayesian evidence representation. It should be stressed that this standpoint is justified in a limited number of cases. In general, the final structure does not fulfil probability requirements. Nevertheless, one can use Bayesian methods to deduce a formula for calculating the support probability for "being a fix" in any point out of the hypothesis universe. Not surprisingly, both approaches yield virtually the same formula. It should be noted that a possibilistic approach itself can be perceived as an extension for the probabilistic, Bayesian concept. Extension is much more flexible in respect of modelling and the ability to process uncertainty.

Modified normalization concept

Measurement and indication data, along with nautical knowledge, can be encoded into belief functions. Both knowledge and data are considered as evidence that is exploited in navigation. Belief functions in nautical applications represent evidence and are subject to combination in order to increase their informative context. Evidence representations and the results of their combinations could include inconsistencies wherever T-norm operations are involved. Inconsistency must be removed to avoid conflicting final results. Conflict arises when belief is greater than the plausibility measure. In the presented applications, the association of two location vectors with T-norm causes the selection of hypothesis frame points situated within a common area. A null result vector means that there are no points within the intersection and might indicate poor quality evidence (Filipowicz, 2014).

It is assumed that evidence representations should be normalized at the initial and the intermediate stages of processing in order to avoid contradictory results. The most popular normalization procedures feature serious disadvantages. The Yager method disables the detection of inconsistency cases. In the Dempster concept, all masses assigned to non-empty sets are increased by a factor that is a function of the total inconsistency mass. It leads the unacceptable proposition that "the higher the inconsistency mass, the greater the probability assigned to non-empty sets" or, referring to position fixing, "the poorer the quality of data, the higher the credibility attributed to the fix". Therefore, the author's proposal (Filipowicz, 2014a) of conversion has been submitted. The suggested transformation cannot be perceived as normalization because it does not yield a belief structure due to a total mass that could be less than one. The proposed conversion features the following properties:

1. Masses attributed to location vectors are not subject to unjustified changes.
2. Conflicts, which are not zero masses assigned to null sets, increase uncertainty.
3. All fuzzy sets are normal, null grades remain unchanged and, subsequently, conflict detection is not impaired.
4. Plausibility value as a primary factor in selecting fixed position remains intact during conversion.
5. Transformation remains basic for the MTE condition, stipulating that belief measures cannot exceed the plausibility value.

The condition specified in point 5 is not straightforward and needs to be proven. The proof was presented by the author. The most important feature of the transformation is that its output contains normal fuzzy sets that proved to be enough to avoid basic conflict. Moreover, plausibility measures regarding the fix remain intact due to proposed conversion. The approach enables one to maintain the value of the plausibility measure, which is the primary factor determining the selection of the final solution.

In the proposed approach, knowledge included in a computational scheme is something that creates a new opportunity. A new standpoint for perceiving the accuracy of the fix is possible when using reasoning mechanisms. Traditional understanding and accuracy estimating are inadequate in most cases. Appropriate expressions are intended for particular observation schemes that include, at most, three measurements. Although a basic set of data (mean errors and constellation of observed objects) are included in accuracy estimation, applying the same mean error measure for different distributions of isolines seems unjustified. The approach does not correlate quality of observations and accuracy of the obtained fix. Quite often, two cases of fixed positions and their accuracy estimations are the same despite different quality of observations. Intersections of isolines in one case can be spread over a much larger area compared to the second case. Thus, the accuracy of one fix seems be different than in another case. Although true, the statement seems to be somewhat contradictory to the state-of-the-art. Supporters of the idea can claim that as long as measurements are random variables, it may happen thus. Under this assumption, accuracy estimations remain valid in both cases.

Unfortunately, in the traditional approach, accuracy estimation does not reflect the real, a posteriori evaluated quality of the fix. Included computational results emphasize the obvious shortcomings of the traditional approach. In the new approach based on MTE, accuracy estimation, along with its imprecision, is embedded into the reasoning scheme. In the proposed approach, the distribution of probabilities of the fix being located within the explored area is embedded into the methodology. Accuracy can therefore be perceived as a cohesive area within which the probability (plausibility) of the fix location is higher than the required threshold value. It is suggested that the area should embrace points with certain percentages of a plausibility value attributed to the fix.

On the unique property of the combination scheme

The concept of exploiting evidence that is meant as encoded facts and knowledge, in supporting decisions in navigation is based on measurement distributions and fuzziness. Introduced confidence intervals define the probabilities of true isolines being located within appropriate strips established along gradient directions. Modified probabilities are incorporated into belief assignments that enable the modelling of uncertain, imprecise data. Imprecision is due to random errors, but systematic deflections occur quite often. This kind of error should be identified and eliminated. The identification of a permanent measurement shift is an important practical nautical issue.

Figure 1 shows an example in which two observations were made for two objects situated at opposite directions from a ship's position. Measurements are imprecise and distorted with random as well as systematic error.

Figure 2 shows two examples in which pairs of observations were made for two objects situated at counter bearings from a ship's position. Each of the observations is marked with small circular shape that is placed on the abscissa axis and assumed to be collinear with gradient directions. The observations' random error distribution are depicted with two bell functions that represent extreme values given the assumed standard deviation. Rectangular shapes emphasizing the interval valued limits

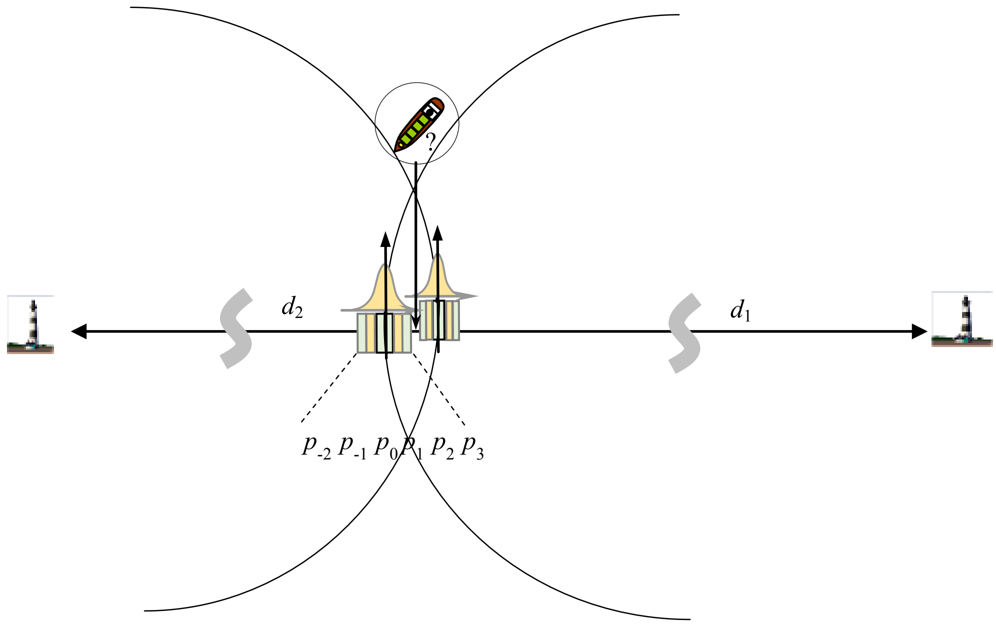

Figure 1. Graphical interpretation of two imprecise measurements, distorted with random and systematic errors, taken for objects at opposite directions

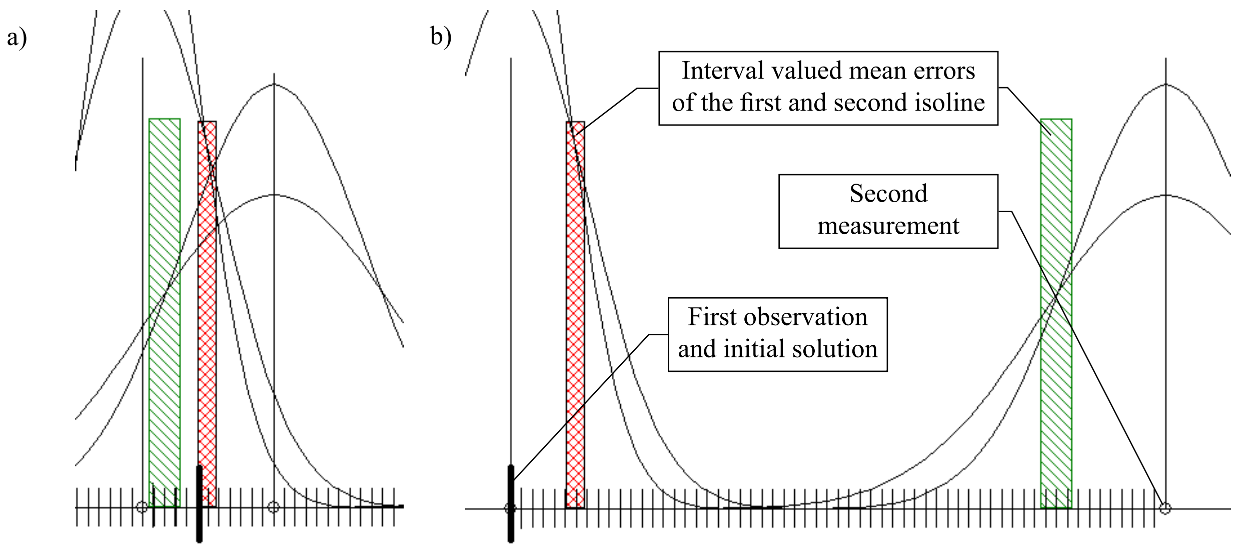

Figure 2. Two cases related to two pairs of observations made for two objects situated at opposite directions. Vertical rectangles refer to interval valued the standard deviations with respect to the measurements

of the mean error are also included. The search space was confined by both isolines, and its discrete points represent the true location of the vessel. The question of which of them best represents the true location is resolved through reasoning based on the results of the evidence combination scheme.

The left-hand side of Figure 2 presents the situation in which the gap between isolines is due to random errors. Case a) presents two observations for which systematic deflection should be excluded because the gap between isolines is smaller than the sum of their mean errors. The statement is rather unlikely for the right-hand side case. The gap can be estimated as the sum of three folded mean errors. Thus, the probability that systematic error was involved is rather high. In order to cover the isolines gap (therefore, to create an artificial case free of systematic error), mean errors were increased during the iterative combination process. The final stage in which the enlarged observation mean errors cover the gap and the association result is presented in Figure 3.

It should be stressed that Figures 2 and 3 remain closely related. Based on the results of combination illustrated in Figure 3, one can reason the solution to the problem presented in Figure 2a. Note that for the latest case, the location of true measurement in between the extreme observations can be easily

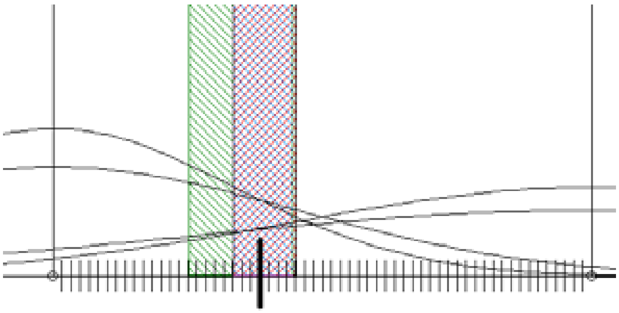

Figure 3. The case presented in Figure 2b with proportionally enlarged observation mean errors. Vertical shapes refer to the interval valued standard deviations with respect to the measurements

Table 1. Summary of data used in the numerical experiment

	Observation 1	Observation 2
Distances	5555.55 m	9259.25 m
Mean errors	55.55 m	92.59 m
Mean error limits	[47.22; 63.89] m	[78.70; 106.48] m
Subjective confidence evaluation	90%	80%
Gap width (see Figure 2 for case a)	107.41 m	
Gap width for case b	555.55 m	

Table 2. The last four iterative combination results

C	δ_1	δ_2	Gap width	$[S^-; S^+]$
3.933	54.07	87.04	141.30	[207.22; 348.33]
4.133	51.48	82.96	134.44	[210.56; 345.00]
4.333	49.07	79.07	128.15	[213.70; 341.85]
4.567	46.67	75.00	121.67	[217.04; 338.52]

evaluated. Therefore, one can reason on the influence of random errors on the final observations' evaluation as, for example, presented in case 2b. The combination results are transferable for the two cases. The systematic error can be estimated as the interval value equal to the observations' gap mean distorted with random deflection. Herein, the scheme of approach was exploited in order to demonstrate the practical aspects of the methodology.

It was proven (Filipowicz, 2014b) that belief and plausibility measures that are calculated based on the results of the iterative combination of two pieces of evidence related to two random variables governed by Gaussian distributions with given approximate standard deviations for which appropriate isolines are separated by a certain Euclidean distance and those obtained from association of evidence related to random variables governed by the same distributions with approximate standard deviations magnified by a certain constant with isolines being separated with distance incremented by the same value are mutually dependent on this constant. The proposition was further exploited in order to calculate the data included in Table 2.

In this chapter, observations were considered that engage two distances made for two objects situated at opposite directions as seen from the observer's position. Both observations resulted in isolines that are assumed to be distorted with random errors and include a systematic shift. Random errors distribution means are supposed to be within the range of ±1% of the measured distance. Possible limits of the estimated mean are within ±15% of their value. Data used in the numerical experiment are gathered in Table 1.

Based upon the presented nautical evidence, a navigator should reason on the quality of measurements and possibly identify the systematic deflection. He is supposed to answer two questions: What is the systematic error of the applied measuring device? and How might random error affect his evaluation? The output generated by implemented software for the above defined numerical example is presented in Table 2, in which the distance units for all data except constant *C* are given in meters. The presented data refer to the last four iterations for which the maximum of the plausibility measure remained high and referred to the same solution. The collected data include the mean errors multiplier *C* with two calculated random deflections δ_i and an interval valued systematic error. Based on the introduced lemma for each multiplier value, random errors were estimated. The evaluation is based on the proposition that enables the migration to the "free from systematic error" case (see both illustrations in Figure 2). Please also note that the direction of random shifts cannot be indicated. The available evidence does not allow a statement of what the signs of random deflection might be; thus, the interval valued permanent errors were calculated taking into account both possible randomness directions (both negative and positive extreme values).

Conclusions

As a result of MTE, approaches towards the theoretical evaluation of tasks including imprecise data are to be reconsidered. It is the navigator who has to handle a set of random points delivered by various navigational aids from which he is supposed to indicate a point as being the position of his ship. Dispersions of points are governed by two dimensional approximate distributions. The fixed position is located somewhere in the vicinity of the indications at hand. This is very similar in the case of measured distances, bearings or horizontal angles. The ship's position is located within the area of the crossings of appropriate isolines. The area of the true position is spanned over the isolines' crossing points provided the available evidence features random errors and might be outside the area once systematic errors prevail. It is supposed that the navigator is able to resolve all dilemmas by applying their knowledge, experience and intuition. MTE delivers a new basis enabling the navigator to formally cope with the problem.

The application of MTE to nautical appliance calibration was also presented in the paper. The hypothesis frame can be reduced in order to guarantee the correctness of a posteriori reasoning in selected nautical applications. Seafarers know where the true measurement is supposed to be located. Observations made for landmarks situated at opposite sides of the ship are examples where such locations can be easily identified. Due to hypothesis frame reduction, the combination inconsistency mass remains small while belief and plausibility are relatively high. It should be emphasised that high inconsistency mass usually indicates poor quality nautical evidence. Yet another reason for a large conflicting mass is a wrongly defined hypothesis frame which, consequently, is not supported by the evidence at hand.

In the presented numerical example, two distance observations distorted with random and systematic errors were considered. The obtained measurement data along with nautical knowledge were encoded into belief structures that were further iteratively combined. Iterations were terminated once a stable solution was achieved. Given this solution, reasoning regarding the combination of systematic deflection free data was carried out. As a result of MTE, the particular distance between isolines due solely to random errors could be calculated. This distance is identified by the hypothesis point with the highest support measures in view of the evidence at hand. It subsequently gives a base for random errors estimations and systematic deflection evaluation. The result fixed error appears interval valued, and the obtained ranges depend on the required threshold probability.

References

1. Ayoun, A. & Smets, P. (2001) Data Association in Multi--Target Detection Using the Transferable Belief Model. *International Journal of Intelligent Systems* 16. pp. 1167–1182.
2. Burrus, N. & Lesage, D. (2004*) Theory of Evidence*. Technical Report No. 0307-07/07/03. Activity: CSI Seminar, Place: EPITA Research and Development Laboratory, Cedex France.
3. Dempster, A.P. (1968) A generalization of Bayesian inference. *Journal of the Royal Statistical Society*, *Series B* 30 (2). pp. 205–247.
4. Denoeux, T. (2000) Modelling vague beliefs using fuzzy valued belief structures. *Fuzzy Sets and Systems* 116, pp. 167–199.
5. Filipowicz, W. (2009) Application of the Theory of Evidence in Navigation. *Knowledge Engineering and Export Systems*. Warsaw: Academic Editorial Board EXIT. pp. 599–614.
6. Filipowicz, W. (2009a) Belief Structures and their Applications in Navigation. *Methods of Applied Informatics* 3. pp. 53–82.
7. Filipowicz, W. (2010) Fuzzy Reasoning Algorithms for Position Fixing. *Measurements Automatics Control* 12. pp. 1491–1495.
8. Filipowicz, W. (2011) Fuzzy Evidence in Terrestrial Navigation. *Navigational Systems and Simulators: Marine Navigation and Safety of Sea Transportation*. A. Weintrit (Ed.) Leiden: CRC Press/Balkema. pp. 65–73.
9. Filipowicz, W. (2011a) Evidence Representation and Reasoning in Selected Applications. *Lecture Notes in Artificial Intelligence*. Jędrzejowicz P, Ngoc Thanh Nguyen, Kiem Hoang (Eds). Berlin, Heidelberg: Springer-Verlag. pp. 251–260.
10. Filipowicz, W. (2012) Evidence Representations in Position Fixing. *Electrical Review* 10b. pp. 256–260.
11. Filipowicz, W. (2014) Fuzzy evidence reasoning and navigational position fixing. *Recent advances in knowledge-based paradigms and applications, advances in inteligent systems and computing*, *234*. Tweedale, J.W., Jane, L.C. (Eds). Heildelberg, New York, London: Springer. pp. 87–102.
12. Filipowicz, W. (2014a) Mathematical Theory of Evidence in Navigation. *Belief Functions: Theory and Applications*. Fabio Cuzzolin (Ed.) Third International Conference, BELIEF 2014 Oxford, UK, Springer International Publishing Switzerland. pp. 199–208.
13. Filipowicz, W. (2014b) Systematic errors handling with MTE. *Procedia Computer Science* 35. pp. 1728–1737.
14. Shafer, G. (1976) *A mathematical theory of evidence*. Princeton: Princeton University Press.
15. Srivastava, R.P., Dutta, S.K. & Johns, R. (1996) An Expert System Approach to Audit Planning and Evaluation in the Belief-Function Framework. *International Journal of Intelligent Systems in Accounting, Finance and Management* 5(3). pp. 165–183.
16. Sun, L., Srivastava, R.P. & Mock, T. (2006) An Information Systems Security Risk Assessment Model under Dempster-Shafer Theory of Belief Functions. *Journal of Management Information Systems* 22(4), pp. 109–142.
17. Yager, R.R. (1996) On the normalization of fuzzy belief structures. *International Journal of Approximate Reasoning* 14 (2–3). pp. 127–153.
18. Yen, J. (1990) Generalizing the Dempster-Shafer theory to fuzzy sets. *IEEE Transactions on Systems, Man and Cybernetics* 20 (3). pp. 559–570.