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ON MINIMIZATION OF NONLINEAR DISTORTION IN SALLEN AND KEY FILTER IN RANGE OF LOW FREQUENCIES

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Summary. In this paper, we show that a mildly nonlinear model of an active filter using the Volterra series and any other related one do not give satisfactory results in evaluation of nonlinear distortion in the range of low frequencies, in which the operational amplifier output voltage saturation is a dominating nonlinear phenomenon in this filter. Therefore, minimization of the filter nonlinear distortion cannot be performed by minimization of the gain-sensitivity product (*GSP*) or the so-called distortion aggravation factor (F_{DAG}) in the aforementioned range of frequencies. This is illustrated, using measured data of filter transfer function distortion, on an example of a low-pass Sallen and Key filter.

Keywords: Sallen and Key filter; modeling nonlinear distortion in low frequencies; operational amplifier output voltage saturation; Volterra series; gain-sensitivity product.

1. INTRODUCTION

In an article [2], a method of simultaneous minimization of nonlinear distortion and noise in Sallen and Key filters has been presented. In this approach, Billam used a filter quality measure for filters with single active element that was called by him a distortion aggravation factor F_{DAG} and introduced in his preceding paper [1]. On this occasion, we point out that the results of analysis of harmonic distortion in active filters, presented in [1], follow from a more general analysis of nonlinear circuits using as a tool the Volterra series. This has been shown, among others, in [3, 8]. So, the results derived in this paper with the use of Volterra series are directly related to the those presented by Billam in [2] and [1].

In [1], Billam assumes that gain of the active filter element does not depend upon frequency. From this, it follows that his method of simultaneous minimization of nonlinear distortion and noise [2] regards the range of smaller frequencies, in which such an assumption is valid. On the other hand, in this range of frequencies, operational amplifier output voltage is the dominating nonlinear phenomenon that decides about the form and level of the filter nonlinear distortion [7]. This paper tries to explain why an approach exploiting the Volterra series and, equivalently, the method of Billam [1] do not lead to satisfactory results in evaluation of active filter nonlinear distortion in the range of low frequencies. The conclusions drawn here were verified experimentally.

2. FILTER NONLINEAR MODEL BASED ON THE VOLTERRA SERIES

It has been shown in [4] that intermodulation distortion of an active filter with a single active element, of which scheme is shown in Fig. 1, can be expressed by a measure called *GSP* (gain-sensitivity product), but only when the Volterra series possibly accurately describes the nonlinear phenomena occurring in this filter. As proved in the paper [5], this occurs when the predominating nonlinear phenomenon in the filter is that related to the so-called *SR* (slew-rate) parameter of an operational amplifier (*OA*). It dominates in the range of higher frequencies.

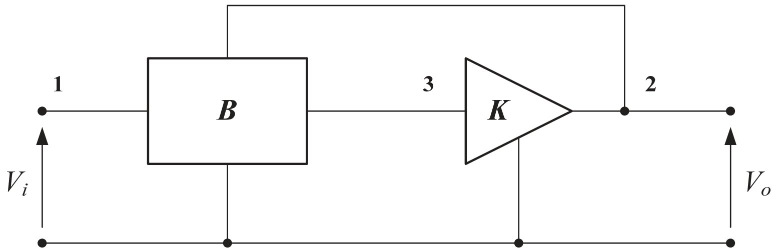

Fig. 1. Model of the filter with the single active element

Using formulas determining the so-called nonlinear transfer functions of the orders one, two and three of a filter having general structure as presented in Fig. 1, and assuming additionally no interaction between the aforementioned orders of filter nonlinearities, it can be shown that the filter nonlinear transfer function of the third-order, $H_3(f,f,-f)$, can be expressed as

$$H_3(f,f,-f) \cong WP(f)\left|H_1(f)\right|^2 H_1(f)GSP(f) \tag{1}$$

where $H_1(f)$ is the filter nonlinear transfer function of the first-order (linear transfer function), but $WP(f)$ and $GSP(f)$ are given by the following formulas

$$GSP(f) = \frac{H_1(f)}{B_1(f)} \tag{2}$$

where $B_1(f)$ is the linear transfer function of the passive block *B* in Fig. 1 from node 1 to node 3 with short-circuited node 2,

$$WP(f)=\frac{K_3(f,f,-f)}{K_1(f)K_1(f)K_1(-f)K_1(f)} \tag{3}$$

where K_3 and K_1 are the third- and first-order, respectively, nonlinear transfer functions of an active element K of a general filter structure shown in Fig. 1.

It has been shown in [4] that, essentially, the function $WP(f)$ does not depend upon a method of realization of the active element K in Fig. 1 with the use of an operational amplifier and resistive circuitry (in a positive or negative feedback loop, or as *OA* without any feedback).

The relative change of the filter transfer function magnitude, caused by nonlinearities, can be expressed by the following formula [4]

$$\left|\frac{\Delta T}{T}\right| \cong \frac{3}{4} AMP^2 \left|\frac{H_3(f,f,-f)}{H_1(f)}\right| \tag{4}$$

where AMP is the amplitude of a single sinusoidal signal at the filter input.

Applying in (4) formulas (1) and (2) leads to

$$\left|\frac{\Delta T}{T}\right| \cong \frac{3}{4} AMP^2 \left|WP(f)\right| \left|H_1(f)\right|^2 \left|GSP(f)\right| \tag{5}$$

On assuming that the function $WP(f)$ does not depend upon frequency, it can be shown that the maximum of the function $\left|\frac{\Delta T}{T}(f)\right|$ given by (5) occurs at the pole frequency f_0 for the filter pole quality factors $Q_a >> 1$. Thus, it follows from (5) that if the nonlinear filter model using the Volterra series description had been also valid in the range of lower frequencies, the relative change of $\left|\frac{\Delta T}{T}(f_0)\right|$ would have been directly proportional to the magnitude of the measure $GSP(f_0)$. We will check the above in the next section.

3. SALLEN AND KEY FILTER

For the purpose of checking the validity of the filter nonlinear model based on the Volterra series presented shortly in the previous section, a low-pass Sallen and Key filter has been designed. Its structure is shown in Fig. 2. The filter pole quality factor was chosen to be $Q_a = 5$, but the pole frequency $f_0 = 1$ kHz. The filter was designed in two variants: first with the following values of $C_2/C_1 = c = 1$ and $R_2/R_1 = r = 1$, and second with such the values: $C_2/C_1 = c = 0.0206$ and $R_2/R_1 = r = 100$. The value of the parameter β in Fig. 2 was chosen to be the same in both variants, namely $\beta = 2.8$.

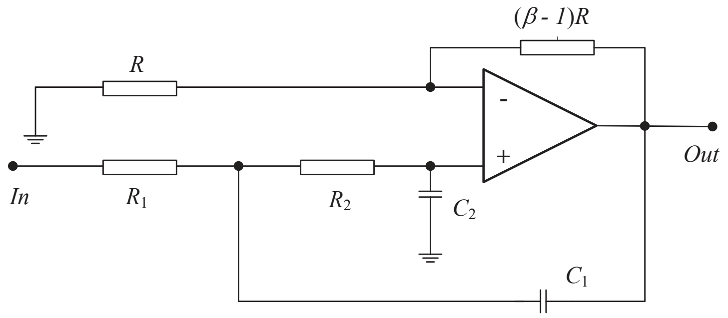

Fig. 2. Low-pass Sallen and Key filter.

In Figs. 3 and 4, the filter transfer function magnitudes $|T|$ versus frequency (for both variants) for two values of the filter input sinusoidal signal amplitudes have been plotted. Moreover, the values (theoretical and experimental ones) of the $|GSP(f_0)|$ and $|F_{DAG}(f_0)|$ measures, and of the following coefficient

$$\gamma \cdot 10^4 = \frac{\left|\frac{\Delta T}{T}(f_0)\right|_{\text{exper.}} * 10^4}{\left(AMP\left|H_1(f_0)\right| = 14\text{V}\right)^2} \tag{6}$$

have been collected. Note that the experimental values mentioned above were calculated with the use of the measured values of $\left|\frac{\Delta T}{T}(f_0)\right|$.

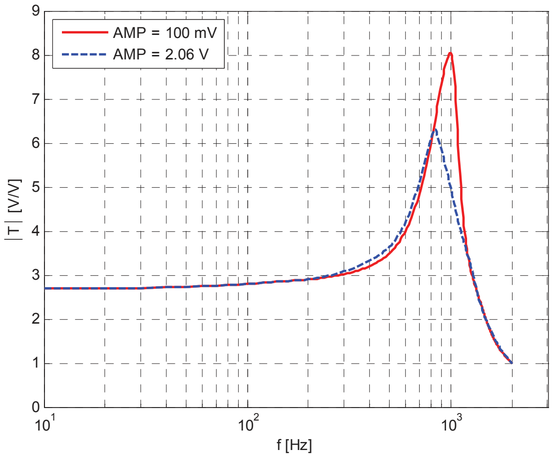

Fig. 3. Graph of $|T|$ dependence upon frequency in variant 1 for two values of input sinusoidal signal amplitude *AMP*.

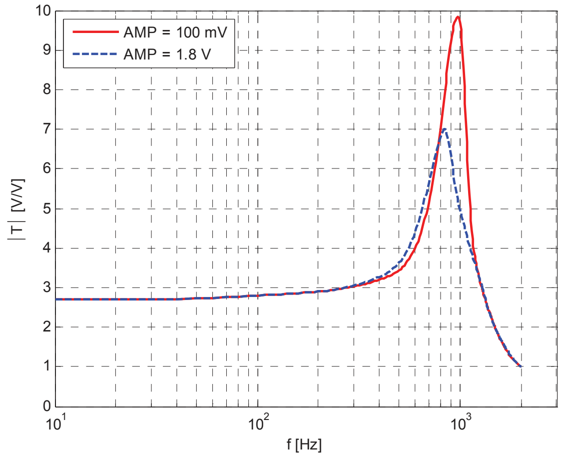

Fig. 4. Graph of $\left|T\right|$ dependence upon frequency in variant 2 for two values of input sinusoidal signal amplitude *AMP*.

Table 1. Values of $\left|GSP\left(f_0\right)\right|$, $\left|F_{DAG}\left(f_0\right)\right|$, and coefficient $\gamma\cdot10^4$.

Parameters *r* and *c*	$\left\|GSP\left(f_0\right)\right\|$		$\left\|F_{DAG}\left(f_0\right)\right\|$		$\gamma\cdot10^4$
	theor.	exper.	theor.	exper.	
$r = c = 1$	42	26.7	15	10	6
$r = 100$ $c = 0.0206$	30	22.4	10.7	8.4	13

The data presented in Table 1 show occurrence of large differences between the theoretical and experimental values indicating that the active filter nonlinear model based on the Volterra series description and, equivalently, that of Billam used in [1] and [2] are not suitable for the range of lower frequencies. Hence, minimization of the filter nonlinear distortion cannot be performed in the above range of frequencies by minimization of such the measures as $\left|GSP\left(f_0\right)\right|$ or $\left|F_{DAG}\left(f_0\right)\right|$. In this context, note that minimization of the filter nonlinear distortion with the use of the factor $\left|F_{DAG}\left(f_0\right)\right|$ was suggested by Billam in [1] and [2].

4. CONCLUSIONS

In this paper, it has been shown that the Volterra series and the method of Billam described in [1] and [2] are not suitable for constructing a nonlinear model of an active filter in the range of lower frequencies.

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O MINIMALIZACJI ZNIEKSZTAŁCEŃ NIELINIOWYCH W FILTRZE SALLENA-KEY W ZAKRESIE NISKICH CZĘSTOTLIWOŚCI

Streszczenie

W niniejszym artykule pokazano, że słabo nieliniowy model aktywnego filtru wykorzystujący szereg Volterry i inne temu podobne modele, nie daje zadowalających wyników w określaniu zniekształceń nieliniowych filtru w zakresie niskich częstotliwości, w którym napięciowe nasycenie na wyjściu wzmacniacza operacyjnego stanowi podstawowe zjawisko nieliniowe w tym filtrze. Konsekwencją tego jest to, że minimalizacja zniekształceń nieliniowych filtru nie może być wtedy oparta o minimalizację takich miar jak, na przykład, iloczyn wzmocnienia i wrażliwości, czy też odwrotność współczynnika sprzężenia w wyżej wspomnianym zakresie częstotliwości. Zostało to wykazane przy pomocy pomiarowych danych zniekształceń funkcji przenoszenia filtru, na przykładzie dolnoprzepustowego filtru Sallena-Key.

Słowa kluczowe: filtr Sallena-Key, modelowanie nieliniowych zniekształceń w zakresie niskich częstotliwościach, napięciowe nasycenie wyjścia wzmacniacza operacyjnego, szeregi Volterry, iloczyn wzmocnienia i wrażliwości