Henryk DUDA*, Bogdan WARDA**

EFFECT OF STRAIGHT-LINE MECHANISM DESIGN ON THE LOAD DISTRIBUTION IN A CYCLOIDAL GEAR

WPŁYW KONSTRUKCJI MECHANIZMU RÓWNOWODOWEGO NA ROZKŁAD OBCIĄŻEŃ W PRZEKŁADNI CYKLOIDALNEJ

Key words: cycloidal gear, straight-line mechanism, load distribution.

Abstract In the cycloidal gear, the torque is transmitted to the planet gears via an eccentric shaft with central bearings mounted on it. A straight-line mechanism is used to output the torque from the planet gears to the output shaft, a mechanism which consists of rotational sleeves mounted on the pins rigidly connected to the output shaft disc. These sleeves roll off in the holes of the planetary wheel. The forces generated in the straight-line mechanism affect the distribution of forces in the cycloid gearing and the amount of force loading the central bearing. The article presents the influence of the number of bolts of the straight-line mechanism on the load distribution in the cycloidal gear. The research carried out with the simulation program has shown that the smaller the number of pins, the greater the fluctuation of the force acting in the central bearing and the greater unevenness of force distributions in the cycloidal gear. This unevenness may cause a decrease in fatigue life of the meshing and central bearings.

Słowa kluczowe: obiegowa przekładnia cykloidalna, mechanizm równowodowy, rozkład obciążenia.

Streszczenie W obiegowej przekładni cykloidalnej moment napędowy jest przekazywany na koła obiegowe za pośrednictwem wałka mimośrodowego z osadzonymi na nim łożyskami centralnymi. Do wyprowadzenia momentu obrotowego z kół obiegowych na wał wyjściowy służy mechanizm równowodowy, który tworzą tuleje osadzone obrotowo na sworzniach sztywno powiązanych z tarczą wału wyjściowego. Tuleje te odtaczają się w otworach koła obiegowego. Siły powstające w mechanizmie równowodowym wpływają na rozkłady sił w zazębieniu cykloidalnym oraz na wielkość siły obciążającej łożysko centralne. W artykule przedstawiono wpływ liczby sworzni mechanizmu równowodowego na rozkład obciążeń w przekładni cykloidalnej. Badania przeprowadzone za pomocą opracowanego programu symulacyjnego pokazały, że im mniejsza liczba sworzni, tym większe wahania siły działającej w łożysku centralnym i większa nierównomierność rozkładów sił w zazębieniu cykloidalnym. Nierównomierność ta może być przyczyną zmniejszenia trwałości zmęczeniowej zazębienia oraz łożysk centralnych.

INTRODUCTION

The cycloidal planetary gear was invented in the late 1920s by Lorenz Braren in Germany and patented in 1931. The characteristic feature of this transmission is the rolling contact in the cyclic gearing, central bearings, and in the straight-line mechanism. This is ensured by cylindrical roller bearings of planetary wheels "1", sleeves mounted on the pivot pins of the fixed wheel "3" and rotary sleeves on the pins of the "2" straight-line mechanism (**Fig. 1**). Obtaining rolling friction at the contact points of the cooperating parts allows one to reduce frictional resistance, achieve high efficiency, and reduce the wear of parts [**L. 15**].

* PhD Student, Lodz University of Technology, Department of Vehicles and Fundamentals of Machine Design, Stefanowskiego 1/15, 90-537 Łódź, Poland.
** Lodz University of Technology, Department of Vehicles and Fundamentals of Machine Design, Stefanowskiego 1/15, 90-537 Łódź, Poland, e-mail: bogdan.warda@p.lodz.pl, tel: +48 426312251.

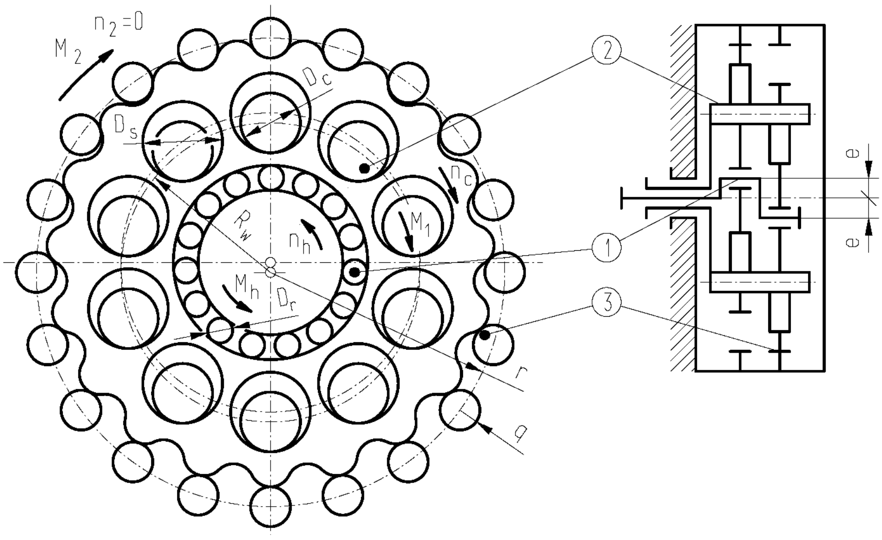

Fig. 1. Cycloidal planetary gear operation principle [L. 4, 5]
Rys. 1. Zasada działania obiegowej przekładni cykloidalnej **[L. 4, 5]**

The torque M_h on the eccentric input shaft forces the movement of the teeth of the cycloid discs on the sleeves of stationary disc and causes the central wheels rotation in the opposite direction to the input shaft, which in connection with the output torque M_1 gives the distribution of forces in the gear shown in **Figure 2**:

– Forces P_j in meshing,
– Forces Q_i in straight-line mechanism, and
– The force R acting on the rolls of the central bearing.

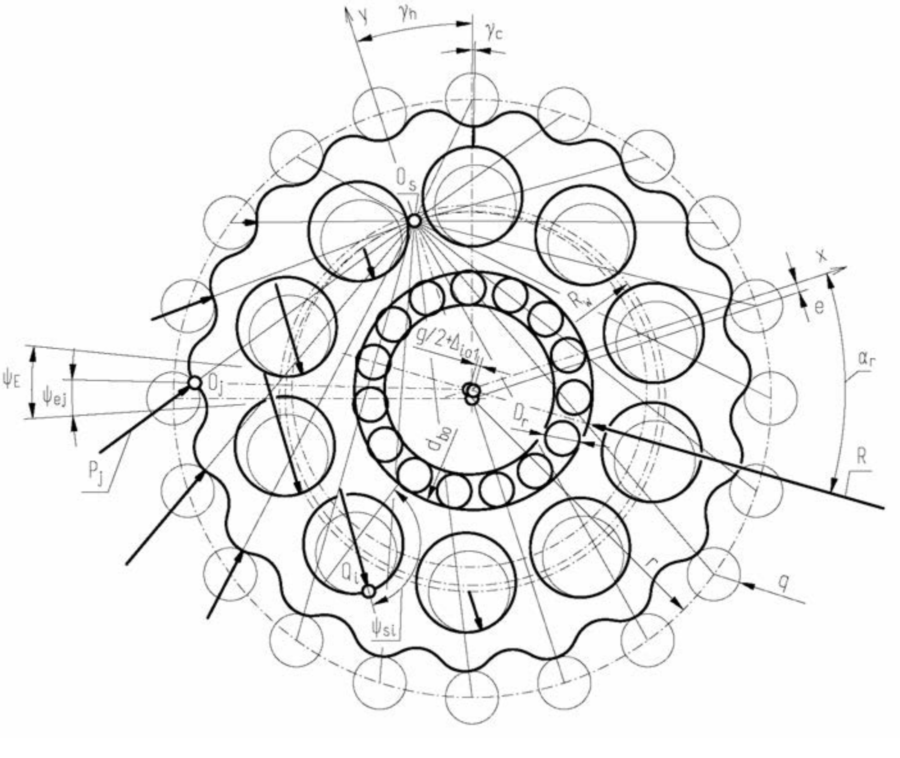

Fig. 2. Distribution of loads in a cycloidal gear
Rys. 2. Rozkłady obciążeń w przekładni cykloidalnej

Cycloidal gearing is created by the planetary wheels cooperating with stationary discs consisted of a set of rolling pins z_k. The gearing of the planetary wheel has external teeth with the number of teeth $z_s = | i_c |$ (where i_c is the transmission ratio), which is in the form of equidistant of shortened epicycloid described by the following equations (1).

$$\begin{aligned} x(\alpha) &= r \cdot cos\alpha + e \cdot cos(z_k \cdot \alpha) - q \cdot cos(\alpha + \gamma) \\ y(\alpha) &= r \cdot sin\alpha + e \cdot sin(z_k \cdot \alpha) - q \cdot sin(\alpha + \gamma) \end{aligned} \tag{1}$$

In the above formulas, e is the eccentricity, q the radius of the cooperating sleeves, and r the radius of their position (**Fig. 2**). The angle α is the angle forming the epicycloid, while the angle γ is called the angle of transfer. The method of determining the values of angles α and γ has been described in [**L. 6**].

Proper meshing cooperation is ensured by the appropriate gearing correction consisting in changing the parameters of the equidistance describing the tooth profile [**L. 4, 5**]. The parametric equations of the corrected teeth are described by Equations (2). The equations include corrected values of the radius of the cooperating sleeves q_k and the radius of their position r_k.

$$\begin{aligned} x_k(\alpha) &= r_k \cdot cos\alpha + e \cdot cos(z_k \cdot \alpha) - q_k \cdot cos(\alpha + \gamma) \\ y_k(\alpha) &= r_k \cdot sin\alpha + e \cdot sin(z_k \cdot \alpha) - q_k \cdot sin(\alpha + \gamma) \end{aligned} \tag{2}$$

In the case of uncorrected meshing, the distribution of forces P_j and Q_i can be determined by analytical methods. For corrected meshing, it is necessary to use numerical methods. The finite element method [**L. 4, 8, 9, 10**] allows one to obtain the most accurate information of the load distribution in rolling pairs of the cycloidal gearbox. It allows for modelling the real features of the gearbox, including the susceptibility of elements or the actual shape of the teeth, but it is very time-consuming. For these reasons, to determine load distributions, analytical methods [**L. 4, 5, 11**] and simplified numerical methods are still used [**L. 5, 7, 12**]. LiXin Xu and YuHu Yang [**L. 12**] determining the distributions of forces in meshing took into account the displacement of the planetary wheel resulting from deformations in the central bearing, but they omitted the way of receiving the torque from the planetary discs. They also did not take into account the meshing correction. Examining the kinematic characteristics of the gear, the authors drew attention to the small periodic fluctuations in the angular velocity of the planetary wheel and the force in the central bearing, resulting from elastic deformation in the meshing. The period of angular velocity changes was $2\pi / z_k$. The author of the work [**L. 11**] examined, among others, the influence of the number of pins on the forces occurring in the straight-line mechanism. He, like the authors of the work [**L. 12**], did not take into consideration the meshing correction.

This work presents the influence of the number of pins in the straight-line mechanism on the distribution of loads in the cycloidal gear. The calculations were carried out using the numerical method [**L. 7**] for gears with corrected meshing.

METHOD FOR DETERMINING LOAD DISTRIBUTIONS ON THE ROLLING PAIRS IN CYCLOIDAL GEAR

To determine the distributions of forces in the straight-line mechanism, the method described in [**L. 7**] was used. This method makes it possible to perform calculations for many variants of data in a short time and allows one to include the following in the calculation:

- The corrected teeth profile of the planetary wheel;
- The displacement of the planetary wheel as a result of deformations in the contact rollers with rings in the central bearing and radial clearance in the bearing: $g/2 + \Delta_{io1}$ (**Fig. 2**); and,
- The number of pins of the straight-line mechanism.

All these factors affect the nature of load distributions on the surface of the teeth of the planetary wheel and the surface of the side holes in the planetary wheel, cooperating with the pins of the straight-line mechanism, as well as the value of the eccentric force.

The following simplifying assumptions were adopted during the determination of the distribution of forces in the cycloidal gearbox:

- Wheel deformations (excluding local deformations in the contact area) are not taken into account;
- Loads are distributed evenly on planetary wheels;
- The directions of P_j forces acting on contact points between teeth and pins of stationary wheel have one intersection point at the rolling point of meshing O_s (**Fig. 2**); and,
- The force at the contact of the tooth of the planet wheel with the j-th roller of the stationary wheel is a function of the deformation δ_{sj}, resulting from the small rotation of the planetary wheel by the angle β_s.

It was assumed that the tooth-to-tooth contact is a Hertzian line contact, and the deformation caused by the rotation of the planetary wheel results in the occurrence of a reaction force equal to the following:

$$P_j = 78000\, \delta_{sj}^{10/9}\, l_e^{8/9} \tag{3}$$

where l_e is the width of the tooth of the planetary wheel (equal to the width of the wheel).

The distribution of forces in the straight-line mechanism was determined assuming that the force Q_i at the contact of the i-th roller with the hole in the planetary wheel is a function of the deformation δ_{ci}, which is the result of both rotation of the planetary wheel by β_s and rotation of the output shaft by β_c. Similar to the contact of the tooth of a planetary wheel with a fixed wheel roller, the force Q_i is equal to the following:

$$Q_i = 78000\, \delta_{ci}^{10/9}\, l_e^{8/9} \tag{4}$$

Based on the above assumptions, the equations of the straight-line of the gear wheel have been formulated. The solution to the equations was achieved numerically, using the Hook and Jeeves method. It made it possible to determine the values of forces P_j, Q_i, and force R for the given geometry of the transmission and the moment loading the output shaft M_1.

SUBJECT OF STUDY

The subject of the research was a cycloidal gear described in the works [**L. 4, 5**]. Gear parameters are shown in **Table 1**.

Table 1. Parameters of the examined Cyclo gear
Tabela 1. Parametry badanej obiegowej przekładni cykloidalnej

Eccentricity	e = 3 mm
The number of teeth of the planetary wheel	z_s = 19
The number of rollers of the co-operating wheel	z_k = 20
Diameter of roller of the co-operating wheel	q = 8.5 mm
Spacing radius of rollers of the co-operating wheel	r = 96 mm
Corrected diameter of roller of the co-operating wheel	q_k = 9 mm
Corrected spacing radius of rollers of the co-operating wheel	r_k = 96.406 mm
Width of the planetary wheel	l_e = 14 mm
Spacing radius of rollers of the straight-line mechanism	R_w = 62 mm
Side hole diameter	D_s = 32 mm
Diameter of roller of the straight-line mechanism	D_c = 26 mm
The number of rollers of the straight-line mechanism	z_c = 3, 4, 5, 6, 10
Diameter of the central hole	d_{bo} = 76.5 mm
Diameter of roller of the central bearing	D_r = 11 mm
Length of roller of the central bearing	L_r = 12 mm
Roller chamfer	r_c = 0.5 mm
The number of rollers of the central bearing	Z_r = 15
Input torque	M_h = 46.37 Nm

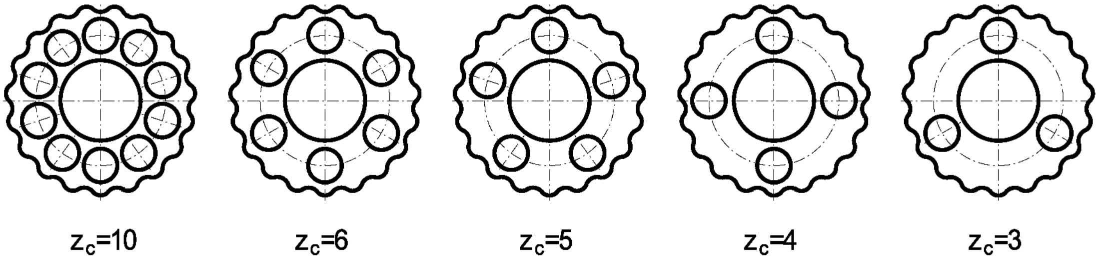

Fig. 3. Arrangement of holes in the planetary wheel
Rys. 3. Rozmieszczenie otworów w kole obiegowym

Calculations of load distributions were carried out using the CYCLOAD computer program for five variants of the straight-line mechanism design differing in the number of pins (**Fig. 3**). The radial clearance value in the central bearing $g = 0$ was used for calculations. Calculations were made for angles determining the position of the drive shaft in the range $\gamma_h = 0°–360°$. In order to obtain sufficiently accurate graphs showing the distributions of forces P_j on the tooth's working surface and Q_i forces on the surface of the holes in the planetary wheel, the value of angle γ_h was changed every 3°.

RESULTS

With the rotation of the input shaft, the forces in the contacts of the rolling elements of the cycloidal gear change. The number of pins in the straight-line mechanism has a decisive influence on the size of these changes. **Figure 4** illustrates the nature of the variability of force R in the central bearing of the planetary wheel as a function of the angle of rotation of the input shaft γ_h for a different number of pins of the straight-line mechanism z_c. It can be noticed that the value of the force R appearing for the angle $\gamma_h = 0$ is repeated after the rotation of the input shaft by an angle:

$$\gamma_{hp} = 2\pi\left(1 - \frac{1}{z_k}\right) \tag{5}$$

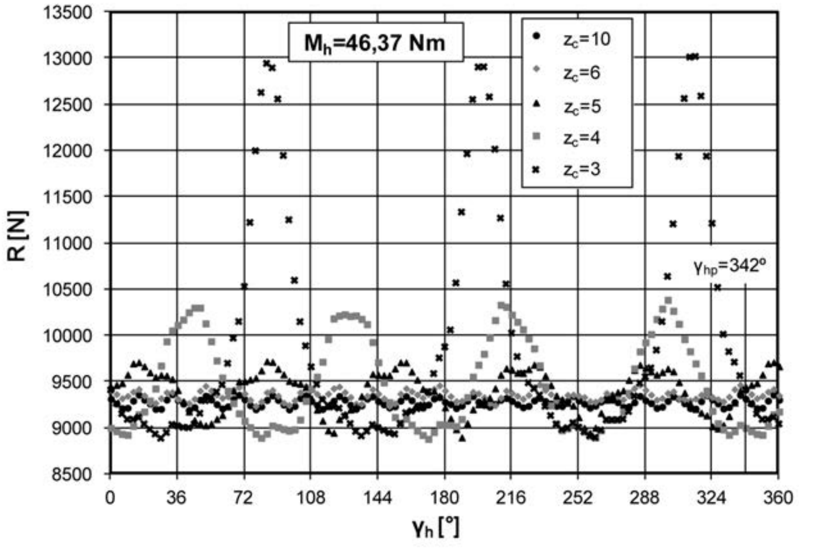

Fig. 4. The force of the eccentricity *R* as a function of the rotation angle of the input shaft γ_h

Rys. 4. Siła oddziaływania mimośrodu *R* w funkcji kąta obrotu wału napędowego γ_h

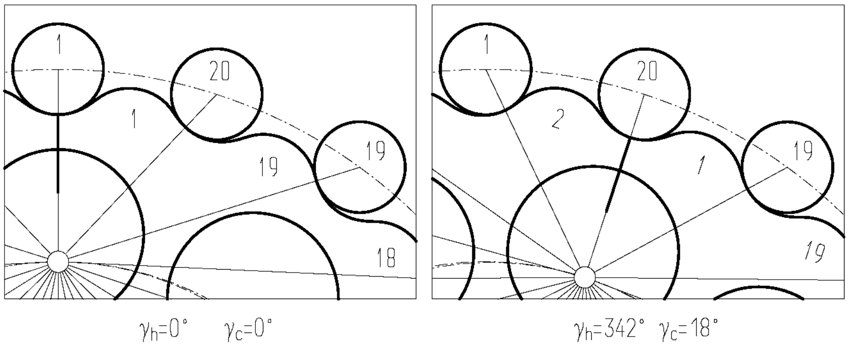

Fig. 5. Position of the planetary wheel for angles $\gamma_h = 0°$ i $\gamma_h = 342°$

Rys. 5. Położenie koła obiegowego dla kątów $\gamma_h = 0°$ i $\gamma_h = 342°$

The planetary wheel takes exactly the same position with respect to the next pin of the stationary gear wheel, as for the angle $\gamma_h = 0$ (**Fig. 5**). The same are also the distribution of forces in meshing and forces in the straight-line mechanism. For considered gear $\gamma_{hp} = 342°$.

The value of eccentric force is subject to fluctuations caused by the asymmetrical arrangement of the holes of the straight-line mechanism relative to the teeth of the planetary wheel (**Fig. 3**). **Figure 4** shows the repeating maxima and minimums of the force value *R*, repeated every angle, depending on the number of pins of the straight-line mechanism:

$$\gamma_{hc} = \gamma_{hp} / z_c \tag{6}$$

Table 2 presents the values of the angles determining the period of force *R* changes, the minimal and maximal value of force and the percentage increase of the value of force *R* in relation to the minimal value for different number of pins of the straight-line mechanism.

Table 2. Fluctuations in force *R* resulting from the geometry of the straight-line mechanism
Tabela 2. Wahania siły *R* wynikające z geometrii mechanizmu równowodowego

z_c	γ_{hc} [°]	R_{min} [N]	R_{max} [N]	increase of value *R*
3	114	8883	13012	46.5%
4	85.5	8878	10382	16.9%
5	68.4	8893	9716	9.3%
6	57	9233	9466	2.5%
10	34.2	9188	9361	1.9%

Fluctuations in eccentric force resulting from the geometry of the straight-line mechanism are most clearly visible for the number of pins $z_c = 3$, where the difference between the smallest and the largest force value exceeds 46%. The maximum load on the central bearing is, in this case, almost 40% greater than the maximum force *R* occurring at the number of pins $z_c = 10$, such as in the cycloidal gear described in the works [**L. 4, 5**]. Irregularity and high values of force *R* have a negative impact on the durability of the central bearing, usually the weakest rolling pair in the cycloidal planetary gear.

The larger the number of pins, the smaller the differences between R_{min} and R_{max}, and the fluctuations of the force *R* resulting from the geometry of the straight-line mechanism are less noticeable in the graph (**Fig. 4**). With a large number of bolts ($z_c = 6, 10$), when the total reaction of the Q_i forces of the straight-line mechanism decreases, the influence of gearing geometry on changes in force *R* is clearly visible. These changes are repeated at an angle corresponding to the angle of pins spacing of the stationary wheel as follows:

$$\gamma_{hk} = 2\pi / z_k \tag{7}$$

The same changes were observed by the authors of the paper [**L. 12**]. These changes affect the irregularities resulting from straight-line mechanism geometry and for a small number of bolts ($z_c = 3, 4, 5$), they are difficult to observe.

Figures 6, 7, and **8** show the distribution of forces Q_i on the surface of the hole in the planetary wheel as a function of the angle ψ_{si} (**Fig. 2**), for the number of bolts $z_c = 10$, 5, and 3. The smaller the number of bolts of the straight-line mechanism, the greater is the maximum force acting on the bolt. The part of the perimeter of the hole cooperating with the bolt, which is under load, is larger. The straight-line mechanism, due to the large radii of curvature at the contact points, is the most durable element of the transmission and even such an increase in force will not significantly reduce its fatigue life [**L. 4**].

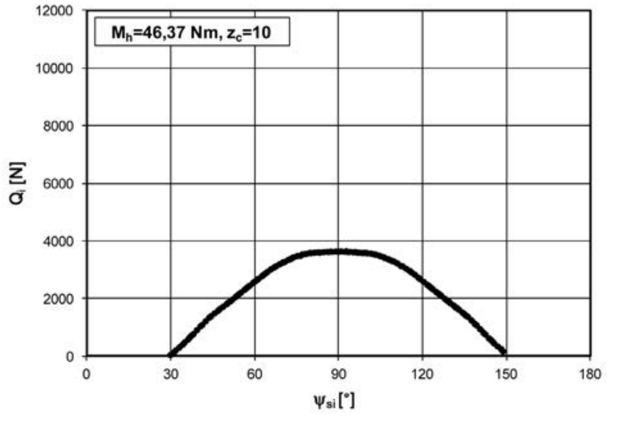

Fig. 6. Load distribution on the surface of the hole in the planetary wheel, $z_c = 10$
Rys. 6. Rozkład obciążenia na powierzchni otworu w kole obiegowym, $z_c = 10$

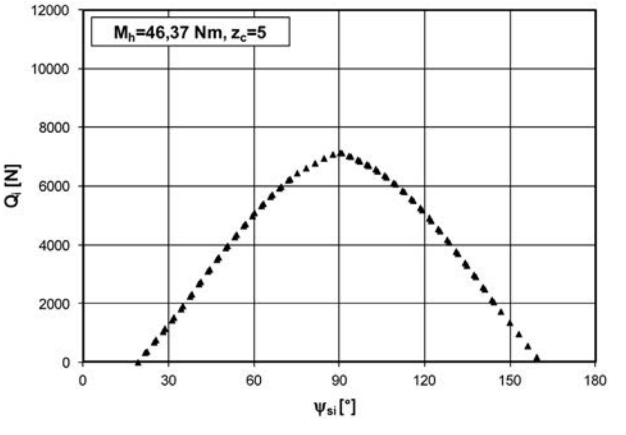

Fig. 7. Load distribution on the surface of the hole in the planetary wheel, z_c = 5
Rys. 7. Rozkład obciążenia na powierzchni otworu w kole obiegowym, $z_c = 5$

Figures 9, 10, and **11** illustrate the distribution of P_j forces on the surface of the tooth of the planetary wheel as a function of the angle ψ_{ej} which describes the location of the points of the equidistant of shortened

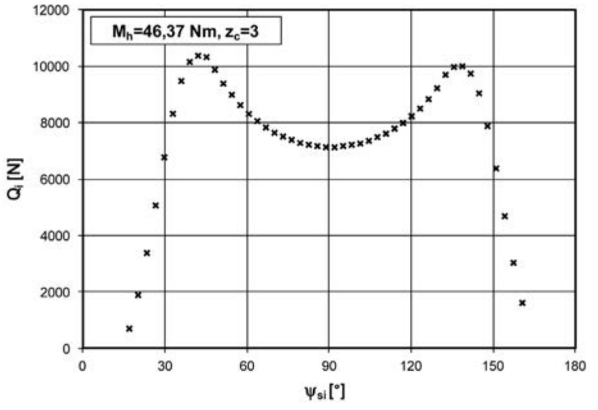

Fig. 8. Load distribution on the surface of the hole in the planetary wheel, $z_c = 3$

Rys. 8. Rozkład obciążenia na powierzchni otworu w kole obiegowym, $z_c = 3$

epicycloid (**Fig. 2**). This angle may vary from $\psi_{ej} = 0$ in the case of a point lying on the base circle to $\psi_{ej} = \psi_E = \pi / z_s$ for a point lying on the addendum circle. The individual points in the graph, for the increasing values of the angles ψ_{ej}, correspond to the forces in the contact of the tooth of the planetary wheel with a roller of a fixed wheel obtained for successive values of rotation angles of the drive shaft γ_h.

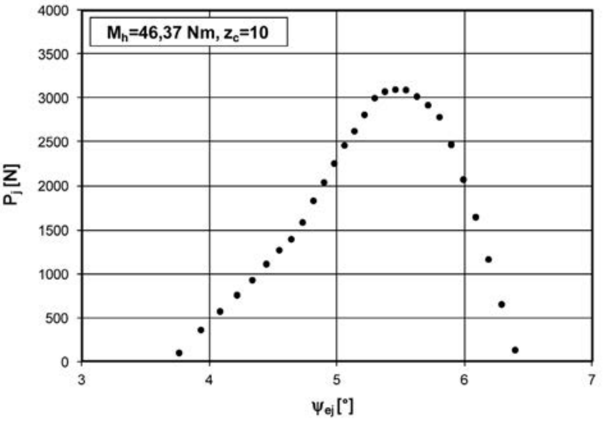

Fig. 9. Load distribution on the tooth surface of the planetary wheel, $z_c = 10$

Rys. 9. Rozkład obciążenia na powierzchni zęba koła obiegowego, $z_c = 10$

With a sufficiently large number of pins of the straight-line mechanism ($z_c = 10$, **Fig. 9**) the load distribution on the tooth surface of the planetary wheel is characterized by high uniformity, despite the fact that the force of the eccentric *R* influence is subject to certain fluctuations. For a smaller number of pins, $z_c = 5$ (**Fig. 10**), and in particular for $z_c = 3$ (**Fig. 11**), the spread of the value of force P_j in the points corresponding to the same values of angles ψ_{ej} is visible. This means that, during the rotation of the drive shaft, the force acting at the same point of the flank of the tooth is not constant but fluctuates due to the irregularity of force *R*. The maximum force P_j for the number of pins $z_c = 3$ is 18% greater than the maximum value P_j forces for $z_c = 10$. Such a difference, as well as the fluctuations in the force on the teeth, can affect the meshing fatigue life.

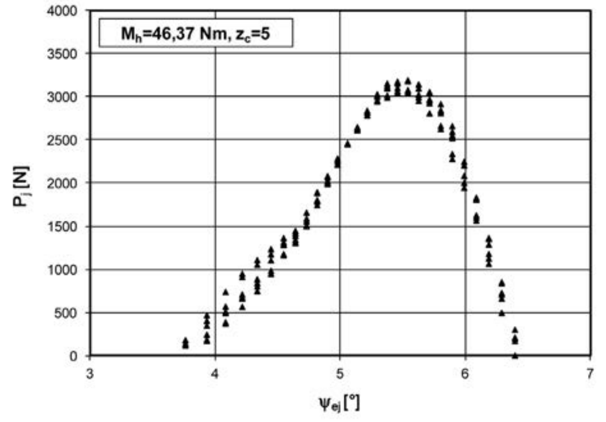

Fig. 10. Load distribution on the tooth surface of the planetary wheel, $z_c = 5$

Rys. 10. Rozkład obciążenia na powierzchni zęba koła obiegowego, $z_c = 5$

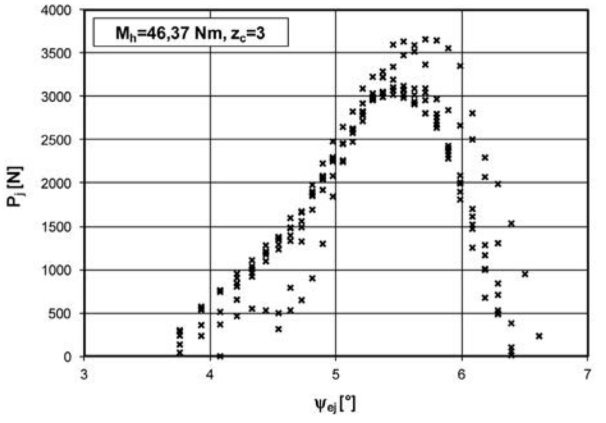

Fig. 11. Load distribution on the tooth surface of the planetary wheel, $z_c = 3$

Rys. 11. Rozkład obciążenia na powierzchni zęba koła obiegowego, $z_c = 3$

SUMMARY

The article describes the influence of the number of pins in the straight-line mechanism on the value of the reaction force in the central bearing of the planetary wheel for selected variants of the design of the straight-line mechanism. The graphs illustrating the distribution of forces on the surface of the tooth of the planetary

wheel and the forces on the surface of the hole of the planetary wheel for the same design variants are also presented. Together with the lower number of pins in the straight-line mechanism, the forces acting on the pins grow, as well as the increase in forces in the cycloid gearing, which is additionally accompanied by fluctuations in value of forces acting in the same points of the tooth flank. The highest load increase results from the small number of pins of the balance mechanism. It is visible primarily in the central bearing of the planetary wheel.

The increase in forces in the rolling pairs caused by the negative effects of a small number of pins on the straight-line mechanism means a decrease in the fatigue life of the cooperating elements. The increase in durability can be achieved by using better materials, appropriate heat treatment, and changes in the gear construction. The central bearing of the planetary wheel, which is usually the weakest element of the transmission, needs special care.

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