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A BIG-BANG BIG-CRUNCH OPTIMIZED GENERAL TYPE-2 FUZZY LOGIC APPROACH FOR MULTI-CRITERIA GROUP DECISION MAKING

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Abstract

Multi-Criteria Group Decision Making (MCGDM) aims to find a unique agreement from a number of decision makers/users by evaluating the uncertainty in judgments. In this paper, we present a General Type-2 Fuzzy Logic based approach for MCGDM (GFL-MCGDM). The proposed system aims to handle the high levels of uncertainties which exist due to the varying Decision Makers' (DMs) judgments and the vagueness of the appraisal. In order to find the optimal parameters of the general type-2 fuzzy sets, we employed the Big Bang-Big Crunch (BB-BC) optimization. The aggregation operation in the proposed method aggregates the various DMs opinions which allow handling the disagreements of DMs' opinions into a unique approval. We present results from an application for the selection of reading lighting level in an intelligent environment. We carried out various experiments in the intelligent apartment (iSpace) located at the University of Essex. We found that the proposed GFL-MCGDM effectively handle the uncertainties between the various decision makers which resulted in producing outputs which better agreed with the users' decision compared to type 1 and interval type 2 fuzzy based systems.

1 Introduction

MCGDM plays an important role in evaluating the utmost decision among a group of humans' interpretation which involves high level of uncertainties. Nowadays, MCDM contributes massively to a group decision makers' evaluation. However, the current multi-criteria decision making with a group of DMs (MCGDM) techniques do not effectively deal with the large number of possibilities that cause disagreement between different judgments and the variety of ideas and opinions among the decision makers which lead to high uncertainty levels.

Research has concentrated on investigating techniques to handle the faced uncertainties in many decision making applications. Fuzzy logic is regarded as an appropriate methodology for decision making systems which is able to simultaneously handle numerical data and linguistic knowledge. Studies in fuzzy decision making have grown rapidly in the utilization of extended fuzzy set theories (i.e., Intuitionistic Fuzzy Sets (IFSs) [1], Hesitant Fuzzy Sets [2], Vague Sets [3], Interval-valued Fuzzy Sets [4]). The work in [5] developed an interactive decision support system for sustainable energy management and the application of fuzzy methods to tackle uncertainties in the data. The work presented in [6] studied the supplier selection which involved several conflicting criteria where the decision maker's knowledge is usually vague and imprecise.

The application of Type-2 fuzzy sets on decision making has been widely applied. In [7], a method was proposed to complement the methods presented in [8] and [9] for fuzzy multiple attribute group decision-making based on interval type-2 fuzzy sets. In [10], a method was proposed which could handle evaluating values represented by non-normal interval type-2 fuzzy sets. The work presented in [11] investigated group decision making problems in which all the information provided by decision makers (DMs) are expressed as interval type-2 fuzzy values in decision matrices.

Recently several researchers have begun to explore the application of general type-2 fuzzy sets and systems. In [12], two methods were proposed for the automatic design of general type-2 fuzzy sets using data gathered through a survey on the linguistic variables. A series of results presented in [13] related to the different levels of uncertainty handled by the different types of Fuzzy Logic Systems (FLSs) including general type-2 fuzzy logic systems. In [14], an approach was presented for uncertain fuzzy clustering using the general type-2 fuzzy C-means algorithm and it was able to balance the performance of type-1 algorithms in various uncertain pattern recognition tasks.

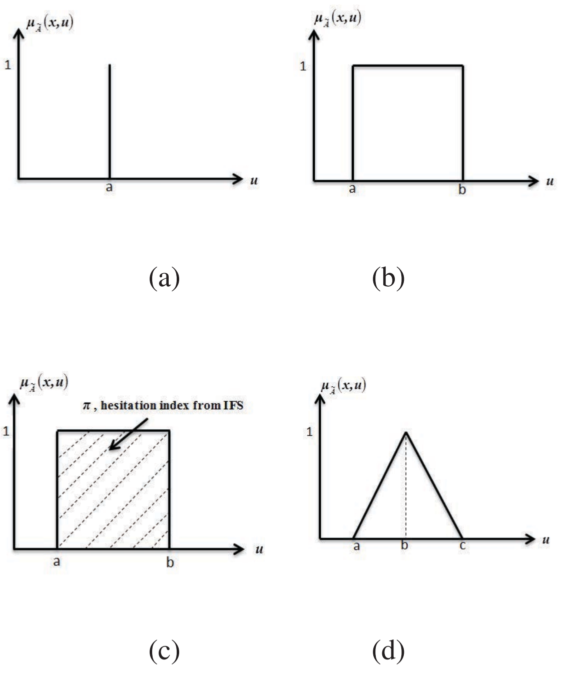


Figure 1. View of the secondary membership function in the third dimension,$x-u$ plane (a) Type-1 Fuzzy Set. (b) Interval Type-2 Fuzzy Set. (c) Interval Type-2 Fuzzy Set with Hesitation index (from IFSs). (d) General Type-2 Fuzzy Set.

In our previous work ([15] and [16]), we developed type-1 (Fig. 1(a)), interval type-2 (Fig. 1(b)) and interval type-2 fuzzy logic with hesitant index (Fig. 1(c)) based on IFSs (Intuitionistic Fuzzy Sets) for MCGDM. Fig. 1 show respectively the secondary membership functions for the type-1, interval type-2, interval type-2 fuzzy sets with hesitant index based on IFSs and general type-2 fuzzy sets. It was shown in [32] that the system based on interval type-2 fuzzy sets with hesitant index based on IFSs could handle the linguistic uncertainties by the interval type-2 fuzzy set Footprint of Uncertainty (FOU). In addition, this combination simultaneously computes the hesitancy from the membership and non-membership degree (of IFSs). However, the interval values with hesitation index cannot fully represent the uncertainty distribution (in the third dimension) associated with the decision makers.

In this paper, we present a general type-2 fuzzy logic based approach for MCGDM (GFL-MCGDM) which is more suited for higher levels of uncertainties.

The optimisation of fuzzy membership functions is crucially needed in fuzzy system to find the best parameters in order to achieve the needed objective. The performance of a fuzzy logic system is very sensitive to the sketch of the fuzzy set membership functions, the base lengths of the membership functions and the location of their peaks [17]. The type of membership functions varies according to the employed system. The subjectivity to interpret the linguistic variables exists because of the deviation of human interpretation. This problem leads to the complexity of the system by the high level of uncertainties. Such uncertainties include linguistic uncertainties where linguistic variables such as 'Distance' and 'Financial' might be interpreted in different ways according to different DMs. The hesitancies, vagueness and confusion might exist internally as well as externally. The internal conflicts such as self-esteem and confidence level can affect the DMs judgment during the assessment. In addition, external circumstances such as the political situation, the circumstances prevailing at that time and the environmental conditions can definitely have an effect on DMs opinions. Thus, in order to sketch subjective membership functions, the system crucially needs an optimization algorithm so as to find the best base lengths of the membership functions and the location of their peaks.

There are many types of optimisation methods to optimize the membership functions which are big bang-big crunch theory [18], [19], [20], [21], genetic algorithm [22], [23], [24], clonal selection algorithm [25] and particle swarm [26], [27], [28].

The work in [19] introduced the Big Bang Big Crunch (BB-BC) theory to solve an optimisation problem. In the Big Bang phase, the system generates random points while in the Big Crunch phase, it shrinks those points to a single representative point via a center of mass or minimal cost approach [19]. It is shown that the performance of the new (BB–BC) method outperforms the classical genetic algorithm (GA) for many benchmark test functions [19]. Thus, we believe that BB-BC is potentially able to optimize fuzzy membership function which is one of the most important parts in a decision system. Different parameters of each fuzzy set used in the fuzzy system might produce different outcomes.

Therefore, the development of fuzzy membership functions in decision system needs a very comprehensive evaluation to aggregate the uncertainties. Involvement number of DMs or experts such as in multi-criteria decision making system (MCDGM) or group decision making (GDM) critically need an optimized membership function in order to present all their opinions to form the best output.

In this paper, we propose general type-2 fuzzy logic based approach for MCGDM (GFL-MCGDM) with the optimized membership functions selected by BB-BC. The decision method utilizes general type-2 fuzzy sets to evaluate the linguistic uncertainties within the DMs' judgments about the linguistic variables. The aggregation operation in the proposed method aggregates the various DMs opinions which allows handling the disagreements of DMs' opinions into a unique approval.

The GFL-MCGDM utilized fuzzy membership functions from BB-BC optimization to maximize the percentage of correlation between decision system and human decision. The proposed system showed agreement between the system output and decision outputs from DMs as quantified by the Pearson Correlation. In addition, the Pearson correlation values given by the BB-BC based on GFL-MCGDM, outperformed the GFL-MCGDM systems based on interval type-2 fuzzy sets, interval type-2 with hesitation index and general type-2 (without BB-BC algorithm).

The rest of the paper is organized as follows, Section II, presents a brief overview of general type-2 fuzzy sets, type-2 fuzzy logic rule based systems, fuzzy multi-criteria group decision making and BB-BC optimization. Section III presents the general type-2 fuzzy logic based approach for MCGDM (GFL-MCGDM). Section IV, presents using BB-BC in GFL-MCGDM. In section V, we present the experiments which took place in the intelligent apartment (iSpace) located at the University of Essex. Finally, Section VI presnets the conclusions and future work.

2 Preliminaries

In this section, we briefly describe a few fundamental theories which were involved in this research.

2.1 General Type-2 Fuzzy Sets

A general type-2 fuzzy set (as shown in Fig. 2), denoted $\tilde{A}$, is characterized by a general type-2 fuzzy membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$and $\mu \in J_x \subseteq [0, 1]$, i.e.,

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) \,|\forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \quad (1)$$

in which $0 \le \mu_{\tilde{A}}(x, u) \le 1$.

$\tilde{A}$ can also be expressed as follows:

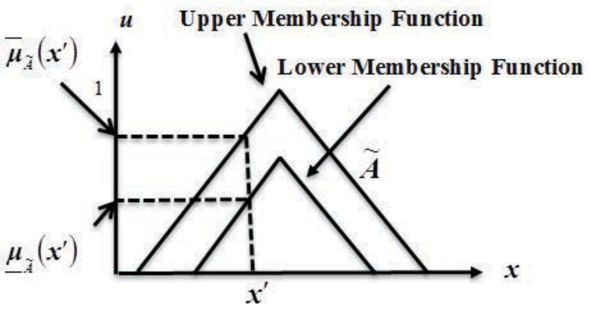
$$\tilde{A} = \int_{x \in X} \int_{u \in J_X} \mu_{\tilde{A}}(x, u) / (x, u) \quad J_x \subseteq [0, 1] \quad (2)$$

where $\iint$ denotes union over all admissible x and $u.J_x$, is called primary membership of $x$in $\tilde{A}$(as shown in Fig.1a), where $J_x \subseteq [0, 1]$for $\forall x \in X$ [29]. The uncertainty in the primary memberships of a general type-2 fuzzy set consists of a bounded region that is called the Footprint of Uncertainty (FOU) [29] which is the aggregation of all primary memberships [14]. According to [6], a general type-2 fuzzy set can be thought of as a large collection of embedded type-1 sets each having a weight to associate with it [30]. At each value of x, say $x = x'$, the 2-D plane whose axes are

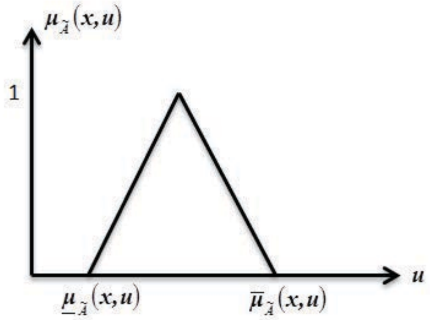
$u$and $\mu_{\tilde{A}}(x', u)$is called a vertical slice of $\mu_{\tilde{A}}(x, u)$ [29]. A secondary membership function is a vertical slice of $\mu_{\tilde{A}}(x, u)$[29]. Hence, $\mu_{\tilde{A}}(x', u)$for $x' \in X$and $\forall u \in J_{x'} \subseteq [0, 1]$ could be written as [29]:

$$\mu_{\tilde{A}}\left(x = x', u\right) \equiv \mu_{\tilde{A}}\left(x'\right) = \int_{u \in J_{x'}} f_{x'}(u)/u \qquad (3)$$

in which $0 \leq f_{x'}(u) \leq 1$. Because $\forall x' \in X$, the prime notation on $\mu_{\tilde{A}}(x')$ is dropped and $\mu_{\tilde{A}}(x)$is referred to as a secondary membership function [31]; it is a type-1 fuzzy set which is also referred to as a secondary set (see Fig. 1b) [32].



(a)



(b)

Figure 2. General Type-2 Fuzzy set (a) The primary membership, J_x. (b) The secondary membership is a fuzzy set.

2.2 Type-2 Fuzzy Logic Rule Based Systems

According to [32], in general type-2 FLSs, the rules will remain the same as in type-1 FLSs but the antecedents and the consequents will be represented by general type-2 fuzzy sets. Consider a type-2 FLS has p inputs $x_1 \in X_1, \ldots, x_p \in X_p$ and c outputs $y_1 \in Y_1, \ldots, y_c \in Y_c$. The *ith* rule in this multiple-input–multiple-output type-2 FLS can be written as follows: R^i: IF x_1 is $\tilde{F}_1^i$ and . . . and x_p is $\tilde{F}_p^i$, THEN y_1 is $\tilde{G}_1^i$ . . . y_c is $\tilde{G}_c^i$ where M is the number of rules in the rule-based.

2.3 Fuzzy Multi-Criteria Group Decision Making

According to [33], MCGDM aims to find a desirable alternative from a set of feasible alternatives based on the decision information on criteria values provided by a group of decision makers. In addition, MCGDM attempts to settle conflicts among the different individual preferences with different alternatives and criteria followed by synthesizing the different individual preferences to unanimous approval. However, the number of criteria, alternatives and diverse categories of a group of decision makers can cause massive ambiguity, hesitation and vagueness.

The MCGDM is described as follows: Let A be a set of alternatives, let$X$be as set of criteria and let D be a set of experts/DMs, where$A = \{a_1, a_2, \ldots, a_e\}$,$X = \{x_1, x_2, \ldots, x_n\}$ and $D = \{d_1, d_2, \ldots, d_m\}$, respectively. A MCGDM problem can be concisely expressed in a matrix format as:

$$D^k = a_r = \begin{array}{c|cccc} & x_1 & x_2 & \cdots & x_n \\ \hline x_1 & x_{11} & x_{12} & \cdots & x_{1n} \\ x_2 & x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ x_n & x_{n1} & x_{n2} & \cdots & x_{nn} \end{array} \qquad (4)$$

In what follows, we state the basic approach to fuzzy MCDM without considering risk attitude and confidence. A general decision making problem with e alternatives $l_r\,(r = 1, \ldots, e)$, n criteria $x_t\,(t = 1, \ldots, n)$ and m experts $z_k\,(k = 1, \ldots, m)$ can be concisely expressed as: $D = |l_r|$. Here D refers to a *DM* (where the entry x_{ij} represents the rating of the rule formed by criteria$x_i$and criteriax_j) where$i = 1, \ldots, n$ and $j = 1, \ldots, n$.

Definition 1. A preference relation P on the set X is characterized by a function $\mu_P : X \times X \to U$, where U is the domain representation of preference degrees. A fuzzy preference relation P on the set X is represented by a complementary/reciprocal matrix: $X = (x_{ij})_{n \times n} \subset X \times X$ with $x_{ij} \geq 0$, $x_{ij} + x_{ji} = 1$, $x_{ii} = 0.5$ for all i, $j = 1, 2, \ldots, n$ where x_{ij} denotes the preferred degree of the criteria x_i over x_j. In particular, $x_{ij} = 0.5$ indicates indifference between x_i and x_j, $x_{ij} > 0.5$ indicates that x_i is preferred to x_j, and $x_{ij} < 0.5$ indicates that x_j is preferred to x_i.

2.4 Optimization Using Big-Bang Big-Crunch Algorithm [18]

Big-Bang Big-Crunch is an optimization method which has been proposed in 2006 by [19]. Nowadays, from the application of this new evolutionary computation algorithm, researchers have found that the main advantage of BB-BC is its high convergence speed and, as a consequence, its low computation time [18]. The Big Bang and Big Crunch (BB-BC) algorithm consists of two steps.

- The *"Big Bang"* phase is where the candidate solutions are randomly distributed over the search space. The initial Big Bang population is randomly generated over the entire search space just like the other evolutionary search algorithms [18]. All subsequent "Big Bang" phases are randomly distributed about the centre of mass1 or the best ?t individual in a similar fashion [18].
- The *"Big Crunch"* phase involves forming a centre or a representative point for further "Big Bang" operations [18]. In this phase, the contraction operator takes the current positions of each candidate solution in the population and its associated cost function value and computes a centre of mass [18]. The centre of mass can be computed as:

$$x_c = \frac{\sum_{i=1}^{N} \frac{1}{f^i} x_i}{\sum_{i=1}^{N} \frac{1}{f^i}} \tag{5}$$

where x_c is the position of the centre of mass, x_c is the position of the candidate, f^i is the cost function value of the ith candidate and N is the population size. Instead of the centre of mass, the best ?t individual can also be chosen as the starting point in the "Big Bang" phase [18]. The new generation for the next iteration "Big Bang" phase is normally distributed around x_c [18]. The new candidates around the centre of mass are calculated by adding or subtracting a normal random number whose value decreases as the iterations elapse [18]. This can be formalized as:

$$x^{new} = x_c + \frac{r\alpha(x_{\max} - x_{\min})}{k} \tag{6}$$

where r is random number; α is a parameter limiting the size of the search space, x max and x min are the upper and lower limits and k is the iteration step [18].

3 GFL-MCGDM

In this section, we describe the proposed GFL-MCGDM and the steps of the system. We also explain how to construct the rule-base and finally how to sketch the membership functions from the surveys.

3.1 The GFL-MCGDM System

In GFL-MCGDM (as shown in Fig. 3), we utilized the fuzzifier and fuzzy logic rule base to construct decision matrices representing decision makers' opinions and judgments. We modified reciprocal matrices (according to Definition 1) by inserting the fuzzy logic rule base. The efficiency of fuzzy logic systems deploying rule bases has been proved in various publications. Thus, we believe this hybridization is able to cope with the competence of the decision system.

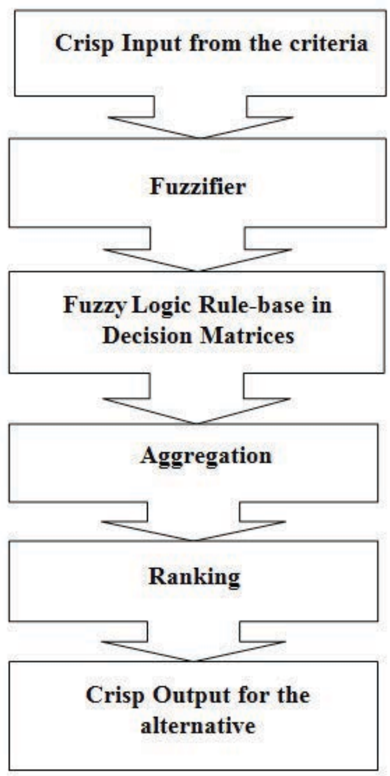


Figure 3. An overview on the proposed GFL-MCGDM.

In addition, the aggregation utilizes a fuzzy weighted average to find an accumulated pairwise comparison decision matrix. At this point, an important limitation of the previous approaches in MCDM comes to light as they lose some of the original decision information in the process of information aggregation and this can cause difficulties in prioritizing the given alternatives [34]. By using a fuzzy weighted average, the proposed system attempts to avoid undistinguished ranking order between the alternatives for the utmost decision, which is proven in [34] by using fuzzy majority. Generally, each x_{ij} calculated in the decision matrix (refer to Eq. 4) should satisfy the following rule according to expert opinions:

$$\text{IF } x_i \text{ is } \widetilde{F}_i \text{ and } x_j \text{ is } \widetilde{F}_j \text{ THEN } \widetilde{a}_r \qquad (7)$$

The fuzzy logic rule-base will then give the membership values in reciprocal matrices. The aggregation parts utilize fuzzy arithmetic averaging operators to compute the membership values in the decision matrix. The normalizations and the priority weights are calculated to determine the ranking for the final outputs.The weighted values allow us to rank the alternatives and the highest ranking can be determined as an output/decision.

In MCGDM, the criteria represent the FLS inputs while the alternatives represent the FLS outputs. We modified the reciprocal decision matrices by inserting the fuzzy logic rule-base. As shown in Fig. 3, the proposed architecture starts by receiving crisp inputs from the criteria which are then fuzzified and then fire the fuzzy rule-base to provide the membership values in decision matrices. The membership value at a given *x'* for a general type-2 fuzzy set is a type-1 fuzzy set in the third dimension.

The aggregation operation in the proposed method aggregates the various DMs opinions which allow handling the disagreements of DMs' opinions into a collective approval. The ranking components in the proposed GFL-MCGDM utilize fuzzy arithmetic averaging operators to compute the membership values in decision matrices. The normalizations and the priority weights will be calculated to determine the final output. The various components of the proposed GFL-MCGDM are discussed in the following subsections.

3.2 The GFL-MCGDM Steps

This section shows the eight steps which were taken to determine the ranking of the outputs. This phase involved the fuzzifier process and decision-making process. The steps below provide an overview of the steps of the proposed GFL-MCGDM:

Step 1: Consider a multi-criteria group decision making problem, let $A = \{l_1, l_2, \ldots, l_e\}$ be a discrete set of alternatives (output parameters), $X = \{x_1, x_2, \ldots, x_n\}$ be a set of criteria (input parameters), and $D = \{z_1, z_2, \ldots, z_m\}$ be a set of DMs. The DM $z_k \in D$ provides his/her judgment based on the rules given and constructs the rule-based reciprocal decision matrix.

Step 2: With the assumption that we have the input values for each criteria, we utilize trapezoidal general type-2 membership function to define the membership degree for each rule defined in the reciprocal decision matrices and we then identify the rule that is fired. Each ${x'_{ij}}^{(r,k)}$ calculated in the decision matrix should satisfy the following rule according to DMs opinions:

$$\text{IF } x_i \text{ is } \widetilde{F}_i \text{ and } x_j \text{ is } F_j \text{ THEN } l_r \qquad (8)$$

$$X^{(r,k)} = \left({x'_{ij}}^{(r,k)}\right)_{\mathrm{n \times n}} \qquad (9)$$

Hence, for each ${x'_{ij}}^{(r,k)}$, we will have $\bar{\mu}_{ij}^{(r,k)}, \underline{\mu}_{ij}^{(r,k)}$, for all $i, j = 1, 2, \ldots, n$.

Step 3: In this step, we define $\tilde{\mu}_{ij}^{(r,k)}$ as follows:

$$\tilde{\mu}_{ij}^{(r,k)} = \frac{\underline{\mu}_{ij}^{(r,k)} + \bar{\mu}_{ij}^{(r,k)}}{2} \qquad (10)$$

Hence, for each entry, we will have $x_{ij}^{(r,k)} = \left(\underline{\mu}_{ij}^{(r,k)}, \tilde{\mu}_{ij}^{(r,k)}, \bar{\mu}_{ij}^{(r,k)}\right)$, for all $i, j = 1, 2, \ldots, n$. In all the operations below please note that all operations on x will be carried on$\underline{\mu}$,$\tilde{\mu}$ and $\bar{\mu}$ independently in decision matrices.

Step 4: Then, we use the min operator to compute the firing strength for each rule. This will lead to construct the fuzzy decision matrices. Based on the DMs/experts $z_k \in D$, we can construct reciprocal decision matrices.

Step 5: The general type-2 fuzzy values of each $x_{ij}^{(r,k)}$ are then aggregated. The aggregated set can be determined by $x_{ij}^{(r)} = \left(v_{ij}^r, w_{ij}^r, y_{ij}^r\right)$ for $(k = 1, \ldots, m)$

where,

$$v^r_{ij} = \min_k \left\{ \underline{\mu}^{(r,k)}_{ij} \right\} \tag{11}$$

$$w^r_{ij} = \frac{1}{m} \sum_{k=1}^{m} \tilde{\mu}^{(r,k)}_{ij} \tag{12}$$

$$y^r_{ij} = \max_k \left\{ \bar{\mu}^{(r,k)}_{ij} \right\} \tag{13}$$

Step 6: Use the fuzzy arithmetic averaging operator to aggregate all $x^{(r)}_{ij} = \left(v^r_{ij}, w^r_{ij}, y^r_{ij} \right)$ corresponding to the$n$criteria.

$$x^{(r)}_t = \frac{1}{n} \sum_{t=1}^{n} x^{(r)}_{ij} \tag{14}$$

Step 7: Find the average of each$x^{(r)}_t$(where$t$ = $1,...,n$), this average is called $x^{(r)}_{tavg}$. Next, normalize the matrix so that each element in the matrix can be written as follows:

$$x^{(r)}_{tnorm} = \frac{x^{(r)}_{tavg}}{\sum_{t=1}^{n} x^{(r)}_{tavg}} \tag{15}$$

Step 8: Find the priority weights, $l^r$of each alternative as:

$$l^r = \frac{1}{n} \sum_{t=1}^{n} x^{(r)}_{tnorm} \tag{16}$$

where $l^r > 0$, $r = 1, \ldots, e$, $\sum_{r=1}^{e} l^r = 1$.

4 Construction of the General Type-2 Membership Function utilizing BB-BC

In this section, we briefly describe the BB-BC steps. Then, we explain how to compute the membership function using BB-BC. Finally, we discuss how to maximize correlation values between decision makers and GFL-MCGDM system.

4.1 The Aggregation of General Type-2 Membership Function

Different meaning interpretions by each DM is a major problem in any decision problem. The various needs have to be measured thoroughly, as they involved a high level of uncertainties especially when the judgements come from a range of different backgrounds (age, origin, sex, level of education, etc.).

From the survey we found that DM's opinion about certain linguistic variables vary among each other and this allowed us to sketch an aggregation of a trapezoidal fuzzy set from the interval values given by DMs (interval values have been sketched as symmetrical triangle fuzzy sets). Table 1 and Table 2 shows two different opinions from two DMs regarding the variable: very young, young, medium, old and very old for the criterion Age.

Table 1. The Meaning of Age by DM 1

Linguistic Variable for Age by DM 1	**Opinion (in the interval from which suitable for reading application)**
Very young	3 years old – 16 years old
Young	16 years old - 27 years old
Medium	27 years old - 40 years old
Old	40 years old - 55 years old
Very old	55 years old - 60 years old

Table 2. The Meaning of Age by DM 2

Linguistic Variable for Age by DM 2	**Opinion (in the interval forms which suitable for reading application)**
Very young	1 years old – 18 years old
Young	18 years old - 25 years old
Medium	25 years old - 45 years old
Old	45 years old - 60 years old
Very old	60 years old - 70 years old

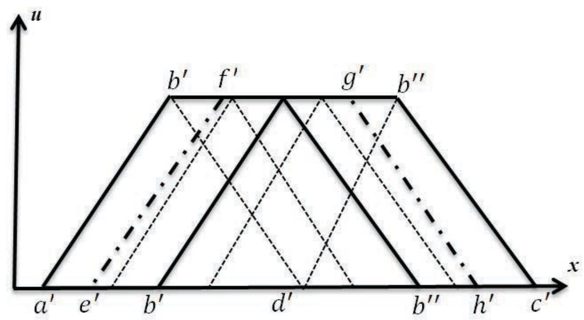
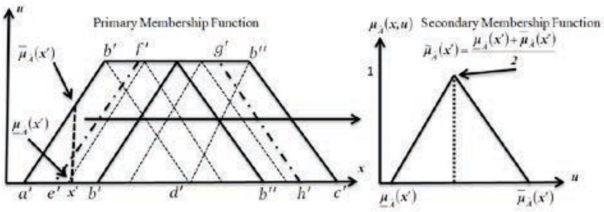


Figure 4. Type-2 Fuzzy Set from DMs' opinion (plotted in thick lines) as generated from the DMs' symmetrical triangular type-1 Fuzzy Sets (plotted in thin dashed lines) and the used type-1 Fuzzy Sets for comparison in thick dashed line.

All fuzzy set representing the linguistic labels for each criterion were modelled in the x-u domain with trapezoidal type-2 fuzzy membership functions (as shown in Fig. 8). Essentially, the linguistic label type-2 fuzzy sets are created by the combination of DMs' opinions (modelled by symmetrical triangular type-1 fuzzy sets as shown in Fig 8). The example of the interval values from Table 1 and Table 2 have been sketched into symmetrical triangular fuzzy sets representing a given linguistic label as shown in Fig. 4. The minimum, maximum and the average values defined by the aggregations of DMs' opinion are demarcated to create the support for each trapezoidal type-2 fuzzy set.



(a) (b)

Figure 5. Generation of General Type-2 Fuzzy Set from DMs' opinion (a) Primary membership function (plotted in thick lines) as generated from the DMs' type-1 Fuzzy Sets (plotted in thin dashed lines) and the used type-1 Fuzzy Sets for comparison in thick dashed line. (b) A secondary membership functions in the third dimension.

Each DM opinion about a given linguistic label is defined by a symmetrical type-1 triangular fuzzy set (as shown in Fig. 5a) which is defined by three points (*a, b, c*). We aggregate$a = \{a_1, \dots, a_m\}$, $b = \{b_1, \dots, b_m\}$ and $c = \{c_1, \dots, c_m\}$ to find the lowest, vertex and upper points of the generated type-2 fuzzy set according to the number of decision makers ($k = 1, \dots, m$). In order to draw the *x-u* domain of the generated general type-2 fuzzy sets for each linguistic variable, the following points have to be defined as follows:

$$a' = \min\{a_1, \dots, a_m\} \tag{17}$$

$$b' = \min\{b_1, \dots, b_m\} \tag{18}$$

$$b'' = \max\{b_1, \dots, b_m\} \tag{19}$$

$$c' = \max\{c_1, \dots, c_m\} \tag{20}$$

$$d' = \frac{b' + b''}{2} \tag{21}$$

Thus, the above equations result in generating the general type-2 fuzzy set for each label from the individual DM opinions (represented as type-1 fuzzy sets). The secondary membership function at each *x'* is a symmetrical triangle (as shown in Fig. 5b).

Thus, the generated general type-2 fuzzy set (in Fig.5a) upper membership function will be formed by points *a', b', b"* and *c'*. At the same time, the lower membership function will be formed by points *b', d' and b"*. The type-1 fuzzy sets (which will be used when comparing the performance of a type-1 fuzzy based system with the proposed system) will consist of the points *e' (*average *of a'* and *b'), f' (*average *of b'* and *d'), g' (*average *of b"* and *d')* and *h' (*average *of b"* and *c')* as shown in Fig. 5a.

In previous studies, from type-1 fuzzy set (*e', f', g', h')* found according to Fig. 4 and 5, we increased the FOU from 5% till 100% to analyse with the GFL-MCGDM system manually. In this paper, we use BB-BC algorithm to find the optimal FOU which will give the higher correlation between the system and the DMs.

4.2 Big-Bang Big-Crunch Steps [19]

The BB-BC algorithm implements the following five steps to optimize the FOUs of the general type-2 fuzzy sets.

Step A: Form an initial generation of *N* candidates in a random manner. Respect the limits of the search space.

Step B: Calculate the cost function values of all the candidate solutions.

Step C: Find the center of mass according to Equation (5). The best ?t individual can be chosen as the center of mass instead of using Equation (5).

Step D: Calculate new candidates around the center of mass by adding or subtracting a normal random number whose value decreases as the iterations elapse. This can be formalized as x^{new}(refer to Equation (6)).

Step E: Return to Step 2 until stopping criteria has been met.

4.3 Computation of the Membership Function using BB-BC Steps

In subsection 4.2, we have shown how we defined the aggregations of DMs opinion for each of the criteria. Thus, we have found 4 values points (*e', f, g' and h'*) for 30 fuzzy sets (6 criteria with each of them have 5 linguistic variables, refer to Table 4) which show the aggregations of type-1 fuzzy sets.

In this analysis, we used parameter v to increase the FOU from 1% to 100% (as shown in Fig. 6). The only parameter we have to optimize is v values which show the percentage of the uncertainty involved in the system. In order to find the optimal v value, we allowed the BB-BC to utilise a random number from 0% to 100%. According to subsection 4.2, we find the optimal v values where the v value for each point for trapezoidal type-2 membership function must be between some points which have been chosen as the maximum point of the trapezoidal shape as shown in Fig. 6:

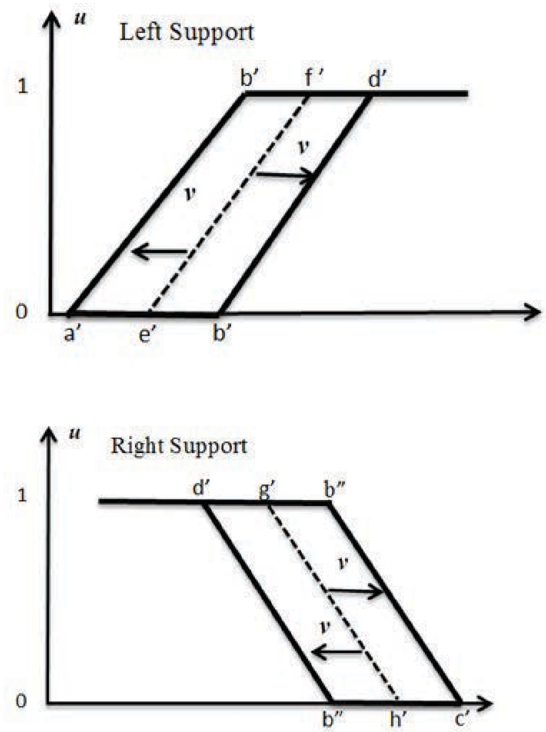


Figure 6. The increasing of FOU for left and right support based on v parameter for BB-BC.

4.4 Maximizing Correlations between Decision Makers and Decision System

The efficiency of the proposed system can be evaluated through the correlation values between the DMs' decision and the output ranking. In this study, Pearson Correlation was used to find the correlation between the DM's decision and the various MCGDM's decisions. Thus, for the proposed GFL-MCGDM based on BB-BC we are using Pearson Correlations as the cost function. The objective of the study is to maximize the Pearson correlation as a Cost Function. The Pearson Correlation which was used to find the correlation between the user's decision and the FL-MCGDM's decision is as follows:

$$\rho_{X,Y} = \frac{COV\,(X,Y)}{\sigma_X\sigma_Y} = \frac{E\,[(X-\mu_X)\,(Y-\mu_Y)]}{\sigma_X\sigma_Y} \quad (22)$$

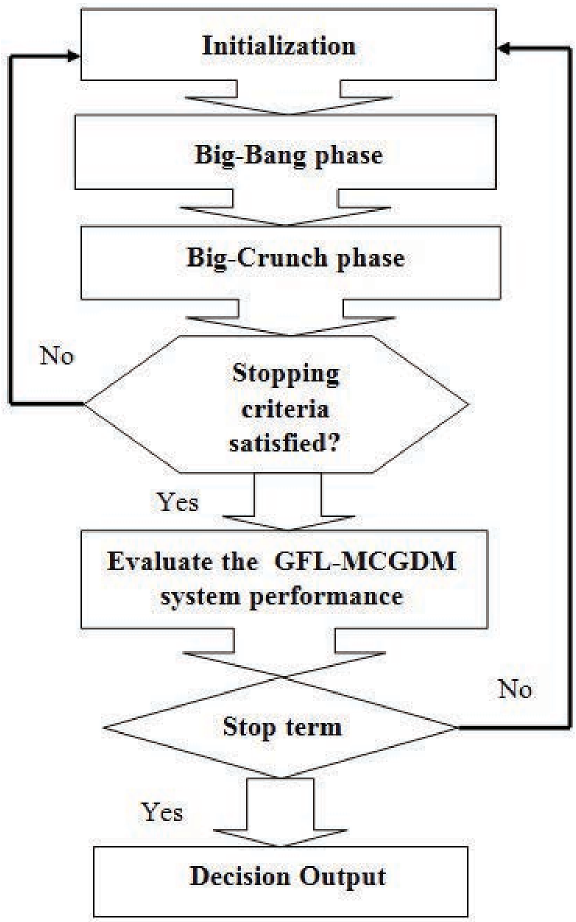


Figure 7. A GFL-MCGDM system based on optimized membership function from BB-BC.

According to Fig. 7, the optimized membership functions, v determined by the BB-BC phases will be applied in GFL-MCGDM system in order to find the highest correlation value (cost function) among the populations generates by BB-BC, showing that the agreement between DMs and the system as decision output.

5 Experiments and Results

5.1 The Experiments setup

Of all the visual tasks performed indoors, reading is considered the most common and continuous task. Effective reading is influenced by various factors with some of these factors being ambiguous and beyond control. According to [35], these factors include the contrast of colour, brilliance and printing of the paper, size and clarity of typeset, length and spacing of the printed line, widths of margin, and the visual acuity of the reader. Another important factor is the illumination provided to the reader. The reading illumination can be designed by the architects and the engineers [35]. Nowadays, ambient intelligence allows for control of lighting in intelligent spaces according to the DMs preferences. Fuzzy Logic-Multi Criteria Group Decision Making (FL-MCGDM) systems are capable of settling the conflicts in the preferred level of lights for reading application by synthesizing various human factors, criteria and alternatives.

The experiments which were conducted in the iSpace involved 15 participants from different backgrounds. The participants were between the ages of 14 to 52 years old and come from different countries. Each trial lasted around 30 minutes and the overall study took approximately 2 weeks to be completed.

5.2 System Generation

In order to generate a rule base for the proposed decision system, a survey has been distributed among all the participants. In the questionnaire, they have been asked about light level preferences based on their opinions, judgments and experiences when reading. A synopsis of the fuzzy rule set for one of the alternative which is '*very low level of ceiling light*' taken from a decision maker is shown in Table 3 based on the linguistic variables showing in Table 4. The rules set for GFL-MCGDM system were obtained as a result of knowledge elicitation from 15 DMs. Later, the rule set taken from decision makers have been constructed in decision matrices (as shown in Equation 4) to perform the needed analysis.

Table 3. Example of One Rule Set for Alternative "*Very Low Level of Ceiling Lights*" from a Decision Maker.

Criteria	**Alternative** ***(Very Low Level of Ceiling Light)***
Time of the day (T)	Afternoon (N)
Ambient luminance (A)	Bright (B)
Text size (X)	Medium (E)
Age (G)	Young (Y)
Distance of eyesight from reading material (D)	Far (F)
Width size of reading material (W)	Medium (M)

According to the first step of our method, after the surveys we construct the reciprocal decision matrices based on the fuzzy rules set (an example is shown in Table 3). 15 DMs evaluated their opinion based on 5 output variables/alternatives (very low, low, medium, high and very high). Thus, we have 75 rules dealing with the preferred lighting level for reading assessment. Consequently, we constructed 75 matrices because each of the rules represents the experts/DMs opinion. 75 matrices are accumulated in this analysis to compute one finest decision for the system. Let us see the following example according to Table 3 (the abbreviation is according to Table 3):

The following is an example of how the experts interpret the above rule set based on the example shown above:

Rule x_{TA}, for alternative Very Low:

IF Time of the Day (T) is Afternoon (N) and Ambient luminance (A) is Bright (B) and Text Size is Medium and Age is Young and Distance of eyesight from reading materialis Far and Width size of reading material is Medium **THEN** Very Low Light Level.

Overall, we have 75 rules from 15 DMs for 5 alternatives. These rules have been constructed in decision matrices in accordance with the reciprocal decision matrix (shown in Equation (4)). For elements x_{AT} we stated the same rules as we utilized the pairwise comparison matrix. Whereas for the value x_{ii} will be 0.5 as according to the Definition 1. Thus each x_{ij} in the decision matrices gives the reciprocal fuzzy membership degree (refer to Definition 1).

Table 4. Fuzzy Linguistic Variables for each Criteria Using for the Reading Application

Criteria	**Linguistic Variable**
Time of the day	Early night Late night Morning Noon Afternoon Evening
Indoor lighting levels from 0 to 1000	Very gloomy Gloomy Medium Bright Very bright
Text size	Very small Small Medium Big Very big
Age	Very young Young Medium Old Very old
Distance of eyesight from the reading material	Very near Near Medium Far Very far
Width size of the reading material	Very small Small Medium Large Very large

Table 5. Rule Sets from a DM in a decision matrix for 'Very Low' alternative based on Table 3.

Criteria	T	A	X	G	D	W
T	-	**NB**	NE	NY	NF	NM
A	NB	-	BE	BY	BF	BM
X	NE	BE	-	EY	EF	EM
G	NY	BY	EY	-	YF	YM
D	NF	BF	EF	YF	-	FM
W	NM	BM	EM	YM	FM	-

5.3 The Experiments

In order to assist in determining the decision making strategy, we have developed a real-world application where the participants were asked to decide on their preferred level of the ceiling lights as the ambient luminance conditions change when they are reading. The application has been deployed in the iSpace which is a purpose-built and fully-furnished two-bedroom apartment at the University of Essex, UK.

The intelligent apartment includes a spacious open plan kitchen and living area, bathroom, master bedroom and a study. It has distributed sensors and actuators which are connected in a homogenous manner over the iSpace network by the use of UPnP middleware. Fig. 8 illustrates the overall architecture of the developed system where we show the communication of the light sensors, the graphical user interface (GUI) and the ceiling lights on the iSpace network.

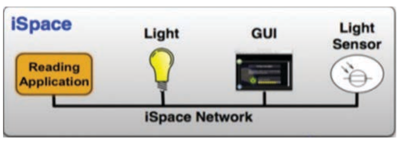


Figure 8. The overall architecture of the Reading Application.

As outlined in Fig. 8, the reading application uses the light sensors which are distributed within the living room area of the iSpace and

whose values are aggregated to account for the perceived ambient luminance. Next, the application employs a GUI displayed on the mobile device Apple iPad. By using this interface, the users can interact with the environment and they are able to change the dimmable ceiling light levels depicted on a scale of [0-10] which represents the percentage of the brightness in numeric format having a range between 0 (lights off) and 100 (maximum brightness). For example, by touching the 7^{th} bar of the scale on the iPad, the user can switch on the ceiling light levels to 70%.

5.4 Employing Fuzzy MCGDM in an Intelligent Environment

The reading application employs a simplified version of our Fuzzy Task Agent [36] where we limited the operation of the intelligent embedded agent to account for logging the users' ceiling light level preferences and some of the criteria that will be used as inputs to the overall system. Fig. 9 demonstrates the complete list of alternatives and the criteria that are effective in the decision making process when reading.

As shown, the alternatives for the preferred level of output ceiling lights can be '*very low*', '*low*', '*medium*', '*high*' and '*very high*'. Moreover, the criteria that may influence the user's preference of the ceiling light levels have been chosen to be the time of day, ambient luminance, age of the user, text size used in the document, distance of eyesight from the reading material and the width of the reading material. All these criteria together with the interaction of the user through the GUI (on an Apple iPad) and the alternatives (preferred ceiling light levels) can be visualized in the photos from the experiments and are shown in Fig. 10.

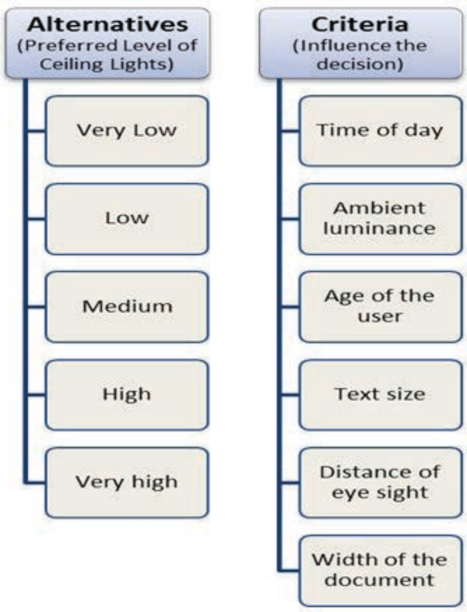


Figure 9. The alternatives and the criteria for the decision making analysis.

Figure 10. Participants making decision on their preferred level of ceiling lights under different criteria.

The real-world experiments have been performed on different days with a total of 15 participants. The participants were asked to be seated on the sofa in the living room of the iSpace. There were two dimmable lights positioned above their seats. Next to them, they had access to a range of reading materials including a dictionary, magazine, book, etc., together with a set of boxes varying in volume which the users were required to use on their laps. The different documents served the purpose of having diverse text size and width whereas the different volume of boxes helped to realise the changing distance of the eyesight from the reading material. Moreover, in order to simulate various lighting conditions, the blinds and the curtains within the living room of the iSpace were operated. For example, closing the curtains meant that the time of day was considered to be evening, night, etc.

During the experiments, the participants were allowed to communicate and interact with the researchers. Their opinions and feelings were also observed. The entire user experience was recorded using a video camera with the participant's permission and the video was then analyzed. From the data obtained, a comprehensive analysis of the users' decisions and opinions was performed.

To be more practical in the decision making analysis, we designed the embedded agent to log some of the criteria such as the time of day and the light sensor value in numeric format as the rest of the criteria (age of the user, text size, distance of eye sight and width of the document) can easily be logged manually. In addition, the preference of the user which is one of the alternatives of the overall system was also logged by the agent in a linguistic label format. The architecture of the software part of the system is included in Fig. 11 where all the relations between the real world, the user interaction, the logs and the overall system FL-MCGDM are clarified.

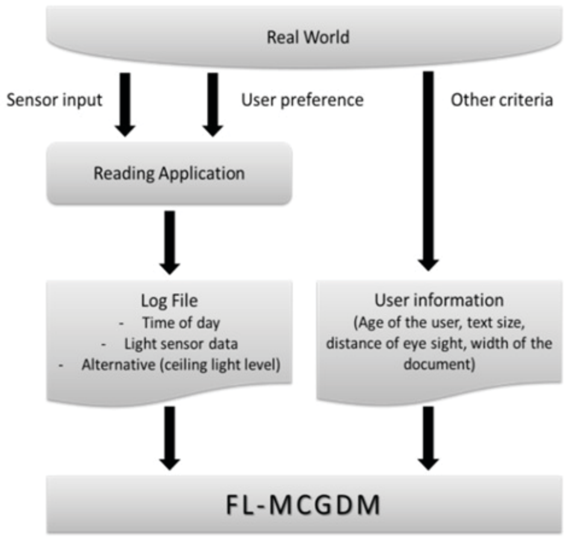


Figure 11. The overall architecture of the criteria and alternative collection for FL-MCGDM.

6 Results

The efficiency of the proposed system can be evaluated through the correlation values between the users' decision and output ranking. The higher the correlation values, the closer the user's decision to the output from the proposed system.

We tested 30 data sets to find the correlation values between the linguistic decision from the user and the output from the proposed GFL-MCGDM system. Examples of three data sets with the input values for each criterion (Time of the Day, Age and Text Size) and the real output decision from the DMs and the proposed systems are shown in Table 6. The ranking from both sides will determine the agreement among one another.

Table 6. Example of Input for Criteria Time, Age and Text Size, Output from Decision Makers and Output from GFL- MCGDM System

Time	**Age**	**Text Size**	**DMs' Decision**	**GFL-MCGDM' Ouput**
22.19	26	8	Very High	Very High
13.34	35	10	Low	Very Low
18.94	26	12	Medium	Medium

Table 7. Pearson Correlation Values for Different Type of Fuzzy Sets without BB-BC

Methods	Pearson Correlation
Type-1 Fuzzy Sets	0.5380
Interval Type 2 Fuzzy Sets	0.5555
Type 2 Fuzzy Sets with Hesitation Index	0.6338
General Type 2 Fuzzy Sets	0.6456

Table 8. Pearson Correlation Values for Type-2 Fuzzy Sets utilizing BB-BC

Methods	Pearson Correlation
Interval Type 2 Fuzzy Sets	0.5555
Type 2 Fuzzy Sets with Hesitation Index	0.6338
General Type 2 Fuzzy Sets	**0.6520**

According to Table 7, it can be observed that type-1 and interval type-2 fuzzy logic based MCGDM gives 0.5380 and 0.5555 correlations to the linguistic appraisal of the DMs (i.e. the DM's decision) whereas interval type-2 fuzzy logic with hesitation index based MCGDM gives a correlation value of 0.6338. Markedly, the GFL-MCGDM system without using BB-BC gives a correlation value of 0.6456.

While using BB-BC (refer to Table 8), interval type-2 fuzzy based MCGDM gives a similar correlation value of 0.5555. Although the interval type-2 fuzzy logic with hesitation index based MCGDM also gives the same correlation values. However, the proposed GFL-MCGDM based on BB-BC gives the highest correlation value of 0.6520. Hence, the proposed system, GFL-MCGDM based on BB-BC was able to model the variation in the group decision making process exhibited by the various decision makers' opinion. In addition, the proposed system showed the highest agreement between the proposed method and the real decision outputs from DMs which outperformed the MCGDM systems based on type-1 fuzzy sets, interval type-2 fuzzy sets and interval type-2 with hesitation index with or without using the optimized membership function by BB-BC algorithm and outperformed GFL-MCGDM system without BB-BC optimization method.

7 Conclusions and Future Work

In this paper, we presented a General Type-2 Fuzzy Logic based approach for MCGDM (GFL-MCGDM). The proposed system aims to handle the high levels of uncertainties which exist due to the varying Decision Makers' (DMs) judgments and the vagueness of the appraisal. In order to find the optimal parameters of the general type-2 fuzzy sets, we employed the Big Bang-Big Crunch (BB-BC) optimization.

We have carried out experiments in the intelligent apartment (iSpace) located in the University of Essex to evaluate various approaches employing group decision making techniques for illumination selection in an intelligent shared environment. The proposed system which utilized BB-BC optimization was able to model the variation in the group decision making process exhibited by the various decision makers' opinions in the intelligent environment. The optimal membership functions evaluated by BB-BC allowed the system to find the highest correlations value between decision makers and the proposed system

In addition, the proposed system showed agreement between the proposed method and the real decision outputs from DMs (as quantified by the Pearson Correlation) which outperformed the MCGDM systems based on type-1 fuzzy sets, interval type-2 fuzzy sets and interval type-2 with hesitation index and also general type-2 without an optimal membership function. The increased correlation value shows that the proposed method is considered to be effective in handling the high level of uncertainties among the DMs and the aggregation phase of the system.

Hence, this shows that the proposed method can play an important role in the production of better Fuzzy MCGDM which is able to better settle conflicts among the different individual preferences with different alternatives and criteria followed by synthesizing the different individual preferences into a unanimous approval.

For future work, we intend to different shapes of general type-2 fuzzy sets evaluated in various real world applications.

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