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Research paper

Conceptual design of reinforced concrete structures using truss-like topology optimization

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Abstract: The paper proposes a procedure for the conceptual design of reinforced concrete (RC) structures under a multiple load case (MLC), based on the truss-like topology optimization method. It is assumed that planar truss-like members are densely embedded in concrete to simulate RC structures. The densities and orientations of the reinforcing bars at nodes are regarded as optimization variables. The optimal reinforcement layout is obtained by solving the problem of minimizing the total volume of reinforcing bars with stress constraints. By solving a least squares problem, the optimized reinforcement layout under the MLC is obtained. According to the actual needs of the project, the zones to be reinforced are determined by reserving a certain percentage of elements. Lastly, a recommended reinforcement design is determined based on the densities and orientations of truss-like members. The reinforcement design tends to be more perfect by adding necessary structural reinforcements that meet specification requirements. No concrete cover is considered. Several examples are used to demonstrate the capability of the proposed method in finding the best reinforcement layout design.

Keywords: conceptual design, reinforced concrete, topology optimization, truss-like material

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1. Introduction

Numerical methods for topology optimization of continuous structures have undergone a rapid development over the past decade. several approaches have been put forward to solve topology optimization problems so far. In 1988, Bendsøe and Kikuchi [1], inspired by the homogenization theory in the field of composite materials, proposed the homogenization method where the design domain can be divided into finite cell, and each cell consists of an individual micro-structure. The sizes and angles of the microstructures in every element are optimized to achieve the optimal structural topology. Later, to improve the efficiency of topology optimization, so-called simp (simplified isotropic material with penalization) was proposed by Bendsøe [2] and others [3, 4]. In 1993, inspired by the idea of biological evolution, the eso (evolutionary structural optimization) method originally proposed by xie and steven [5] is built on the basic criterion: gradually removing the inefficient elements, the structure evolves towards an optimum. in 2003, Wang et al. [6] put forward the level set method for structural topology optimization. In addition, new advances have been made in the past several years. Guo et al. [7] proposed the moving morphable components method. Similar to this method, there is the moving morphable bars method [8].

Topology optimization has become an effective design tool that can be utilized in a wide range of engineering fields, spanning from industrial products to structural engineering. It has been applied to optimize the design of plane structures, space structures and prager structures [9, 10]. In general, a RC structure can be divided into b-regions and d-regions in practical design processes. The approach for b-region design is maturely established and can be easily achieved by the traditional bending theory and a general shear design method. While in the structural design for d-regions, traditional approaches for slender beams are inappropriate. How to achieve a proper analysis and design for complex stress components such as corbels, walls or deep beams with openings, pile caps, and beam-column joints has been an enormous challenge for decades.

At present, the strut-and-tie method is a basic tool for analysis and design of RC structures, which has been incorporated in different codes of practice. Structural topology optimization has been used to generate strut-and-tie model (STM). Bołbotowski and Sokół [11] developed a new method to generate STMS on the basis of a modification of the classic ground structure approach. Some scholars adopted the eso method to generate STMS [12–16]. Shobeiri et al. [17, 18] proposed a method to generate STMS in RC structure based on the beso algorithm. However, the reinforced concrete is regarded as a single material in these methods. Bruggi [19] generated STMS by solving the problem of minimum flexibility with volume constraints based on the simp method. Xia et al. [20] proposed a program to evaluate the topology optimization results of generating STMS. Qiao et al. [21] established STMS using the mmc method. However, these methods fail to consider the differences between the characteristics of reinforcing bars and concrete. Considering the different mechanical properties for the tensile (steel) and compressive (concrete) regions, Victoria et al. [22] put forward a method to generate more efficient STMS. Du et al. [23] studied structural topology optimization involving bi-modulus materials with asymmetric properties in tension and compression, and it can be used for generating STMS. Amir and

Sigmund [24] presented a new method to obtain the optimal reinforcement layout, in which the strain softening damage model is used to represent the concrete, and the ground structure is used to simulate the reinforcement. But, the ground structure has limitations. The initial truss layout is pre-defined, with a certain subjectivity, and has a significant impact on the final truss topology. Luo and Kang [25] proposed a topology optimization algorithm based on bi-material model to obtain the optimal reinforcement layout. Marco [26] proposed a procedure for the automatic preliminary design of reinforced concrete structures based on the eso method. Yang et al. [27] put forward a method to optimize the reinforcement design of RC structures under a single load case (SLC) using truss-like material model.

The paper proposes a procedure for the conceptual design of RC structures under an MLC, based on the truss-like topology optimization. The planar truss-like members are densely embedded in concrete to simulate RC structure. The densities and orientation of the truss-like members at nodes are taken as design variables. The optimization problem is to minimize the total volume of reinforcing bars with stress constraints. Firstly, as per the fully stressed criterion based on bi-phase material, the optimal reinforcement layout under each SLC is obtained. Secondly, by solving a least squares problem, the optimized reinforcement layout under the MLC is obtained. Then, calculate the volume of reinforcing bars in all elements and arrange them in descending order. According to the actual needs of the project, the zones to be reinforced are determined by reserving a certain percentage of elements. Lastly, a recommended reinforcement design is offered based on the densities and angles of reinforcing bars in the zones. The algorithm in this paper neither penalizes intermediate densities nor removes inefficient elements. Therefore, no numerical instability exists in optimization iterations. Several examples are used to demonstrate the capability of the proposed method in finding the best reinforcement layout design. Obtained solutions are able to suggest useful resulting reinforcement layouts.

2. Finite element analysis and reinforcing bars volume

The planar truss-like material model with two families of orthotropic members is adopted to simulate reinforcing bars embedded in concrete. It is assumed that the densities and orientations of the two families of reinforcing bars are ρ_1, ρ_2, and α, respectively. Then, the elastic matrix of the planar truss-like material can be denoted as follows [28]

$$\boldsymbol{D}_s(\rho_{1j}, \rho_{2j}, \alpha_j) = E_s \sum_{b=1}^{2} \rho_{bj} \sum_{r=1}^{3} s_{br} g_r(\alpha_j) \boldsymbol{A}_r \tag{2.1}$$

where: E_s is elastic modulus of reinforcing bars; s_{br}, g_r and $\boldsymbol{A}_r$ are refered to the existing literature above; J is the set of all nodes in the design domain.

The first row and first column element of the elastic matrix $\boldsymbol{D}_s(\rho_{1j}, \rho_{2j}, \alpha_j)$ can be written as follows:

$$D_{11} = \frac{E[(\rho_1 + \rho_2) + (\rho_1 - \rho_2)\cos 2\alpha]}{2} \tag{2.2}$$

The directional stiffness of the continuum of reinforcing bars along θ can be denoted as:

$$S(\theta) = D_{11}(\theta; \rho_1, \rho_2, \alpha) = E_s[(\rho_1 + \rho_2) + (\rho_1 - \rho_2)\cos 2(\alpha - \theta)]/2 \tag{2.3}$$

The element stiffness matrix of the bi-phase material structures can be denoted as:

$$\boldsymbol{k}_e = \int_{V_e} \boldsymbol{B}^T\,(\boldsymbol{D}_s + \boldsymbol{D}_c)\,\boldsymbol{B}\,\mathrm{d}V = \boldsymbol{k}_e^s + \boldsymbol{k}_e^c \tag{2.4}$$

where $\boldsymbol{B}$ is geometry matrix; $\boldsymbol{D}_c$ and $\boldsymbol{D}_s$ are the elastic matrix of concrete and continua of reinforcing bars; $\boldsymbol{k}_e^s$ and $\boldsymbol{k}_e^c$ are the element stiffness matrix of the concrete and continua of reinforcing bars, respectively.

The structural stiffness matrix $\boldsymbol{K}$ of the bi-phase material structures can be obtained as follow:

$$\boldsymbol{K} = \sum \left(\boldsymbol{k}_e^s + \boldsymbol{k}_e^c\right) \tag{2.5}$$

Solving the structural stiffness equation, we get nodal displacement vector $\boldsymbol{U}$.

Regardless of the relative slip between concrete and reinforcing bars, the strain at any node j of a truss-like structure is defined as the average strain of the elements around node j, namely:

$$\boldsymbol{\varepsilon}_j = \frac{1}{n_j} \sum_{e \in S_j} \boldsymbol{B}_j \boldsymbol{U}_e \tag{2.6}$$

where S_j and n_j are the set of elements and the number of elements around node j, respectively.

The node stress column vector is calculated according to the following formula:

$$\boldsymbol{\sigma}^s = \boldsymbol{D}_s \varepsilon = \begin{bmatrix}\sigma_x^s & \sigma_y^s & \tau_{xy}^s\end{bmatrix}^T, \qquad \boldsymbol{\sigma}^c = \boldsymbol{D}_c \varepsilon = \begin{bmatrix}\sigma_x^c & \sigma_y^c & \tau_{xy}^c\end{bmatrix}^T \tag{2.7}$$

The total volume of reinforcing bars is calculated by:

$$V = \sum_{j \in J} \sum_{e \in S_j} \int_{V_e} N_j \,\mathrm{d}V \sum_b \rho_{bj} \tag{2.8}$$

The finite element analysis used in the algorithm regards concrete and reinforcing bars as linear elastic.

3. Optimal reinforcement layout under single load case

To obtain the optimized continuum of reinforcing bars under an MLC, it is first necessary to optimize the reinforcement layout under each SLC. The densities and orientations of the reinforcing bars at nodes are taken as design variables. The optimization problem is to minimize the total volume of reinforcing bars with stress constraints. To give full play to the strength of materials, the reinforcing bars and concrete at any point in the structure should be in a state of full stress as far as possible. In other words, to prevent the failure of concrete in the tensile (compression) zone, an appropriate amount of reinforcement is arranged in the tension (compression) zone, so that the principal stress (principal strain) of concrete and reinforcement is equal to the allowable stress (strain) of the material. Only in this way can the strength of reinforcing bars and concrete be fully utilized. Therefore, the formulation of optimization problem for RC structure under each SLC can be written as

$$\tag{3.1} \begin{cases} \text{find } \overline{\rho}_{bj},\ \overline{\alpha}_j \\ \min\ \overline{V} \\ \qquad \left|\sigma_b^s\right| \le \sigma_p^s \\ \text{s.t.}\ \ \left|\sigma_c^c\right| \le \sigma_{pc}^c \\ \qquad \sigma_t^c \le \sigma_{pt}^c \end{cases} \qquad \begin{array}{l} b = 1,2 \\ j = 1,2,\ldots,J \end{array}$$

where $\overline{\rho}_{bj}$ and $\overline{\alpha}_j$ represent the densities and orientations of reinforcing bars (namely, the principal stresses) at node j under each SLC, respectively; $\overline{V}$ is the volume of reinforcing bars under each SLC; σ_p^s denotes the permissible stress of reinforcing bars; σ_{pc}^c is permissible compressive stress of concrete; σ_{pt}^c is permissible tensile stress of concrete; σ_b^s denotes the stress along reinforcing bars under load case l; σ_c^c and σ_t^c denote principal stress in concrete under each SLC.

Considering the equilibrium along the directions of principal stress in the composite material, the optimal densities of reinforcing bars under each SLC can be optimized as per the fully stressed criterion based on bi-phase material

$$\tag{3.2} \overline{\rho}_{bj}^{k+1} = \frac{\sigma_{bj}^k - \sigma_p^c}{\sigma_p^s - \sigma_p^c}, \qquad \begin{array}{ll} \sigma_p^c = \sigma_{pt}^c, \ \text{if } \varepsilon_b^k \ge 0 & b = 1,2 \\ \sigma_p^c = \sigma_{pc}^c, \ \text{if } \varepsilon_b^k < 0 & j = 1,2,\ldots,J \end{array}$$

4. Optimal reinforcement layout under multiple load case

The formulation of reinforcement optimization problem for reinforced concrete structure under the MLC can be written as

$$\tag{4.1} \begin{cases} \text{find } \rho_{bj},\ \alpha_j \\ \min\ V \\ \qquad \left|\sigma_{bl}^s\right| \le \sigma_p^s \\ \text{s.t.}\ \ \left|\sigma_{cl}^c\right| \le \sigma_{pc}^c \\ \qquad \sigma_{tl}^c \le \sigma_{pt}^c \end{cases} \qquad \begin{array}{l} b = 1,2 \\ j = 1,2,\ldots,J \\ l = 1,2,\ldots,L_c \end{array}$$

where ρ_{bj} and α_j represent the densities, orientations of reinforcing bars at node j under the MLC, respectively; V is the volume of reinforcing bars under the MLC.

For the optimization problem of minimum volume of a stress-constrained structure under a SLC, it is easy to determine the principal stress direction of a truss-like structure. The optimized the continuum of reinforcing bars can be obtained by arranging the members along the principal stress directions and updating the densities as per fully stressed criterion based on bi-phase material. However, for the optimization design under an MLC, the fully stressed criterion does not apply. There is a principal stress direction in each case. It is impossible to determine the member orientations of the optimized structures under the MLC based on the principal stress directions. Therefore, we need to consider other methods.

As we know, in a truss-like structure, the more material is arranged along the direction of the member, the greater the structural stiffness in that direction. When the external load remains unchanged, the stress at this point (the stress along the direction of the truss-like member, namely, the principal stress) is smaller. To satisfy the stress constraint conditions of the optimization problem, the stiffness along any direction at any point in the optimized structure under the MLC should be no less than the stiffness envelope value of the optimized structure under each SLC. In addition, to make the structural volume V minimum, the stiffness along any direction at any point in the structure should not exceed the required amount. Namely, the directional stiffness under the MLC is as similar as possible to the maximum directional stiffness of the optimal structure under every SLC along all directions. Therefore, the optimization problem for RC structures under the MLC can be transformed into

$$\begin{cases} \text{find } \; \rho_{bj},\ \alpha_j \\ \min \; \|S(\theta) - S_{\max}(\theta)\| \end{cases} \tag{4.2}$$

where $S_{\max}(\theta)$ denotes the maximum stiffness under all SLCs.

According to Eq. (2.1), once the optimal reinforcement layout under each SLC is obtained, the elastic matrix $\boldsymbol{D}\big(\overline{\rho}_{1l}\ \overline{\rho}_{2l},\ \overline{\alpha}_l\big)$ $(l = 1, 2, \ldots, L_c)$ of the optimal structure under the SLC at every node is determined. The elastic matrix $\boldsymbol{D}(\rho_1, \rho_2, \alpha)$ of the optimal structuresunder the MLC can be estimated based on the elastic matrix $\boldsymbol{D}(\overline{\rho}_{1l}, \overline{\rho}_{2l}, \overline{\alpha}_l)$ $(l = 1, 2, \ldots, L_c)$. It is assumed that reinforcing bars with densities y_1 and y_2 are arranged along two mutually orthogonal directions φ and $\varphi + \pi/2$ in the optimized structure under the MLC. The directional stiffness of the continuum of reinforcing bars along θ can be denoted as:

$$S(\theta) = D_{11}(\theta; y_1, y_2, \varphi) = E\left[(y_1 + y_2) + (y_1 - y_2)\cos 2(\varphi - \theta)\right]/2 \tag{4.3}$$

The differences δ^2 between the stiffness of the optimal structure under the MLC and the maximum stiffness under all SLCs over all directions is defined as [28]:

$$\begin{aligned}\delta^2 &= \|S(\theta) - S_{\max}(\theta)\|_2^2 = \|D_{11}(\theta; y_1, y_2, \varphi) - S_{\max}(\theta)\|_2^2 \\ &= \int_0^{\pi} \left[D_{11}(\theta; y_1, y_2, \varphi) - S_{\max}(\theta)\right]^2 \mathrm{d}\theta \\ &= \int_0^{\pi} \left[D_{11}(\theta; y_1, y_2, \varphi)\right]^2 \mathrm{d}\theta - 2\int_0^{\pi} \left[D_{11}(\theta; y_1, y_2, \varphi) \cdot S_{\max}(\theta)\right] \mathrm{d}\theta \\ &\quad + \int_0^{\pi} [S_{\max}(\theta)]^2 \,\mathrm{d}\theta\end{aligned} \tag{4.4}$$

The extremum condition is obtained by the differentiation of Eq. (4.4) with respect to $(y_1 + y_2)$, $(y_1 - y_2)$ and φ:

$$\begin{aligned}\frac{\partial \delta_1^2}{\partial(y_1 + y_2)} &= \frac{\pi E^2}{2}(y_1 + y_2) - \pi E^2 J_0 = 0 \\ \frac{\partial \delta_1^2}{\partial(y_1 - y_2)} &= \frac{\pi E^2}{4}(y_1 - y_2) - \pi E^2 (J_1 \cos 2\varphi + J_2 \sin 2\varphi) = 0 \\ \frac{\partial \delta_1^2}{\partial \varphi} &= -2\pi E^2 (y_1 - y_2)(-J_1 \sin 2\varphi + J_2 \cos 2\varphi) = 0\end{aligned} \tag{4.5}$$

This leads to:

$$y_1, y_2 = J_0 \pm 2(J_1 \cos 2\varphi + J_2 \sin 2\varphi) \tag{4.6}$$

$$\varphi = \begin{cases} \dfrac{1}{2} \arctan \dfrac{J_2}{J_1} & \text{if } \; J_1 \cos 2\varphi_1 + J_2 \sin 2\varphi_1 > 0 \\ \dfrac{1}{2} \arctan \dfrac{J_2}{J_1} + \dfrac{\pi}{2} & \text{if } \; J_1 \cos 2\varphi_1 + J_2 \sin 2\varphi_1 < 0 \end{cases} \tag{4.7}$$

where:

$$\begin{aligned}J_0 &= \frac{1}{\pi E} \int_0^{\pi} S_{\max}(\theta)\,\mathrm{d}\theta \\ J_1 &= \frac{1}{\pi E} \int_0^{\pi} \left[S_{\max}(\theta) \cos 2\theta\right] \mathrm{d}\theta \\ J_2 &= \frac{1}{\pi E} \int_0^{\pi} \left[S_{\max}(\theta) \sin 2\theta\right] \mathrm{d}\theta\end{aligned} \tag{4.8}$$

5. Demonstration of optimal reinforcement layout

The optimal reinforcement layout derived by the numerical algorithm developed in this paper is a type of non-uniform anisotropic continuum. Reinforcing bars in concrete are distributed continuously. Firstly, to demonstrate the reinforcement layout, crossed lines are adopted. The densities and orientations of reinforcing bars are presented using two short lines at every node. The orientations and lengths of the two lines represent the angles and densities of two families of reinforcing bars at every node. A few lines that are too long are cut short to make the figure distinguishable. Secondly, calculate the volume of reinforcing bars in all elements and arrange them in descending order. According to the actual needs of the project, determine the zone to be reinforced by reserving a certain percentage of elements. Lastly, a recommended reinforcement design is determined based on the densities and angles of reinforcing bars as follow: Draw lines from nodes with larger densities to the finite element boundary. The orientation at the point where the line intersects the element boundary is calculated by interpolating the orientations of the nodes at both ends of the element boundary as the orientations of the next line segment in adjacent elements. The next line segment is drawn in adjacent elements along the orientation calculated above. This process is repeated from one element to another until the line reaches the design domain boundary, creating a polyline. Only the polylines within the zone to be reinforced are retained to represent the reinforcement arrangement.

6. Optimization approach and procedure

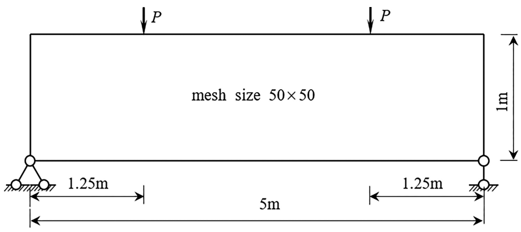
The optimization problem is to minimize the total volume of reinforcing bars with stress constraints. The densities and orientations of reinforcing bars at nodes are regarded as design variables. The procedure for topology optimization problem of the reinforcement layout design under MLCs involves the following steps:

1. The design domain is divided into finite elements.
2. Set the iteration index $k = 0$; Initial design values are assigned to design variables; The element retention ratio is set.
3. Finite element analysis is performed to obtain nodal displacement vector $\boldsymbol{U}$.
4. The stress vectors of reinforcing bars and concrete is calculated according to Eq. (2.7).
5. The Optimal reinforcement layout under each SLC is determined by Eq. (3.2).
6. The optimal densities and orientations of reinforcing bars under the MLC is obtained as per Eq. (4.6) and Eq. (4.7), respectively.
7. Return to step (3) if the relative change in the maximum densities of reinforcing bars in two successive iterations is larger than a given small positive value (10^{-2} in this study) or the loop iterations are less than 10. Otherwise, the iterations are terminated.
8. Optimal continua of reinforcing bars are illustrated.
9. Calculate the volume of reinforcing bars in all elements and arrange them in descending order. The Zones to be reinforced are determined.
10. The Zones to be reinforced are determined.

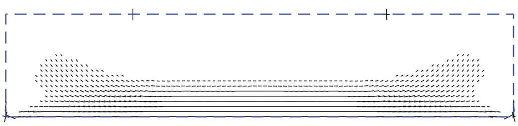
7. Numerical examples

Three examples are presented in this section. The concrete grade is C30. Young's modulus of reinforcing bars and concrete are $E_s = 210$ GPa and $E_c = 7.15$ GPa, respectively. Poisson's ratio of concrete is $\nu = 0.2$; the compressive and tensile strengths of concrete are $\sigma^c_{pc} = 14.3$ MPa and $\sigma^c_{pt} = 1.43$ MPa, respectively. The compressive and tensile strengths of reinforcing bars are $\sigma^s_p = 360$ MPa [27]. Four-node rectangular elements are adopted. Crossed lines at nodes are used to denote the optimal layout of the reinforcing bars. The orientations and the lengths of the two lines represent the orientations and densities of two families of reinforcing bars at every node. A few lines that are too long are cut short to make the figure recognizable. No concrete cover is considered.

Example 1: In this example, the layout design of reinforcing bars in a simply supported beam is considered. The geometry and dimensions of the design domain are shown in Fig. 1a. The beam is acted by two point load $P = 300$ kN. Crossed lines are drawn in Fig. 1b to demonstrate the optimal layout of the reinforcing bars. The zones to be reinforced are determined in Fig. 2a and Fig. 3a according to different retention ratios (RR).

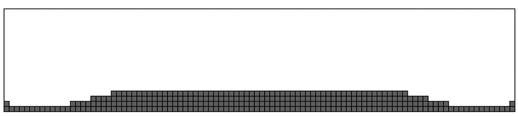


(a) Mechanics model

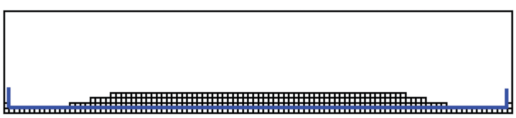


(b) Optimal layout of reinforcing bars

Fig. 1. Mechanics model and optimal layout of reinforcing bars of example 1



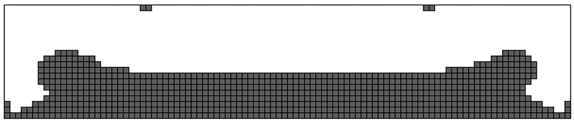
(a) Zone to be reinforced



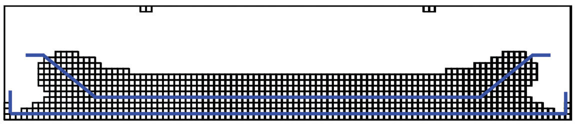
(b) Recommended reinforcement design

Fig. 2. Optimal layout of reinforcing bars of example 1 (RR = 15%)

The recommended reinforcement designs are shown in Fig. 2b and Fig. 3b, respectively. It is possible to observe the presence of reinforcing bars in the bottom part of the beam to absorb tensile stresses generated by bending. On the sides, diagonal reinforcing bars are present to resist shear stresses in the beam. The shear reinforcing bars have angle measured to be around 45°.



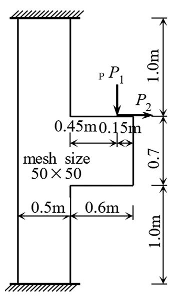
(a) Zone to be reinforced



(b) Recommended reinforcement design

Fig. 3. Optimal layout of reinforcing bars of example 1 (*RR* = 40%)

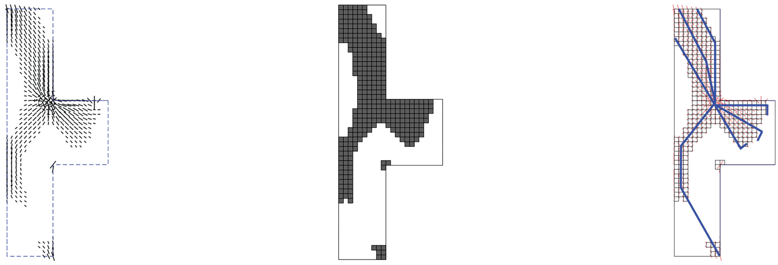
Example 2: In this example, the layout design of reinforcing bars in a 2-D corbel is considered. The geometry and dimensions of the design domain are shown in Fig. 4a. The corbel is acted by two independent load sets P_1 and P_2. 708 rectangle four-node plane stress elements are used. Crossed lines are drawn in Fig. 4c, 5a and 6a to demonstrate the optimal layout of reinforcing bars under the SLC1 (P_1 = 500 kN, P_2 = 0 kN), SLC2 (P_1 = 0 kN, P_2 = 350 kN) and the MLC (P_1 = 500 kN, P_2 = 300 kN), respectively. The Optimal distribution of steel and concrete, as shown in Fig. 4b [26], was obtained on the basis of the ESO method under the SLC1 (P_1 = 500 kN, P_2 = 0 kN). The zones to be reinforced are determined in Fig. 4d, Fig. 5b and Fig. 6b according to different *RR*. The recommended reinforcement designs are shown in Fig. 4e, Fig. 5c and Fig. 6c, respectively.



(a) Mechanics model

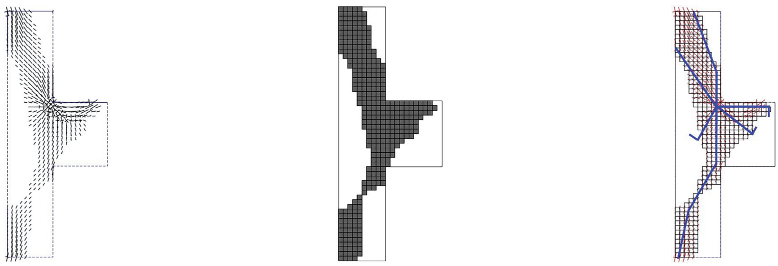
(b) Optimal distribution of steel and concrete based on the ESO method [26]

Fig. 4. Optimal layout of reinforcing bars under SLC1 of example 2



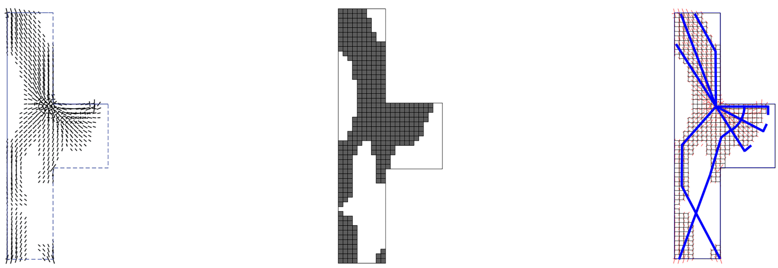
(c) Optimal reinforcement layout under SLC1 | (d) Zone to be reinforced (RR = 45%) | (e) Recommended reinforcement design (RR = 45%)

Fig. 4. Optimal layout of reinforcing bars under SLC1 of example 2



(a) Optimal reinforcement layout under SLC2 | (b) Zone to be reinforced (RR = 50%) | (c) Recommended reinforcement design (RR = 50%)

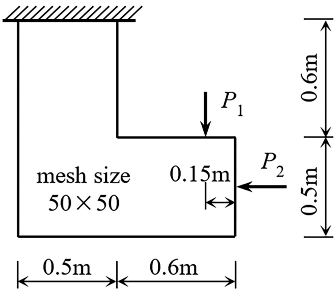
Fig. 5. Optimal layout of reinforcing bars under SLC2 of example 2



(a) Optimal reinforcement layout under MLC | (b) Zone to be reinforced (RR = 55%) | (c) Recommended reinforcement design (RR = 55%)

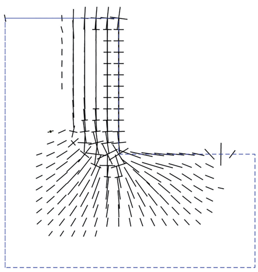
Fig. 6. Optimal layout of reinforcing bars under MLC of example 2

Example 3: In this example, the layout design of reinforcing bars in a 2-D corbel with a ledge support is considered. The geometry and dimensions of the design domain are shown in Fig. 7a. The corbel is acted by two independent load sets $P_1 = 500$ kN and $P_2 = 350$ kN. 340 rectangle four-node plane stress elements are used. Crossed lines are drawn in Fig. 7c, 8a and 9a to demonstrate the optimal layout of the steel bars under

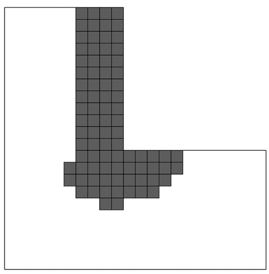


(a) Mechanics model

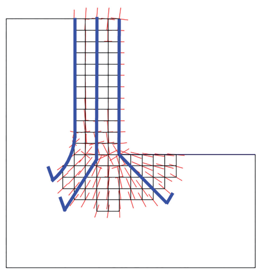
(b) Optimal distribution of steel and concrete based on the ESO method [26]



(c) Optimal reinforcement layout under SLC1

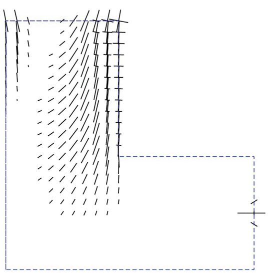


(d) Zone to be reinforced (RR = 245%)

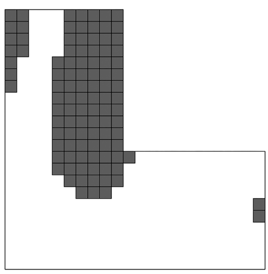


(e) Recommended reinforcement design (RR = 25%)

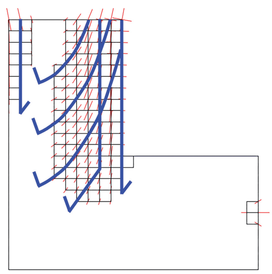
Fig. 7. Optimal layout of reinforcing bars under SLC1 of example 3



(a) Optimal reinforcement layout under SLC2



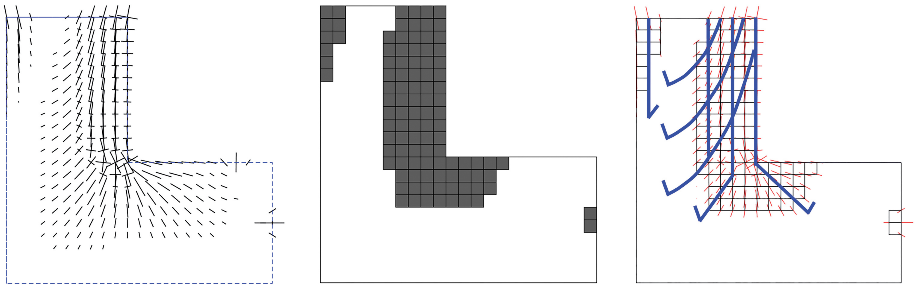
(b) Zone to be reinforced (RR = 30%)



(c) Recommended reinforcement design (RR = 30%)

Fig. 8. Optimal layout of reinforcing bars under SLC2 of example 3

the SLC1 (P_1 = 250 kN, P_2 = 0 kN), SLC2 (P_1 = 0 kN, P_2 = 400 kN) and the MLC (P_1 = 250 kN, P_2 = 400 kN), respectively. The Optimal distribution of steel and concrete, as shown in Fig. 7b [26], was obtained on the basis of the ESO method under the SLC1 (P_1 = 500 kN, P_2 = 0 kN). The zones to be reinforced are determined in Fig. 7d, Fig. 8b and Fig. 9b according to different *RR*. The recommended reinforcement designs are shown in Fig. 7e, Fig. 8c and Fig. 9c, respectively.



(a) Optimal reinforcement layout under MLC

(b) Zone to be reinforced (*RR* = 30%)

(c) Recommended reinforcement design (*RR* = 30%)

Fig. 9. Optimal layout of reinforcing bars under MLC of example 3

8. Conclusions

A numerical algorithm is presented in this paper that can generate the optimal reinforcement layout of RC structures under an MLC. The planar truss-like material model with two families of orthotropic members are densely embedded in concrete to simulate reinforced concrete structure. The densities and orientations of the truss-like members at nodes are taken as design variables. The optimization problem is to minimize the total volume of reinforcing bars with stress constraints. Compared with the ESO method, the algorithm in this paper obtained the optimal reinforcement layout with a small number of elements and fewer iterations. No numerical instability exists in optimization iterations. In addition, the zones to be reinforced can be determined according to the actual needs of the project. The orientation of the reinforcement is more accurate. Obtained solutions are able to suggest useful resulting reinforcement layouts.

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