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DYNAMIC BALANCE RESEARCH OF PROTECTED SYSTEMS

I. Naumeyko, M. Alja'afreh
Kharkiv National University of Radio Electronics; e-mail: pmkaf@kture.kharkov.ua
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Abstract. The dynamic models of the complex ergatic objects' behavior, presented in the form of differential equations and their systems were studied. The stability and other properties are researched. The methods of analysis and reduce of harmful factors and their impact on people were theoretically proved. The methods of analysis and critical points removal in dynamic models of hazards distribution are offered.

The object of study is the system of the harmful external factors protection. Subject of research is the system of two nonlinear differential equations as a model of technical systems with protection. The object of protection is described by logistic equation. and defense system - by non-linear differential equation with a security functions of rather general form. This paper describes critical modes analysis and stationary states' stability of protected systems with harmful influences. Numerical solution of general problem and also the analytical solution for the case of fixed expected harmful effects have been obtained. Various types of general models for "Man-machine-environment" systems were studied. Each of describes some kind of the practically important quality of object in an appropriate way. And all together they describe the object in terms of it's safe operation. Their further detailing process results to either well-known, or some new subsystems' models. Systems with "fast" protection at a relatively slow dynamics of the object were studied. This leads to the models with small parameter and asymptotic solutions of differential equations. Some estimates for protection cost in different price-functional and for different functions in the right part of equation, which describes the dynamics of defense were obtained. For calculations, analysis and graphical representations some of mathematical packages was applied.

Key words: Non-linear system, singular points, eigenvalues, asymptotic behavior, first approximation, linearization.

INTRODUCTION

One of the most important elements of Ukrainian economy has always been and remains to be industrial production, which is not safe at all. And it is pretty out of date both morally and physically. In this regard, the devices and their integrated systems of protection the staff and surrounding population are particularly important today [1, 2]. It is known that safety and efficiency are conflicting criteria. Their junction is possible only in a complex supersystem [3]. This approach allowed to consider a model of "Human-Machine-protected Environment" as a known model of competition of two factors – safety and efficiency [4, 5].

In this context, let us first examine the general "Man-Machine-Environment" system. Input information for this system is the information from the higher-level system (targets, instructions, etc.); output of this system is the labor result.

When the system operates, it's internal state changes. The element "Human" has three functional parts: control the "Machine", object of influence by the environment and the "Machine" either.

The element "Machine" fulfills a major technological function – impact on the subject of labor and a side function – to change parameters of environment.

Different types of common models of "Human-Machine Environment" were studied in this paper, each of which describes in a proper way some of practically important object's quality. And all together describe an object in terms of its safe functioning [6]. Their further detailing leads to well-known and as well as to some new models of subsystems [7]. This work is on quantitative analysis of an important model - protection of person from harmful effects from external environment and from impact of the "Machine "subsystem.

THE ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

In works [8, 9] a model of dynamic system that describing a situation where primary subsystem "produces" a harmful factor, and second sub-system - protection - is trying to reduce it completely, or at a reasonable price. As the base model – the basis for modification – a system of ordinary differential equations was taken. It describes fundamental laws of competition [10], and also known in ecology as a model of coexistence of species [11 – 14].

We need to check that despite the model has been simplified [15], basic characteristics and dependents on the system should be available. Based on the results obtained in the course of work the bifurcation of protection system must be analyzed, i.e. we should figure out a scenario of stability's loss [16] and protective effectiveness.

We follow [4] to introduce the basic assumptions directly following from everyday experience. They are evident, i.e. they do not require additional justification and only need to be formalized. Below they are called the Axioms [4].

Consider the Bio Impact U:

$$U = \int_0^T u(t)dt \qquad (1)$$

where: the intensity of harmful factor u is an alternating function of time t; T is total exposition time.

The total Bio Impact may also depend on the intensity of the other harmful factor *v*. Similarly, $V(t,v,u)$.

In the first approximation the additive property is provided [6]:

$$\text{«overall harmfulness»} = k_1U + k_2V,$$

where k_i are the weights.

Axioms (natural positions):

1. Auto cumulativeness.

The harmful effects is growing faster than its value *u*.

2. Mutual cumulativeness.

The harmful effects grow as other factors together with *u* are growing.

3. In regular situation $\frac{\partial}{\partial t}u \le 0$. In critical situation (positive feedback) $\frac{\partial}{\partial t}u > 0$.

4. Protection $z(t)$ can be controlled programmatically or adaptively, depending on the value $u(t)$.

5. The cost of protection $C=C(z)$ it is natural to consider as a steady increasing function of its intensity.

Let *f* and *g* be smooth functions, steadily increasing in both arguments,

$$f(0,V) = g(V,0) = 0, \forall U,V .$$

Then it is natural to suppose

$$U' = f(U,V)\,;\; V' = g(U,V) .$$

It is rather general case for a system of differential equations describing behavior of the object to have under conditions $u \ge 0$, $z \ge z_0$ the form of:

$$\begin{cases} u'(t) = \alpha u(t) - \beta z u(t) \\ z'(t) = F(u(t), z(t)) \end{cases}, \qquad (2)$$

where: z_o is stationary protection; $F(u, z)$ is protection ability function.

OBJECTIVES

First, we conduct a formal description of the model researched in this work. Harmful effects can be written in the first approximation, as the integral (1).

Protection ability function $F(u, z)$ from (2) in this work is considered in either of the following general enough forms:

1) $F(u(t), z(t)) = \gamma u(t)$;
2) $F(u,z) = \gamma u - \delta z$;
3) $F(u,z) = \gamma_1 u + \gamma_2 u^2 - \delta_1 z - \delta_2 z^2$.

Solution of the differential equations system (2) is not always possible to be found analytically. That is why for finding protection functions and harmful impact effects some numerical methods are used to solve this system of differential equations. So, the objective is to study for stability the system (2) under different forms of protection ability function and values of the subsystem protection parameters (α, β, γ). It is also necessary to evaluate the cost of protectionability for different functions $F(u,z)$.

THE MAIN RESULTS OF THE RESEARCH

1. Methods for studying stability.

The theorem of linearization establishes a relation of phase portrait of a nonlinear system in the neighborhood of the stability point with phase portrait of its linearization [16, 17].

The origin of coordinates is a simple fixed system's point for

$$\dot{y} = X(y), y \in S \subseteq R^2 ,$$

if its corresponding linearized system is simple.
This definition extends the meaning of simplicity on fixed points of nonlinear systems. It can be used also in the case when singular point, which interests us is not at origin of coordinates; then we have to enter local coordinates.

Let nonlinear a system $\dot{y} = Y(y)$ have a simple fixed point y=0. Then in a neighborhood of the origin phase portraits of this system and its linearization are qualitatively equivalent, unless a fixed point of linearized system is not a center [17].

The theorem on linearization forms the basis of one of the main methods of investigation the non-linear systems – the method of investigating stability in linear approximation.

By applying in practice the linearization theorem the significant simplification in calculations are achieved since with linear terms of new system it is more convenient to carry out a qualitative analysis than with nonlinear.

Application of theorems on linearization are similarly considered in analysis of environmental models and competition in economic systems [18 – 20].

2. The problem of fast and slow variables.

Dynamical systems include a large number of processes with different time scales meanwhile the hierarchy of these times is such that they differ greatly [20, 21].

The level of detailing in modeling of studied phenomena depends on purpose of modeling. However in any case the problem of modeling is to build a model of phenomenon having as smaller number of variables and arbitrary parameters as possible and at the same time to correctly reflect properties of phenomena.

Accounting for time hierarchy process lets the reducing of the number of differential equations. "Very slow" variables do not change on time scales of these processes and can be regarded as constant parameters. For "fast" variables instead of differential equations the algebraic equations for their steady-state values can be written. As "fast" variables reach their steady-state values almost instantly compared with "slow" ones [8].

3. Research algorithm.

1. We find, if possible, an analytical solution of the system (1) using the functions in the standard mathematical set [22]. If a solution cannot be found in a general way, then let us solve it by numerical methods (in default package is proposed to use a fairly universal method by Adams [19]), by using inner functions, suppose *NDSolve*.

2. Once the solution of (1) is found, let us analyze function of hazard: at what times its value exceeds the value of stationary protection, i.e. protection system activates. By finding these time intervals, we make a decision – to increase the impact on the harmful factor (so, the cost of protection system increases), to leave system unchanged, or there is an opportunity to reduce the cost of protection system, suppose by reducing stationary protection.

3. By choosing a solution we repeat steps 1-2 until we go beyond restrictions (time of system's work or its cost).

4. Analytical model's study.

Let us study a system of differential equations (1) with a small parameter ε:

$$\begin{cases} u'(t) = \varepsilon\alpha u(t) - \beta z(t)u(t) \\ \varepsilon z'(t) = \gamma u(t) - \delta z(t) \end{cases}. \tag{3}$$

The difference of this system from the previously considered is the quasi-stationary harm. Let us solve the system (3) using asymptotic method for $\varepsilon^0, \varepsilon^1, \varepsilon^2$.

To start with, we write out the system (3), taking into account the dependence of functions $u(t,\varepsilon)$ and $z(t,\varepsilon)$ on time and small parameter.

Let us solve system (3) for the case ε^0 (zero approximation).

Let us write functions' asymptotic of $u(t,\varepsilon)$ and $z(t,\varepsilon)$.

$$u(t,\varepsilon) = u_0(t) + \overset{=}{o}(\varepsilon), \quad z(t,\varepsilon) = z_0(t) + \overset{=}{o}(\varepsilon).$$

System (3) for zero approximation becomes as follows:

$$\begin{cases} u_0'(t) = -\beta u_0(t)z_0(t) + \overset{=}{o}(\varepsilon) \\ 0 = \gamma u_0(t) - \delta z_0(t) + \overset{=}{o}(\varepsilon) \end{cases}. \tag{4}$$

When doing substitutions $u_0(t) = \frac{\delta}{\gamma} z_0(t)$, we get:

$$\frac{\delta}{\gamma} z_0'(t) = -\beta z_0^2(t). \quad z_0(t) = \frac{1}{\beta t}, \quad u_0(t) = \frac{\delta}{\beta\gamma t}.$$

Protection and harm functions obtained for zero approximation have the form of:

$$z(t) = \frac{1}{\beta t}, \quad u(t) = \frac{\delta}{\beta\gamma t}.$$

Now let us solve system (3) including member with ε^1.

Similarly, we write functions' asymptotic of $u(t,\varepsilon)$ and $z(t,\varepsilon)$.

$$u(t,\varepsilon) = u_0(t) + \varepsilon u_1(t) + \overset{=}{o}(\varepsilon^2),$$
$$z(t,\varepsilon) = z_0(t) + \varepsilon z_1(t) + \overset{=}{o}(\varepsilon).$$

System (3) for the first approximation has the form of:

$$\begin{cases} u_0'(t) + \varepsilon u_1'(t) = \varepsilon\alpha u_0(t) - \beta u_0(t)z_0(t) - \\ -\varepsilon\beta(u_1(t)z_0(t) + u_0(t)z_1(t)) + \overset{=}{o}(\varepsilon^2) \\ \varepsilon z_0'(t) = \gamma u_0(t) + \varepsilon\gamma u_1(t) - \delta z_0(t) - \varepsilon\delta z_1(t) + \overset{=}{o}(\varepsilon^2) \end{cases}. \tag{5}$$

Members of sum with multiple factors ε with level 2 and higher are converted to a remainder term $\overset{=}{o}(\varepsilon^2)$.

Let us write system (5) in details by grouping terms standing by ε^0 and by ε^1.

$$\begin{cases} u_0'(t) = -\beta u_0(t)z_0(t) \\ 0 = \gamma u_0(t) - \delta z_0(t) \\ u_1'(t) = \alpha u_0(t) - \beta(u_1(t)z_0(t) + u_0(t)z_1(t)) \\ z_0'(t) = \gamma u_1(t) - \delta z_1(t) \end{cases} \left| \begin{array}{l} \varepsilon^0 \\ \varepsilon^0 \\ \varepsilon^1 \\ \varepsilon^1 \end{array} \right. \tag{6}$$

We provide the replacement

$$u_0(t) = \frac{\delta}{\gamma} z_0(t).$$

Substitute into the first equation of system (6) and solve differential equation:

$$\frac{\delta}{\gamma} z'_0(t) = -\beta z_0^2(t).$$

In result of differential equation solution it was found the function $z_0(t)$. And then, with its help the function $u_0(t)$ was also found:

$$z_0(t) = \frac{1}{\beta t}, \quad u_0(t) = \frac{\delta}{\beta\gamma t}. \tag{7}$$

To find functions $z_1(t)$ and $u_1(t)$ let us make a substitution in the third and fourth equation of system (6) functions $z_0(t)$ and $u_0(t)$ from (7). The obtained system has a form of:

$$\begin{cases} u_1'(t) = \alpha\frac{\delta}{\beta\gamma t} - \beta(u_1(t)\frac{1}{\beta t} + \frac{\delta}{\beta\gamma t} z_1(t)) \\ -\frac{1}{\beta t^2} = \gamma u_1(t) - \delta z_1(t) \end{cases} \tag{8}$$

We provide replacement

$$u_1(t)=\frac{1}{\gamma}(\delta z_1(t)-\frac{1}{\beta t^2}).$$

Let us substitute it into the first equation of system (8) and solve differential equation:

$$\frac{1}{\gamma}(\delta z_1{}'(t)+\frac{2}{\beta t^3})=\alpha\frac{\delta}{\beta\gamma t}-\beta(\frac{1}{\gamma}(\delta z_1(t)-\frac{1}{\beta t^2})\frac{1}{\beta t}+\frac{\delta}{\beta\gamma t}z_1(t)).$$

In result of solution of this differential equation function $z_1(t)$ was found and with its help, the function $u_1(t)$ too:

$$z_1(t)=\frac{1}{2\beta}(\alpha-\frac{2\ln t}{t^2\delta}),$$

$$u_1(t)=\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta-2\ln t-2).$$

These obtained protection functions and functions of harm for the first approximation have the form of:

$$z(t)=\frac{1}{\beta t}+\varepsilon\frac{1}{2\beta}(\alpha-\frac{2\ln t}{t^2\delta}),$$

$$u(t)=\frac{\delta}{\beta\gamma t}+\varepsilon\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta-2\ln t-2).$$

Now we solve the system (3) including members with ε^2.

We write in details asymptotic behavior of function $u(t,\varepsilon)$ and $z(t,\varepsilon)$.

$$u(t,\varepsilon)=u_0(t)+\varepsilon u_1(t)+\varepsilon^2u_2(t)+\overset{=}{o}(\varepsilon^3),$$

$$z(t,\varepsilon)=z_0(t)+\varepsilon z_1(t)+\varepsilon^2z_2(t)+\overset{=}{o}(\varepsilon).$$

System (3) for the second approximation has the form of:

$$\begin{cases}u_0{}'(t)+\varepsilon u_1{}'(t)+\varepsilon u_2{}'(t)=\varepsilon\alpha u_0(t)+\varepsilon^2\alpha u_1(t)-\\-\beta u_0(t)z_0(t)-\varepsilon\beta(u_1(t)z_0(t)+u_0(t)z_1(t))-\\-\beta(u_2(t)z_0(t)+u_1(t)z_1(t)+u_0(t)z_2(t))+\overset{=}{o}(\varepsilon^3).\\ \varepsilon z_0{}'(t)+\varepsilon^2z_1{}'(t)=\gamma u_0(t)+\varepsilon\gamma u_1(t)+\varepsilon^2\gamma u_2(t)-\\-\delta z_0(t)-\varepsilon\delta z_1(t)-\varepsilon^2\delta z_2(t)+\overset{=}{o}(\varepsilon^3)\end{cases}\qquad(9)$$

Members in the sum with multiple factors ε with level 3 and higher transfer to remainder term $\overset{=}{o}(\varepsilon^3)$ as before.

We write system (9), grouping the terms with ε^0, ε^1 and ε^2.

$$\begin{cases}u_0{}'(t)=-\beta u_0(t)z_0(t) & \varepsilon^0\\ 0=\gamma u_0(t)-\delta z_0(t) & \varepsilon^0\\ u_1{}'(t)=\alpha u_0(t)-\beta(u_1(t)z_0(t)+u_0(t)z_1(t)) & \varepsilon^1\\ z_0{}'(t)=\gamma u_1(t)-\delta z_1(t) & \varepsilon^1\\ u_2{}'(t)=\alpha u_1(t)-\beta(u_2(t)z_0(t)+u_1(t)z_1(t)+u_0(t)z_2(t)) & \varepsilon^2\\ z_1{}'(t)=\gamma u_2(t)-\delta z_2(t) & \varepsilon^2\end{cases}\qquad(10)$$

Let us provide the replacement

$$u_0(t)=\frac{\delta}{\gamma}z_0(t).$$

Substitute into the first equation of system (10) and solve differential equation:

$$\frac{\delta}{\gamma}z'_0(t)=-\beta z_0{}^2(t).$$

In result of solving the differential equation it was found the function $z_0(t)$ and with its help, it was found the function $u_0(t)$.

$$z_0(t)=\frac{1}{\beta t},\quad u_0(t)=\frac{\delta}{\beta\gamma t}.\qquad(11)$$

In order to find functions $z_1(t)$ and $u_1(t)$ we are doing replacement in the third and the fourth equation of system (10) for the functions $z_0(t)$ and $u_0(t)$ from (11). Newly obtained system has the form of:

$$\begin{cases}u_1{}'(t)=\alpha\frac{\delta}{\beta\gamma t}-\beta(u_1(t)\frac{1}{\beta t}+\frac{\delta}{\beta\gamma t}z_1(t)),\\ -\frac{1}{\beta t^2}=\gamma u_1(t)-\delta z_1(t).\end{cases}\qquad(12)$$

Now, we make the replacement

$$u_1(t)=\frac{1}{\gamma}(\delta z_1(t)-\frac{1}{\beta t^2}).$$

Let us substitute it into the first equation of system (12) and solve the differential equation:

$$\frac{1}{\gamma}(\delta z_1{}'(t)+\frac{2}{\beta t^3})=\alpha\frac{\delta}{\beta\gamma t}-\beta(\frac{1}{\gamma}(\delta z_1(t)-\frac{1}{\beta t^2})\frac{1}{\beta t}+\frac{\delta}{\beta\gamma t}z_1(t)).$$

In the result of differential equation's solution the functions $z_1(t)$ and $u_1(t)$ were found.

$$z_1(t)=\frac{1}{2\beta}(\alpha-\frac{2\ln t}{t^2\delta}),$$

$$u_1(t)=\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta-2\ln t-2).\qquad(13)$$

In order to find functions $z_2(t)$ and $u_2(t)$ let us make a replacement into the fifth and sixth equation of system (10) for functions $z_0(t)$, $u_0(t)$, $z_1(t)$, $u_1(t)$ from (11)–(13). The obtained system has a form of:

$$\begin{cases} u_2{}'(t) = \dfrac{1}{4t^2\beta\gamma\delta}(t^2\delta(t^2\alpha^2\delta - 2\alpha - 4t\beta(\gamma u_2(t) + \\ \quad \delta z_2(t))) - 4\ln t - 4\ln^2 t)\times \\ \quad \dfrac{1}{2\beta}(\alpha - \dfrac{2\ln t}{\beta t^2}) = \gamma u_2(t) - \delta z_2(t). \end{cases} \tag{14}$$

Making the replacement

$$u_2(t) = \frac{1}{\gamma}(\delta z_2(t) + \frac{1}{2\beta}(\alpha - \frac{2\ln t}{\beta t^2})),$$

by substitution into the first equation of system (14) we solve the differential equation:

$$\frac{1}{\beta\delta\gamma t^4}(5 - 6\ln t + \beta\delta^2 t^4 z_2{}'(t)) =$$
$$= \frac{1}{4\beta\delta\gamma t^4}(4 - 2\alpha\delta t^2 + \alpha^2\delta^2 t^4 - 12\ln t - \tag{15}$$
$$-4\ln^2 t - 8\beta\delta^2 t^3 z_2(t)).$$

As a result of differential equation solving (15) function $z_2(t)$ was found and with its help the function $u_2(t)$ was also determined:

$$z_2(t) = \frac{1}{12\beta\delta^2 t^3}(36 - 6\alpha\delta t^2 + \alpha^2\delta^2 t^4 - 12\ln t +$$
$$+12\ln^2 t),$$
$$u_2(t) = \frac{1}{12\beta\delta\gamma t^3}(24 - 6\alpha\delta t^2 + \alpha^2\delta^2 t^4 +$$
$$+12\ln t + 12\ln^2 t).$$

The obtained protection functions and the functions of harm for the second approximation have a form of:

$$z(t) = \frac{1}{\beta t} + \varepsilon\frac{1}{2\beta}(\alpha - \frac{2\ln t}{t^2\delta}) + \varepsilon^2\frac{1}{12\beta\delta^2 t^3}(36 - 6\alpha\delta t^2 +$$
$$+\alpha^2\delta^2 t^4 - 12\ln t + 12\ln^2 t),$$
$$u(t) = \frac{\delta}{\beta\gamma t} + \varepsilon\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta - 2\ln t - 2) +$$
$$+\varepsilon^2\frac{1}{12\beta\delta\gamma t^3}(24 - 6\alpha\delta t^2 + \alpha^2\delta^2 t^4 + 12\ln t + 12\ln^2 t).$$

Next we minimize the function $u(t)$ in parameters β and ε for zero, first and second approximation in order to calculate the price of protection system. And also to see, how adequately is to regard ε to be small.

To do this, we first have to define for which time interval the hazard function reaches an acceptable result (for us it is not critical the stabilization time for normal, non-critical, system's indicators). In the case when running time of protection system is a critical parameter it is obligatory to define, under which values of β parameter, the harm function takes acceptable values. At the same time we are limited by the time response of protection system.

Let us take for specification of system parameters in (3) so close to the real values:

$$\alpha = 0.2,\ \gamma = 0.5,\ \delta = 2,\ z_0 = 12.$$

Let the cost of stationary protection be $c_0 = 1200$.

As the value of stationary protection is $z_0 = 12$, then we have to find time t, after which protection function will take the value less than $z_0 = 12$ and relatively, the appropriate parameters' values β and ε.

For $z(t) = \dfrac{1}{\beta t}$ the parameters equal:

t=2.01701, β = 0.0413154.

For $z(t) = \dfrac{1}{\beta t} + \varepsilon\dfrac{1}{2\beta}(\alpha - \dfrac{2\ln t}{t^2\delta})$:

we have t=4.55546, β =0.0182969, ε =0.000734976.

For

$$z(t) = \frac{1}{\beta t} + \varepsilon\frac{1}{2\beta}(\alpha - \frac{2\ln t}{t^2\delta}) + \varepsilon^2\frac{1}{12\beta\delta^2 t^3}(36 - 6\alpha\delta t^2 +$$
$$+\alpha^2\delta^2 t^4 - 12\ln t + 12\ln^2 t),$$

we have t=1.18948, β =0.0700591, ε =0.000101185.

To evaluate the protection cost we use the function:

$$\tilde{C}(T) = \int_0^T c(z - z_0)dt + C_0,$$

where: C_0 – cost of stationary protection; z_0 – value of stationary protection; $c(z)$ – costs function, which can take forms of a), b) and c) given below.

Let us integrate by taking T=6.5 (time during which protection system will take the value less than z_0) and record the obtained results:

a) $c(z) = z$; $\tilde{c} = \int_0^T c(z(t) - z_0)dt + C_0$ = 73+1200 =1273;

b) $c(z) = z^2$; $\tilde{c} = \int_0^T c(z(t) - z_0)dt + C_0$ =544+1200=1744;

c) $c(z) = z\ln z;\ \tilde{c} = \int_0^T c(z(t) - z_0)dt + C_0$ = 221.33+1200 = 1421.33.

CONCLUSIONS

1. So, the main results of the work are:

– for the system (3) the asymptotic method was first used and the direct formulas for solution were obtained;

– the value of parameter ε was obtained by numerical solution of original system and this means legitimacy of asymptotic approach: smallness of parameter ε is confirmed;

– the found expressions and values for intensity of protection allow us to determine its optimal value.

2. In addition to receiving the solutions in closed analytic form and their research, this approach allows to obtain real estimates for the cost of protection and even to reduce this cost in time when the intensity of harmful factor u does not exceed the threshold of dynamic protection $c(z(t)\text{-}z_0)=0$.

3. The asymptotic approach appears to be very useful. So, it will be next applied to the models protection systems that are more complicated than (2).

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