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Determination of the limit errors of parameters of periodic signal harmonics caused by the quantization

Abstract

A method for determining amplitude and phase errors of signal harmonic components caused by A/D converter quantization is presented. The method is realized by a computer algorithm which uses output probes of the converter as the input data. Performed simulation studies indicate that the method is correct. It is possible to precisely determine the harmonic parameter measurement errors caused by the quantization.

Keywords: sampling A/D converter, ADC, periodic signal, quantization error, numerical methods, algorithm.

1. Introduction

The scientific literature related to quantization errors in A/D converters is very extensive. The paper [1] is one of the earliest papers in which the influence of sampling system quantization errors on the measurement accuracy is analysed. The spectrum of quantized signals is studied there. One of the conclusions claims that in precise measurement systems in which probes are gathered measurement errors more than in non-synchronized systems. There are some publications that extend the theory included in [1] e.g. work [3].

The measurement uncertainty related to Discrete Fourier Transform is analysed in [2]. The expressions that describe the measurement errors of the fundamental harmonic amplitude and phase are presented there. It is assumed that the quantization errors of subsequent probes are characterized by the values that are uncorrelated. The formulas show that the more samples are gathered, the less error is made.

The author in his earlier work described how to determine the amplitude and phase errors of the fundamental component of a sinusoidal signal caused by the quantization using numerical methods. In this paper, the author would like to present a method for determining the same kind of errors but for arbitrary periodic signals.

2. Description of quantization error

The parameters of the harmonic number k of the input signal can be represented by a complex number using Fourier transform (1). The parameters of the harmonic component number k of the output signal can be determined by using DFT (2). These expressions are presented below:

$$\underline{U}_k = \frac{2}{T}\int_0^T U(t)e^{-\mathrm{j}k\omega t}dt, \tag{1}$$

$$\underline{U}_{k,D} = \frac{2}{N}\sum_{i=0}^{N-1} pr_i e^{-\mathrm{j}k\frac{\omega}{f_p}i}, \tag{2}$$

where: $U(t)$ – continuous in time input signal, pr_i – quantized probes of the input signal, $\underline{U}_k$ – value of the harmonic number k of the input signal, $\underline{U}_{k,D}$ – value of the harmonic number k of the output signal, f_p – sampling frequency, N – number of gathered probes, ω, T – angular velocity and the period of the input signal, t – time.

The relation between the quantities described above can be presented in the following form

$$\underline{U}_k + \Delta_{\underline{U}_k} = \underline{U}_{k,D}, \tag{3}$$

where $\Delta_{\underline{U}_k}$ – measurement error of the harmonic number k caused by the quantization.

Theoretical considerations and numerical studies presented in this paper regard the situation when the quantization is the only source of the measurement error in an A/D converter.

3. Measurement method – example

A sinusoidal signal was applied to the input of a 3-bit-resolution A/D converter with 1.05 V reference voltage. 20 samples were taken synchronously in one period of the signal. The value of the fundamental harmonic parameters measurement error, $\Delta_{\underline{U}_1}$, was supposed to be $\pm 0{,}5$ LSB for the real and imaginary part. At this moment, it was not possible to determine the accurate value of the error. Combination of $\Delta_{\underline{U}_1}$ and $\underline{U}_{1,D}$ gives the set of values which correspond to the input signal values ($\underline{U}_1$). This set is presented in Fig. 1.

The next step of the method was transforming the complex values of the input signals (Fig. 1) into a time dependent series. The result is shown in Fig. 2. The set of the gathered samples is placed on the foreground of the set of the time dependent signals.

The next transformation (see Figs. 2 and 3) was connected with elimination of the signals which did not correspond to the acquired samples. If the input signal did not cross all quantization ranges that corresponded to all probes, that signal was neglected (see Fig. 3).

The time series presented in Fig. 3 were transformed back to the complex plane. As a result, we got the set marked with black colour in Fig. 4. This set is presented on the foreground of the starting set of the input signals shown in Fig. 1. The variable $\underline{U}_{1,M}$ represents the mean complex value of the determined set.

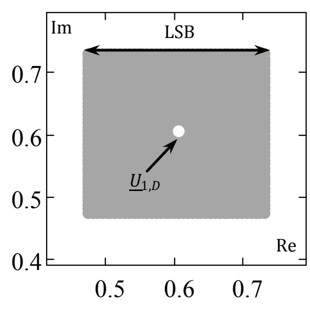

Fig. 1. The starting set representing the input signals $\underline{U}_1$

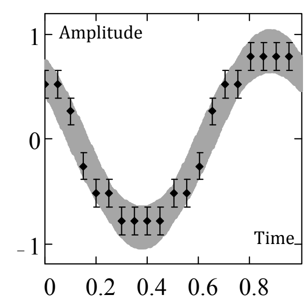

Fig. 2. Time dependent input signals on the background of the gathered samples

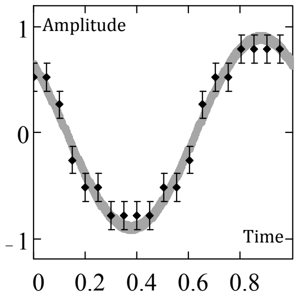

Fig. 3. Input signals that correspond to the gathered probes

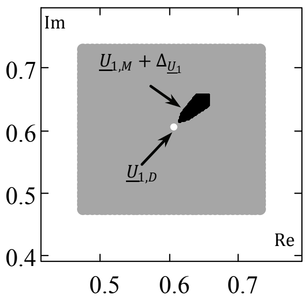

Fig. 4. A new set of input signals on the foreground of the starting set from Fig. 1

The enlarged final set that corresponds to the measurement errors is presented in Fig. 5. The limit errors $\Delta_{Re,1}$ and $\Delta_{Im,1}$ of the real and imaginary part of the value that represent the fundamental harmonic of the input signal are marked. It is easy to compute the approximate amplitude and phase limit errors according to the real and imaginary part errors. One may use the following expressions:

$$\Delta_{Ampl,1}=\sqrt{{\Delta_{Re,1}}^2+{\Delta_{Im,1}}^2},$$

$$\Delta_{\varphi,1}=\text{arctg}\frac{\Delta_{Ampl,1}}{U_{1,M}},$$

where: $\Delta_{Ampl,1}$ – amplitude measurement limit error, $\Delta_{\varphi,1}$ – phase measurement limit error, $U_{1,M}$ – modulus of the final set mean value.

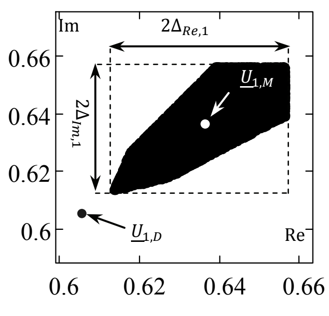

Fig. 5. Enlarged final set of the input signals presented in Fig. 4 which is a combination of the measurement error $\Delta_{\underline{U}_1}$ and the mean value $\underline{U}_{1,M}$

The presented method is not difficult to understand. However, using this method by hand is very complicated or almost impossible especially when there are many samples with smaller quantization ranges. To overcome this problem, a computer algorithm was designed. It is described in the next section.

4. Computer algorithm

4.1. The model of input signals

The relation between input and output signals in an ideal A/D converter that has some finite measurement resolution may be written in the form of the set of equations

$$pr_i-\frac{1}{2}\Delta_q<\sum_{k=1}^{M}A_k\sin(k\omega t+\varphi_k)\leq pr_i+\frac{1}{2}\Delta_q, \qquad (4)$$

where: i – variable which takes values from 0 to N-1, Δ_q – quantization error of an A/D converter, M – number of harmonics in the input signal, A_k, φ_k – value of the amplitude and phase of the harmonic number k.

The variable N is the number of gathered probes in one period of the signal. The quantization error of the ADC is described by the following known formula

$$\Delta_q=\frac{U_r}{2^{k_{bit}-1}},$$

where: U_r – reference voltage of the converter, k_{bit} – bit resolution of the converter.

A periodic signal that consists of M harmonics and fulfils the set of equations (4) can be represented as a point P_X in the following form

$$P_X=(A_1,\varphi_1,\ldots,A_M,\varphi_M).$$

The sets $\left(pr_i-\frac{1}{2}\Delta_q, pr_i+\frac{1}{2}\Delta_q\right\rangle$ are not empty and they are open at the left side. Therefore, if a signal P_X fulfils equations (4), then it is possible to choose such a point P_Δ with appropriately small coordinates that the combination P_X+P_Δ also fulfils equations (4). It means that there exists an infinite set of signals that fulfils (4). The author assumes that this set is bounded and continuous.

4.2. Working principle

Function $\text{f}:\mathbb{R}^{2M}\to\mathbb{R}$ is used to find periodic signals which correspond to the set of probes $\boldsymbol{K_D}$ gathered at the output of the A/D converter. It is represented by the following expression

$$\text{f}(P_X)=\left|f_{\text{A/C}}(P_X)\cap\boldsymbol{K_D}\right|. \qquad (5)$$

The function $f_{\text{A/C}}$ processes the signal in the same manner as the ideal A/D converter with the finite resolution. It transforms the input continuous signal represented as P_X into the set of samples $\boldsymbol{K}$.

Let us assume that probes $\boldsymbol{K_D}$ are acquired in a real experiment and represent the M harmonic signal. The function f returns the maximal value equal to N when the set $\boldsymbol{K}$ is equal to $\boldsymbol{K_D}$. The function f returns the values in the range from 0 to N.

In order to find all the signals which give the maximum in the function f, mapping the computer algorithm must be used. The working principle of the algorithm is illustrated in Fig. 6.

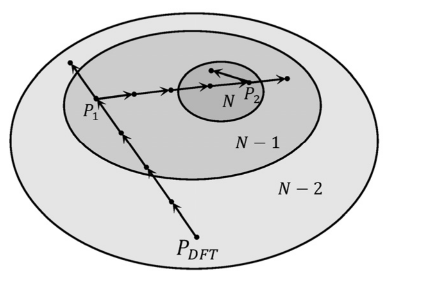

Fig. 6. Illustration explaining the working principle of the algorithm

An exemple set of input signals is presented in Fig. 6. It is divided into three subsets which give the values equal to $N-2$, $N-1$, N in the function f mapping. The point from which the search of the maximum is stared is computed by DFT and is designated as P_{DFT}. Further, the direction in which the point moves is randomly chosen. The directions of the steps are represented by the arrows in Fig. 6. The point is moving by making steps with the constant length. When it enters the area with the lower value than the value of the area in which it was one step before, it moves back one position and it chooses randomly the direction of the movement again. This situation corresponds to points P_1 and P_2 on the exemplar Fig. 6. In the area which gives maximum in the f mapping, a few thousand steps are performed. Each step point is written into the memory and this information is used to calculate the mean value of the set and the limit errors of the parameters of all periodic signal harmonics.

5. Chosen results of simulation studies

The coordinates of the vector which is used to find the maximum of the function f have different values. They depend on the dimensions of the solution set. These dimensions were assessed.

It was assumed that the amplitude value of each component of the sampled signal was in the range $\pm\Delta_q$ relating to the result computed by the DFT. Therefore, the maximum value by which the amplitude coordinates of the algorithm searching point were changed was equal to $0.001 \cdot \Delta_q$. This value corresponded to all coordinates of the amplitudes.

The assessment of the maximum changes of phase coordinates was a little bit more difficult. We had to find the interval t_Δ by which we could move the input signal but at the same time the probe had to maintain its original quantized values. One can do this by finding the maximum slope of the signal by using the output probes set. If the difference between adjacent probes of the maximum slope is designated as d_{max}, the interval of interest may be computed from the following formula

$$t_\Delta = (T/N)\Delta_q/d_{max}.$$

Different phase shifts for different harmonic components correspond to the same time interval t_Δ. It was assumed that the change of the phase coordinate of the algorithm searching point for the harmonic number i was equal to $0.001 \cdot 2^i \pi t_\Delta/T$.

Simulation studies were performed for a D/A converter with 8-bit resolution and with 1.05 V reference voltage value. The simulated signal consisted of three harmonics with the following parameters: 500 mV, 36°, 100 mV, 180° and 400 mV, 240°. In Figs. 7-8, the bars represent the measurement limit errors of the first and third harmonic amplitude value.

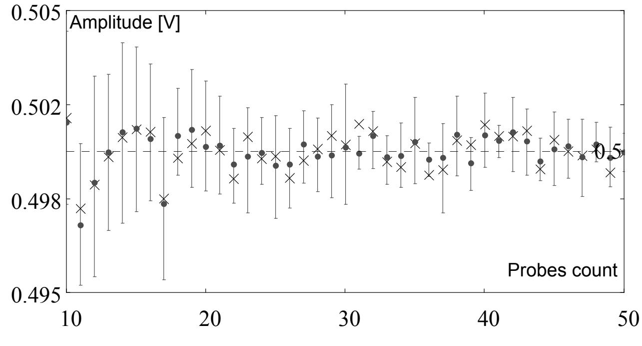

Fig. 7. The first harmonic amplitude value and its measurement errors

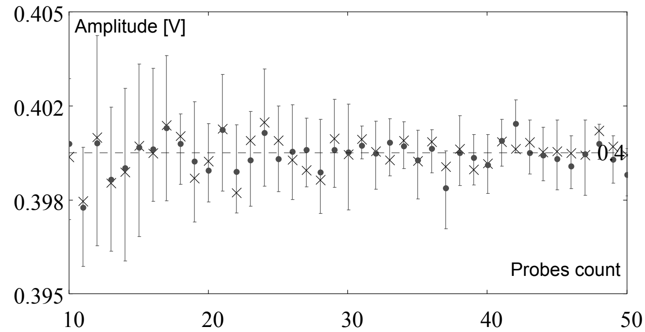

Fig. 8. The third harmonic amplitude value and its measurement errors

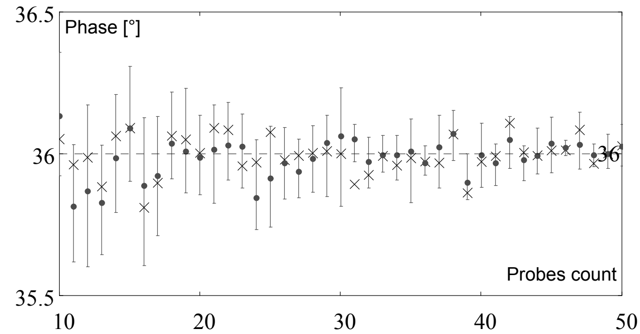

Fig. 9. The first harmonic phase value and its measurement errors

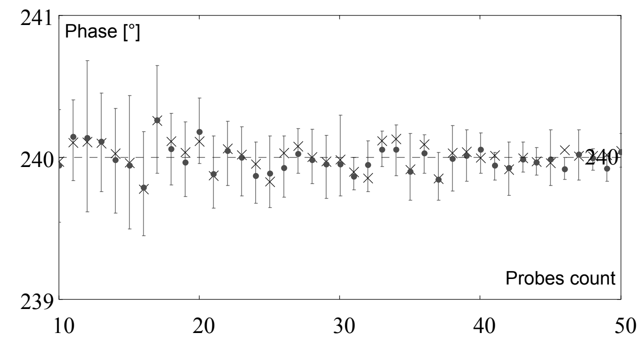

Fig. 10. The third harmonic phase value and its measurement errors

The dots represent the mean value of the amplitude. for each component. The crosses represent the results returned by DFT. The horizontal axis represents the probes count gathered synchronously in one period of the input signal. Fig. 9-10 are analogous to Fig. 7-8 but correspond to the phase measurements.

6. Conclusions

Figs. 7-10 show the relation between the values determined by the evaluated method and DFT. Each determined final set of signals include the signal applied to the input of the A/D converter. The values returned by DFT not always correspond to the determined appropriate input signal set.

When the method is used in a real experiment, the harmonic amplitudes returned by DFT with values higher than Δ_q need to be taken into account. In other case, the algorithm may not find the maximum of the function f which is equal to N.

The measurement path of digital instrumentation attenuates or amplifies as well as shifts in phase the measured signal in a natural way. When two voltage signals are measured correctly by the same instrument, their complex voltage ratio may be independent from amplitude and phase errors connected with this instrument.

Therefore, quantization may be the most important source of the measurement error.

The method evaluated in this paper can be used to accurately determine the measurement errors of periodic signal harmonic parameters caused by A/D converter quantization.

The author would like to study the following model of an input signal

$$U(t) = \sum_{k=1}^{M} A_k sin(k\omega t + \varphi_k) + p(t),$$

where $p(t)$ is the polynomial representing the effect of lower harmonics on the signal. When using the above model it is necessary to determine the polynomial rank and assess the values of its coefficients. Then the algorithm should be redesigned to include the search of the set of coefficient values.

7. References

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Received: 21.08.2015 *Paper reviewed* *Accepted: 02.10.2015*

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