

# An approach to in-situ compressive strength of concrete

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**Abstract.** The paper presents the problem of estimating in-situ compressive strength of concrete in a comprehensive way, taking into account the possibility of direct tests of cored specimens and indirect methods of non-destructive tests: rebound hammer tests and ultrasonic pulse velocity measurements. The paper approaches the discussed problem in an original, scientifically documented and exhaustive way, in particular in terms of application.

**Key words:** testing concrete in structures, cored specimens, non-destructive testing, basic curve, expression of uncertainty measurements, conformity criteria, characteristic in-situ compression strength.

## 1. Introduction

The problem of the evaluation of the compressive strength of concrete in-situ (in a structure) is one of the main tasks of the diagnostics of constructed features, which is associated with the quality control for the purposes of construction of concrete structures as well as ensuring the safety of the existing concrete structures.

The most important issues in the procedure of the evaluation of the compressive strength of concrete in a structure include:

- the substantive and formal basis for the evaluation, including the conformity criteria,
- the preparation of the data inputs for the evaluation, based on the results of direct testing of the strength of cored specimens taken out of a structure (a reference method) or an indirect in-situ tests using non-destructive testing methods [5],
- scaling – the determination of the relationship between the compressive strength of concrete and the result of the indirect quantity measurement,
- the evaluation of the total uncertainty of data inputs for the evaluation,
- the estimation of the characteristic strength and class of concrete in a structure.

## 2. Substantive and formal basis for the evaluation of in-situ compressive strength

The evaluation of in-situ compressive strength is based on the conformity criteria, similarly to the control of concrete production.

The conformity criteria were adopted in 1977 as the basis for the classification of concrete compressive strengths [22]. Treating the strength of concrete as a random variable with

a normal distribution, the quantile of this distribution, equal to 0.05, was adopted as the characteristic strength  $f_{ck}$  that determined the class of the concrete. At the same time, a requirement was established for the conformity criterion for the strength of a given concrete on the basis of testing of a series set of the individual  $n$  results of control specimens to be estimated for a confidence level  $\gamma$  of at least 0.50.

The approach to the problem requires both particularity of an adequate material model [3] of concrete composite and multicriterial optimization [8] methods of its assessment in situ.

The conformity criteria are expressed in the form of mathematical inequalities of the following type:

$$f_{ck} \geq f_{m(n)is} + k_n s_n \quad (1)$$

where  $f_{m(n)is}$  and  $s_n$  are, respectively, the mean value and the standard deviation of the compressive strength of concrete in the tested population  $n$  of specimens.

According to the rules of mathematical statistics [23], the coefficients  $k_n$  in the formula (1) can be determined using the methods of:

- 1) statistical inference,
- 2) OCC (operational characteristic curve) functions (curves),
- 3) Bayes inference (Bayesian methods).

It should be noted, however, that the coefficients  $k_n$  determined in different ways do not lead to comparable criteria.

When using the method of statistical inference, the coefficients  $k_n$  in the conformity criteria are determined explicitly depending on the assumed required confidence level  $\gamma$  and the number  $n$  of the specimens, taking into account the nature of the standard deviation (known  $\sigma$  or estimated  $s_n$ ). Prior to the introduction of the standard [16], the values  $k_n$  determined in this way were used, among others, in the Polish standard pertaining to concrete [23].

The analysis of the applied conformity criteria suggests that the numerical values  $k_n$  that were adopted in them were

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determined by the OCC method by L. Taerwe [13, 14]. The adoption of the values  $k_n$  in the conformity criteria according to this method results in the circumvention of the requirement pertaining to the confidence level  $\gamma$ .

In turn, the conformity criteria adopted in Eurocode [21] were explicitly determined using the Bayesian inference, emphasizing that their confidence level is close to 0.75.

In the cases of a known estimator of this quantity  $s_n$ , the values of the coefficients  $k_n$  obtained using the three above-mentioned methods, designated in publications [1, 6, 13, 14], are given in Table 1.

Table 1. The summary of the selected coefficients  $k_n$  in the conformity criteria set by different methods [1]

determination method		number $n$ of specimens				
		3	6	9	15	$\infty$
1) statistical inference for a confidence level	0.25	1.25	1.33	1.37	1.42	1.54
	0.50	1.94	1.75	1.71	1.68	1.64
	0.75	3.15	2.34	2.14	1.99	1.76
	0.90	5.31	3.09	2.65	2.33	1.86
2) OCC functions		2.67	1.87	1.67	1.48	–
3) Bayesian inference		3.37	2.18	1.99	1.82	1.64

As the data in Table 1 suggests, the values of the coefficients  $k_n$  are very different, especially for small numbers of specimens  $n \leq 15$ . The value of the coefficient  $k_n = 1.48$  used for this number of specimens is, thus, a conscious departure from the principle of estimating the conformity criterion with a confidence level of at least 0.5. According to M. Holický and M. Vorlíček [6], the confidence level is below 0.3 for such a coefficient. Paradoxically for  $n = 6$  and  $n = 3$ , the values  $k_n$  determined with the use of the OCC method provide the confidence level of 0.5.

Unfortunately, the values  $k_n$  are adopted in the same manner in the conformity criteria for the evaluation of the strength of concrete in a structure.

### 3. The preparation of data inputs for the in-situ concrete strength evaluation

The preparation of data inputs for the evaluation of the strength of concrete in a structure includes calculating statistical measures of the obtained in-situ test result populations. These can be test results for the compressive strength  $f_{i, is}$  of cored specimens taken from a structure (a direct method) or results of the estimation of the strength of concrete  $f_{i, is, X}$  using an indirect in-situ method.

The statistical measures of the obtained test result populations: the mean in-situ compressive strength  $f_{m(n)is}$ , the standard deviation of concrete  $s_n$ , and the lowest in-situ compressive strength test result  $f_{is, lowest}$ .

The determination of the measures of the compressive strength test result populations for concrete in a structure obtained using the indirect method requires prior performance of the so-called scaling. Scaling consists in the determination of a relationship between the compressive strength of concrete and the result of the measurement of the indirect quantity  $X_i$ . The quantities  $X_i$  can include the number of rebounds  $R_i$  of a hammer striking the concrete surface for the sclerometric method or the speeds of the propagation of longitudinal waves  $V_i$  for the ultrasonic method.

There are several questionable issues in the data input preparation procedures. In the direct method, they are related to the consideration of the impact of certain factors on the result of the determination of the strength of concrete in cored specimens. In the indirect method, they are related to the scaling method. In both the methods, it is important to estimate the uncertainty of the determined statistical measures of the obtained test result populations. These issues are discussed below.

### 4. The impact of some factors on the result of the determination of the strength of concrete in cored specimens

As far as the reliability of the obtained in-situ compressive strength tests results is concerned, the application of an appropriate testing procedure and taking into account the impact of the factors related to the properties of the concrete and the testing methodology and conditions is required.

These factors may include: concrete moisture content and void age, drilling direction relative to the casting, imperfections of concrete, reinforcing rods in the specimen, core diameters, the ratio of the core diameter to its height, the flatness of the loaded core end surfaces and method of their adjustment, and cracks and hollows in concrete, the impact of the ratio of the aggregate size to the core diameter on the strength of the core specimen and the impact of the drilling on the structure of the cored specimen.

Between 1992 and 1999, the Building Research Institute (ITB) in Warsaw completed extensive research on the impact of some of these factors on the determined compressive strength of concrete in cored specimens (more than 2000 specimens were tested). These tests were related to the influence of the specimen geometry, including the positioning of their axes relative to the casting, the method of preparation of the loaded specimen surfaces and the moisture content of concrete depending on the specimen storage conditions prior to the compressive test.

Particularly important were the results of these tests of cored specimens that confirmed the possibility of performing tests on specimens with diameters equal to or close to 50 mm, proposed by F. Indelicato [7, 8] and so far scarcely used, as well as the original method of testing sandwich specimens (specimens made of two parts).

These tests are widely discussed in the monograph [3]. The most important findings and conclusions of these tests are presented below:

- the mean compressive strengths of moist cored specimens (saturated with water by immersion in water for five days prior to the compressive testing) are lower by approx. 25% in relation to the strength of specimens stored in the dry air conditions, regardless of the diameter of the specimen and the ratio of the height of the specimen to its diameter;
- unless this results from additional requirements, cored specimens should be tested in the air-dry state;
- the mean core compressive strengths obtained for cores with the axis perpendicular to the direction of the casting (concrete placement) are less than the core strengths obtained from cores with the parallel axis, irrespective of the diameter of the cored specimen and the end surface smoothing method;
- given the small value of the difference (from 1 to 9%), it is reasonable to take into account the impact of this factor on the determined concrete strength only in some particular cases;
- in the case of imperfections occurring in heterogeneous (sandwich) cored specimens, the strength of concrete for such specimens with the height equal to the diameter does not differ in practice from the strength of the homogeneous specimens from cores from the same concrete (the differences do not exceed 2%);
- it is reasonable to use sandwich cored specimens for the diagnosis of the strength of concrete in thin components and floor layers;
- the mean strengths of concrete determined on cored specimens with a diameter of 100 mm or 80 mm are virtually no different from those for standardized cube specimens with a 150 mm side length and tested at the same age. Hence the conversion factor of 1.0 is preferred regardless of the class of the designed concrete;
- for the cored specimens with the diameter of 50 mm taken from C20/25 or higher-class concretes, the conversion factor of 1.1 is justified; it is not recommended to take such specimens from weaker concretes;
- the tests did not show a significant underestimation of the strength for cores taken from a C20/25 and higher-class concrete whose diameter did not satisfy the requirement of being at least triple the minimum aggregate size in the concrete;
- the concrete strengths for cored specimens with the height equal to  $2d$  are lower than the strengths for the specimens with the height of  $d$ ; in the tested cored specimens with diameters from 50 to 150 mm, the differences reached 32%, which means they were higher than those for the specimens made in moulds;
- it is reasonable to prefer cored specimens with heights equal to their diameters;
- compared to the strengths of the cored specimens with end surfaces smoothed by grinding, the strengths for cored specimens with mortar end caps are lower – differences of up to 13% – while in the case of sandbox caps the differences are as high as 22%;
- recommending the smoothing method by grinding the cored specimen end surfaces as the reference method is fully justified;

- it is reasonable to limit the performance of *in-situ cores* at measurement locations situated at a distance of less than 10 cm in the clear from the edge of the concrete component.

Table 2 includes a summary of correction factors that result from research conducted by the Building Research Institute and take into account the impact of the selected factors in relation to the compressive strength of the reference cored specimen. For the reference specimen, a specimen taken from cores performed perpendicularly to the direction of the casting was chosen, with the diameter of  $d = 100$  mm, the height of  $h = d$ , with ground homogeneous (not sandwich-type) end surfaces, and tested in the air-dry state.

Table 2. The summary of the correction factors of the cored specimen strengths

No	correction from cored specimens	value $k$
1	from cores perpendicular to the casting for specimen of the parallel cores	1.02–1.09
2	with a diameter of 50 for 100 mm specimens:	1.05–1.10
3	with diameter of 80 for 100 mm specimens	1.02–1.05
4	$h = 2d = 200$ mm for specimens $h - d = 100$ mm	1.25
5	$h = 2d = 300$ mm for specimens $h = d = 100$ mm	1.16
6	smoothed with mortar for specimens with ground surfaces	1.10–1.13
7	with sandbox caps for specimens with ground surfaces	1.22
8	sandwich type (made of two parts) for homogeneous specimens	1.02–1.05
9	tested in the water-saturated state for air-dry state specimens	1.25

## 5. Scaling in the indirect methods of the in-situ compressive strength testing

The classical accurate scaling that satisfies the principles of the regression and correlation analysis is rarely used in diagnostic practice due to stringent requirements set by the standard. This applies in particular to the requirement that the required data pairs (at least 18 of them) “are obtained in the testing programme” in the whole “measuring range of interest” and “evenly spaced within the limits that are covered by the data”.

In the experience of the Building Research Institute, such an interval should be in the range  $\pm 30\%$  of the mean strength value of concrete in the population consisting of at least 30 data pairs.

Approximate scaling (calibration) involves correction (a simple parallel shift or a shift with rotation) of the basic (base) regression graphs, recommended by standards or ob-

tained as a result of independent accurate scaling or held by the person performing the tests.

The basic (base) regression functions are presented in the form of mathematical equations or graphs – scaling curves (straight lines). These graphs are defined as lower envelopes of the various relationships obtained by researchers between the quantities  $X$ , measured via direct methods, and the compressive strength of concrete –  $f_i$  (determined on cored specimens with the diameter of 100 mm).

Examples of graphs and equations of the base regression functions are shown in Fig. 1 for sclerometric tests and in Fig. 2 for ultrasonic tests.

The basic curves marked with the letter A are recommended in the standard [20], and those marked with the letter B have been recommended in Poland by the Building Research Institute for many years [3]. Whereas the graphs A and B are similar for sclerometric tests, then for ultrasonic testing, both the curve A and its scope are not consistent with the physical interpretation. The works related to the future amendment of the standard [20] are considering new basic curves shown in the graphs in Fig. 1 and 2 in the form of graphs marked with the letter C.

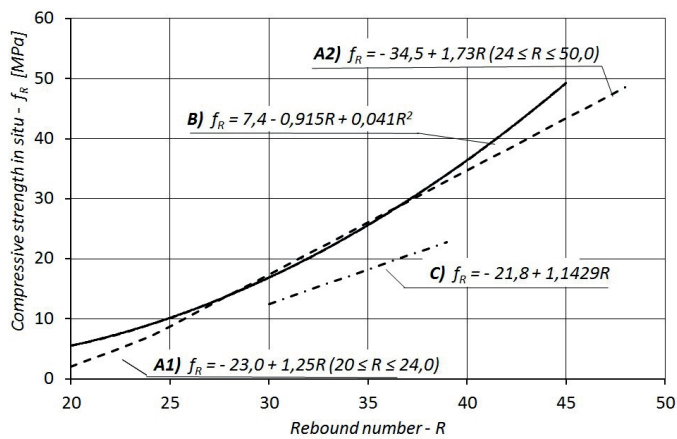


Fig. 1. Examples of the basic scaling graphs for the sclerometric method

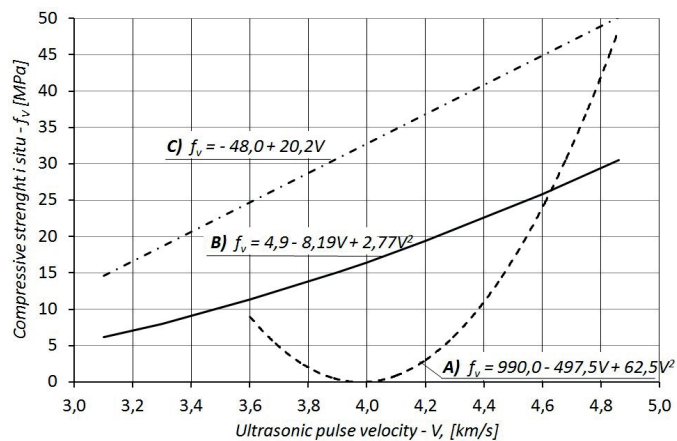


Fig. 2. Examples of the basic scaling graphs for the ultrasonic method

The approximate scaling procedure is discussed below with respect to sclerometric tests.

Scaling means the determination of the relationship between the property being measured – the rebound number  $R$  – and the compressive strength  $f_{is,R}$ . The approximate scaling procedure includes:

- the assumption of a regression curve that is hypothetical to a given concrete,
- making an in-situ measurement of the rebound number  $R$  in  $n$  measurement points and the determination of the strength of concrete  $f_{is,R}$  on the basis of the adopted basic (base) regression curve,
- cutting out cored specimens at the same locations and the direct determination of the compressive strength  $f_i$ ,
- the calculation of the differences  $\delta f_{is} = f_{is,R} - f_i$ , and their mean value  $\delta f_{m(n)}$  and the standard deviation  $s_{n,\delta f_{is}}$  (hereinafter called  $s_n$ ),
- the determination of the basic curve shift parameter for the position corresponding to the lower limit of its confidence range from the formula

$$\Delta f = \delta f_{m(n)} - k s_n, \quad (2)$$

where the coefficient  $k$  depends on the number of measurement locations  $n$

- adoption of the equation of corrected basic scaling curve in the form of

$$f_{is,R} = f_R - \Delta f. \quad (3)$$

A debatable issue in the applied procedure is the way of determining the coefficient  $k$  in the formula (2).

The coefficient  $k$  is defined in the way used in the proposal by L. Taerwe [13, 14] for the conformity criteria set by the standard [16]. For the number of specimens  $n = 9$  required for scaling, in accordance with Table 1, the value  $k = 1.67$  is adopted.

As shown by L. Brunarski [3] and I. Skrzypczak [12], such a way of determining the shift parameter  $\Delta f$  leads to an excessive reduction and is not justified in the regression analysis.

For example, in the case of the calculated shift:  $\delta f_{m(n)} = 8$  MPa and  $s_n = 5.0$  MPa, the value of the second segment can be  $k s_n = 1.67 \cdot 5.0 = 8.57$  MPa and thereby nullify the effect of correction.

According to the authors, in compliance with the principles of the regression analysis, the correct value  $\Delta f$  should be estimated by the formula

$$\Delta f = \delta f_{m(n)} - t_{v,p} \frac{s_{rest}}{\sqrt{n}}, \quad (4)$$

where:

$s_{rest}$  – the residual standard deviation, otherwise known as the standard error of the estimate,

$t_{v,p}$  – Student’s  $t$ -distribution statistics, adopted for the number of the degrees of freedom  $v = n - 2$  and the assumed significance level  $\alpha$ .

The residual standard deviation is

$$s_{rest} = \sqrt{\frac{1}{n-2} \sum_{i=1}^n (\delta f_{is} - \delta f_{m(n)})^2} = s_n \sqrt{\frac{n-1}{n-2}}, \quad (5)$$

hence the formula (4) can be described in a form analogous to the formula (2)

$$\Delta f = \delta f_{m(n)} - t_{v,p} \frac{s_n}{\sqrt{n}} \sqrt{\frac{n-1}{n-2}} = \delta f_{m(n)} - k s_n \quad (6)$$

where:  $k$  – coefficient whose values are calculated by the formula

$$k = t_{v,p} \sqrt{\frac{n-1}{n(n-2)}}. \quad (7)$$

The relationship must be determined with the assumption of the possibility of a 10% underestimation of the concrete strength being determined, i.e. at the significance level of 10%.

The values of the coefficients  $k$  calculated from the formula (7), depending on the number of specimens  $n$  and the corresponding Student's  $t$ -distribution statistics  $t_{v,p}$  for the numbers of degrees of freedom  $\nu = n - 2$  and the desired confidence level  $p = 0.80$  are given in Table 3.

Table 3  
The values of the coefficients calculated in accordance with the formula (7)

$n$	3	4	5	6	7	8	9	12
$t_{v,p}$	1.89	1.64	1.53	1.48	1.44	1.42	1.40	1.36
$k$	1.54	1.00	0.79	0.68	0.60	0.54	0.50	0.41

In the above-described case:  $\delta f_{m(n)} = 8$  MPa and  $s_n = 5.0$  MPa, the value of the second segment reducing the shift is much smaller – it is  $k s_n = 0.50 \cdot 5.0 = 2.5$  MPa.

According to the authors, it is safer to perform scaling for a number lower than the applied number  $n = 9$  rather than use the basic curves without scaling. Hence, Table 3 provides appropriate values of the coefficient  $k$  for the number of specimens being less than 9.

The next step should be the justification of the proposed formula (4).

In the case of a linear regression function defined by the formula

$$f_R = a_1 + a_2 R_i, \quad (9)$$

the graph of the function is in the straight form.

According to the classical regression analysis, the fitting area where the determined straight line of the linear regression is located with a determined probability is limited by the curves described by the hyperbole equation

$$f_R^{est} = a_1 + a_2 R_i \pm t_{v,p} s_{rest} \sqrt{\frac{1}{n} + \frac{(R_i - R_m)^2}{\sum (R_i - R_m)^2}}. \quad (10)$$

In this equation,  $t_{v,p}$  is the Student's  $t$ -distribution statistics assumed for the number of the degrees of freedom  $\nu = n - 2$  and the assumed confidence level  $p$ . The values of the statistics  $t_{v,p}$  for a certain confidence level  $p_n = 0.80$  are included in Table 3.

The width of the confidence interval increases as the distance from the centre point of the regression straight line grows. If we assume a roughly constant width, i.e. accept a fixed value  $R_i - R_m$ , it is possible to describe the limit curves of the confidence interval by means of straight line equation

$$f_R^{est} = a_1 + a_2 R_i \pm t_{v,p} \frac{s_{rest}}{\sqrt{n}}. \quad (11)$$

In accordance with the principles of the regression analysis, a curve that limits its confidence interval from the bottom should be adopted as the corrected best-fit regression curve.

Hence, in the adopted equation of the corrected basic scaling curve

$$f_{is,R} = f_R - \Delta f, \quad (12)$$

the shift parameter should be defined using the formula recommended by the authors

$$\Delta f = \delta f_{m(n)} - t_{v,p} \frac{s_{rest}}{\sqrt{n}}, \quad (13)$$

in which  $s_{rest}$  was specified above with the formula (5).

## 6. Estimating the uncertainty of the estimated mean strength value

The determined mean compressive strength value of concrete in a structure, regardless of the testing method, is only an estimator of the quantity being determined. Hence, there appears the question of estimating its uncertainty at a given confidence level.

The uncertainty of the calculated value  $f_{m(n)is}$  – the mean strength of concrete in a structure – should be estimated as a general rule. This should be done regardless of the method in which it is obtained.

In the applied procedure, the uncertainty must be taken into account for the evaluation of strength using the indirect methods. Unfortunately, the method in which the uncertainty is taken into account was not explicitly specified.

This requirement will be met if the adopted scaling curve is the lower limit curve of the confidence interval of the relationship determined using the accurate scaling method.

For the curve determined using the approximate scaling (calibration) method, the requirement is met through the adoption of the proposed definition of the basic curve shift parameter  $\Delta f$  recommended by the authors.

If these requirements are not met, the calculated mean strength value  $f_{m(n),is,R}$  should be reduced by the value of the expanded uncertainty  $U_p$  estimated as described below.

If the direct testing method is applied, the values  $f_{m(n)is}$  (hereinafter indicated as  $f_m$  for short), calculated from the formula

$$f_m = \frac{4F}{\pi d^2} 10^{-3} + df, \quad (14)$$

are a function of estimates of three input quantities:

- the mean value of the destructive forces  $F$  of the specimen,
- the mean dimension of the diameter  $d$  of the specimens,
- the correction  $df$  related to the random variability of the concrete strength of the tested series of specimens, or in other words – with the mean value confidence interval,

The combined standard uncertainty  $u_c(f_m)$  is determined by the formula

$$u_c(f_m) = \sqrt{u_F^2(f_m) + u_d^2(f_m) + u_{df}^2(f_m)}. \quad (15)$$

The components of the combined standard uncertainty associated with the subsequent input quantities are

$$u_F(f_m) = \left| \frac{\partial f_m}{\partial F} \right| u(F), \quad (16)$$

where:  $u(F)$  – the standard uncertainty of the first quantity equal to  $0.58 \delta F$  ( $\delta F$  – absolute error of the indications of the force) is type B;

$$u_d(f_m) = \left| \frac{\partial f_m}{\partial d} \right| u(d), \quad (17)$$

where:  $u(d)$  – the standard uncertainty of the second quantity equal to  $0.58 \delta d$  ( $\delta d$  – the absolute error of the specimen diameter measurement) is type B;

$$u_{df}(f_m) = \left| \frac{\partial f_m}{\partial (df)} \right| u(df), \quad (18)$$

where:  $u(df)$  – , the standard uncertainty of the third quantity is type A and determined by the standard deviation of the mean value  $\bar{s}_n = S_n/\sqrt{n}$ .

For given components, the resulting (effective) number of the degrees of freedom  $\nu_{eff}$  of the combined standard uncertainty is

$$\nu_{eff} = \frac{u_c^4(f_m)}{u_{df}^4(f_m)} (n - 1). \quad (19)$$

The values of the coverage factor  $k_{\nu_{eff}, p}$ , corresponding to the effective number of the degrees  $\nu_{eff}$ , is read from the Student's t-distribution tables. The confidence level  $p = 0.75$ , according to Eurocode [21], is recommended in this case by the authors [3].

The estimation procedure is completed by the determination of the expanded (total) uncertainty from the formula

$$U_{pf} = k_{\nu_{eff}, p} u_c(f_m), \quad (20)$$

At an assumed confidence level  $p$ , the mean core compressive strength of concrete in a structure is located in the interval  $f_m \pm U_p$ .

For the indirect tests, e.g. the sclerometric method, the mean compressive strength values of concrete in a structure  $f_{m(n)is}$  (hereinafter identified as  $f_m$ ), calculated from the formula

$$f_m = a_1 + a_2 R + a_3 R^2 + df + d(fR), \quad (21)$$

are a function of estimates of three input quantities:

- the mean value of the rebound number  $R$  at the measurement location (dimensionless quantity),
- the correction  $df$  related to the random variability of concrete strength at the measurement location,
- the correction  $dfR$  related to the uncertainty of the adopted relationship.

The combined standard uncertainty of the output quantity  $u_c(f_m)$ , determined by the formula

$$u_c(f_m) = \sqrt{u_R^2(f_m) + u_{df}^2(f_m) + u_{dfR}^2(f_m)}, \quad (22)$$

The components of the quantity, which appear in the formula (22), are determined as follows:

$$u_R(f_m) = \left| \frac{\partial f_m}{\partial R} \right| u(R), \quad (23)$$

where:  $u(R)$  – the standard uncertainty of the first quantity equal to  $0.58 \delta R$  ( $\delta R$  – the absolute error of the reading of the rebound number  $R$ ) is type B;

$$u_{df}(f_m) = \left| \frac{\partial f_m}{\partial (df)} \right| u(df), \quad (24)$$

where:  $u(df)$  – the standard uncertainty of the second quantity is type A and determined by the standard deviation of the mean value  $\bar{s}_n = S_n/\sqrt{n}$ ;

$$u_{dfR}(f_m) = \left| \frac{\partial f_m}{\partial (dfR)} \right| u(dfR), \quad (25)$$

where:

$u(dfR)$  – the uncertainty determined by the limit error of the scaling curve, defined  $s_{rest} \leq 0.12 f_m$  is the standard uncertainty of type B with the value  $u(dfR) = 0.58 \cdot 0.12 f_m = 0.07 f_m$ .

For given components, the resulting (effective) number of the degrees of freedom  $\nu_{eff}$  of the combined standard uncertainty is determined by an equation identical to (19).

Proceeding further similarly to the direct testing, calculation is performed for the expanded (total) uncertainty  $U_p$ , which defines the interval  $f_m \pm U_p$  in which the mean concrete compressive strength value determined by measuring the number of rebounds  $R$  is located for the assumed confidence level  $p$ .

## 7. Estimation of the characteristic strength and class of concrete in a structure

The statistical measures of the result populations based on the in-situ testing: the mean value  $f_{m(n),is}$  and the standard deviation of strength  $s_n$  and the lowest in-situ compressive strength test result  $f_{is,lowest}$  must meet certain requirements.

For the direct testing of concrete compressive strength of cored specimens taken from the structure:

- the spread of the obtained results, including the standard deviation  $s_n$  and the value  $f_{is,lowest}$  should indicate that they represent one population of concrete,
- the optimal population comprises 15 results; smaller populations  $n \geq 3$  are allowed to estimate the characteristic strength.
- In the case of the indirect determination of the compressive strength of concrete on the basis of *in-situ* measurements of the rebound number  $R$ :
- the mentioned quantities represent the populations of the strength values estimated on the basis of sclerometric testing in at least 15 measurement locations,
- the standard deviation should be a value calculated in a given population, but the value of at least 3 MPa is assumed.

The method used for determining the characteristic compressive strength of concrete in a structure  $f_{ck,is}$  depends on the result number  $n$ , equal to or greater than 15, or less than 15.

If the compressive strength of concrete in a structure was determined on the basis of a least 15 core specimen test results or the results of the indirect concrete testing, the value  $f_{ck,is}$  that is smaller out of the two values determined from the formulas

$$f_{ck,is} = f_{m(n),is} - k_2 s_n, \quad (26)$$

$$f_{ck,is} = f_{is,lowest} + 4, \text{ MPa}, \quad (27)$$

where:

$k_2$  – a constant ratio, with a value equal to 1.48,

$s_n$  – the greater of the two values: the standard deviation determined on the basis of the test results or its assumed value equal to:

- in the case of the direct core strength testing  $s_n = 2.0$  MPa,
- in the case of the indirect strength testing using the sclerometric method  $s_n = 3.0$  MPa.

If the compressive strength of concrete in a structure was determined on the basis of the results of testing of 3 to 14 specimen cores,  $f_{ck,is}$  is adopted using the smaller of the two values determined from the formulas

$$f_{ck,is} = f_{m(n),is} - k s_n, \quad (28)$$

$$f_{ck,is} = f_{is,lowest} + 4, \quad (29)$$

where:  $k$  – the coefficient depending on the number  $n$  of the cored specimens, as specified in Table 4.

Table 4  
Coefficients for the formula (28) [20]

$n$	3 ÷ 6	7 ÷ 9	10 ÷ 14
$k$	7	6	5

The validity of the values adopted for the coefficients in the formula (28) raises objections.

A better solution is to continue to use the formula (26) in the case of the number of specimens in the smaller populations  $n < 15$ . It would be enough to adopt the values of the coefficients  $k_2$  determined using the OCC operational characteristic curve autocorrelation (similarly to the 1.48 parameter adopted for 15 specimens) given in Table 5.

Table 5  
Values of the coefficient  $k_2$  [14]

$n$	$k_2$	$n$	$k_2$	$n$	$k_2$
3	2.67	7	1.80	11	1.58
4	2.40	8	1.73	12	1.55
5	2.13	9	1.69	13	1.53
6	1.87	10	1.63	14	1.50

As it can be easily ascertained, in the case of the number of specimens  $n = 14$ , the adoption of the value of the coefficient  $k = 5$  in the formula (28) is tantamount to adopting the standard deviation  $s_n$  limited to the value of 3.33 MPa ( $1.50 \cdot 3.33 = 5$  MPa) in the formula (25). On the other hand, in the case of the number of specimens  $n = 3 - s_n$  limited to the value of 2.62 MPa ( $2.67 \cdot 2.62 = 7$  MPa).

Therefore, according to the authors, recommending a uniform method of estimating  $f_{ck,is}$  for  $n$  greater than 3, from the formulas (26) and (27) is accurate and more reliable than the method specified in the standard.

The characteristic compressive strength of concrete in a structure  $f_{ck,is}$ , directly determined on the basis of core testing, can be used to estimate the associated strength class of concrete set by the standard.

The classes of concrete strength are indicated by the symbol  $Cf_{ck,cyl}/f_{ck,cube}$ . The values of the characteristic strength are provided after the letter C:  $f_{ck,cyl}$  – determined based on the standardized cylindrical specimens with the diameter of 150 mm and the height of 300 mm and  $f_{ck,cube}$  – based on the cubic specimens with the side length of 150 mm.

The classification includes 16 classes of concrete compressive strengths. During conformity checks of the designed concrete in cubic specimens, the minimum characteristic strengths  $f_{ck,cube}$  for standard cubic specimens are as follows:

$$f_{ck,cube} = 10, 15, 20, 25, 30, 37, 45, 50, 55, 60, 67, 75, 85, 95, 105 \text{ and } 115 \text{ MPa (N/mm}^2\text{)}.$$

Based on many years of experience and expertise, it was found that the characteristic compressive strength of concrete in a structure  $f_{ck,is}$  may be lower by 15% compared to the provided values  $f_{ck,cube}$ .

Thus, the estimation of the short-term (actual) class of concrete in a structure uses the condition that the minimum value  $f_{ck,is}$  of concrete of a given class is

$$f_{ck,is} = 0.85 f_{ck}. \quad (30)$$

These values are given in Table 6 with the accuracy of up to  $\pm 1.0$  MPa.

Table 6

Min values  $f_{ck, is}$  corresponding to the concrete strength classes [20]

class of strength of concrete	min value $f_{ck, is}$	class of strength of concrete	min value $f_{ck, is}$
C8/10	9	C45/55	47
C12/15	13	C50/60	51
C16/20	17	C55/67	57
C20/25	21	C60/75	64
C25/30	26	C70/85	72
C30/37	31	C80/95	81
C35/45	38	C90/295	89
C40/50	43	C100/115	98

The described procedure allows for an estimation of the short-term (actual) characteristic strength of concrete in a structure. Also, it allows for such an estimation of the strength class for concrete in a structure.

In the case of doubt whether the compressive strength of concrete in a structure meets the conformity criteria based on the standardized tests during the production of concrete, the more stringent evaluation procedure is used.

Such an evaluation can be performed in disputes, provided that 15 or more concrete core test results are available.

The compressive strength of concrete in a structure is considered to be equivalent to the conformity conditions set by the standard if two conditions (two inequalities) resulting from the formulas (26) and (27) are met

$$f_{m(n), is} \geq 0.85f_{ck} + 1.48 s_n. \quad (31)$$

$$f_{is, lowest} \geq 0.85f_{ck} - 4, \quad (32)$$

where:

$f_{ck}$  – the characteristic compressive strength corresponding to the class of the designed concrete.

These conditions were presented above in the corrected form in relation to the obviously erroneous formulas provided by the standards in the form

$$f_{m(n), is} \geq 0.85(f_{ck} + 1.48 s_n), \quad (33)$$

$$f_{is, lowest} \geq 0.85(f_{ck} - 4). \quad (34)$$

The evaluation of the satisfaction of the standardized conformity criteria is not used if there are less than 15 core test results.

However, in such a case, it is alternatively allowed, by mutual agreement, to perform testing of concrete using an indirect non-destructive method (e.g. sclerometric testing) in conjunction with the direct method for which at least three cored specimens are required.

It is then necessary to meet the following three conditions (inequalities):

- with respect to the results of concrete strength testing using the sclerometric method

$$f_{m(n), is, R} \geq 0.85f_{ck} + 1.48 s_n, \quad (35)$$

$$f_{is, lowest, R} \geq 0.85f_{ck} - 4, \quad (36)$$

- with respect to the core compressive test results

$$f_{is, lowest} \geq 0.85f_{ck} - 4. \quad (37)$$

## 8. Summary

The paper focuses on the debatable nature, or even errors, of some of the procedures used to determine in-situ compressive strength of concrete (in a structure). The word “used” refers to the procedures resulting from the provisions of the standard [19] and standards associated therewith [16–19].

The paper presents proposals for corrections and original alternative procedures. These proposals are based on the results of many years of research and practical experience of the Building Research Institute (ITB) in Warsaw.

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