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NUMERICAL AND ANALYTICAL APPROACHES FOR ROLL MOTION ANALYSIS IN REGULAR LONGITUDINAL WAVES

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ABSTRACT

In this study numerical and analytical approaches were investigated in terms of accuracy of their results, practicality of solution and ability to reproduce the main features of the parametric roll phenomenon such as loss of stability and bifurcations in parametric roll motion analysis of ships. In general, single-degree-of-freedom analytical approach is based on reducing number of degrees of freedom from 3 to 1 by using the quasi-static Froude-Krylov assumption, incorporating heave and pitch effects by means of a time varying restoring moment. On the other hand, numerical approaches to motion of six and four degrees of freedom are based on three dimensional diffraction/radiation and potential flow theories. In summary, this paper reveals that analytical approaches are sufficiently adequate to obtain accurate practical results for this relatively complex phenomenon.

Keywords: Parametric roll motion; bifurcations; panel method; AQWA

INTRODUCTION

The first publications about parametrically excited roll motion appeared in 1930s and 1940s by Watanabe and Kempf [1,2], respectively. Roll motion of a ship in longitudinal waves have been studied by a number of researchers including Graff, Heckscher [3], Kerwin [4], Paulling and Rosenberg [5]. The first experimental observation of parametric roll was done by Paulling et al in San-Francisco Bay [6]. Although its theoretical existence has been known for a long time, parametric roll attracted a great deal of interest in recent years because of the incidents that resulted in damages [7,8]. Results of studies about Post-Panamax C11 class containership which sustained extensive loss and damage to deck-stowed containers in October 1998 showed that Post-Panamax containerships tend to experience parametric roll motion in extreme weather [7]. These casualties led designers, researchers and regulatory authorities to initiate further research and investigations. Among the researchers Spyrou [9], Neves, Rodrigues [10] and Bulian *et al.* [11] focused attention on nonlinear aspects and effects of changing tuning factors on parametric roll motion. In addition, some researchers focused attention on probabilistic properties of parametric roll [12,13,14,15,16]. The state of the art in development of methodology and regulations for assessment of ship intact stability, can be found in [17].

Ships sailing in longitudinal waves are excited by the hydrostatic and hydrodynamic forces. Differing the geometry of ship's wetted hull form with respect to wave crest position has an important role on roll motion. In regular waves, the excitation is periodical with a finite period and certain ratios of encounter and natural frequencies which may lead to loss of stability. The most dangerous situation usually occurs in the first parametric resonance region in which wave length is approximately equal to the ship length at an encounter frequency that is twice greater than the roll natural frequency. In this case, the variation of restoring moment causes the roll angle to increase drastically unless other factors such as damping come into play. This particular state is called the parametric roll resonance phenomenon. A detailed explanation of the physics behind parametric roll has been given in [12] and assessment of parametric roll resonance in design of container carriers has been presented in the guide of American Bureau of Shipping [13].

In the literature, parametric roll resonance phenomenon has been investigated with the use of both numerical and analytical approaches. Both the solution methods have their own advantages depending on a purpose of application. Numerical approaches are suitable for solving coupled motions in time domain. Neves and Rodriguez used a two-dimensional analysis for a set of equations for coupled heave, pitch and roll motion with 2nd and 3rd order nonlinearities describing the restoring action [18]. Levadou and Van't Veer used coupled nonlinear equation of motion in time domain with 3 and 5 degrees of freedom (DOF) [19]. The nonlinear excitations are incorporated by pressure integration over the actual wetted surface while diffraction forces are considered linear. Hydrodynamic effects are calculated in the frequency domain by a 3D panel code and are incorporated in the time domain by adopting the impulse response function method. France et al. [7] and Shin et al. [12] used a similar approach with a hybrid singularity based on Rankine source in the near field and transient Green's function in the far field.

On the other hand, analytical approaches have some practical advantages such as possibility of determining roll amplitudes and bifurcations above the stability threshold. An alternative simplified approach is possible for reducing number of DOF from 3 to 1 with the quasi-static Froude-Krylov assumption [20,21]. Based on these assumptions, both direct and indirect effects of waves may be evaluated. It is believed that the quasi-static approximation may be a good model which is able to assess the main features of the phenomenon such as loss of stability and bifurcations [21].

In this paper, both numerical and analytical approaches were applied to predict large roll amplitudes in time and frequency domain. 6 -DOF and 4- DOF time domain simulations based on 3D panel code and single- DOF time domain simulation considering heave and pitch effects by means of time varying restoring moment were used for numerical DOF approaches. 6 DOF and 4 DOF time domain simulation approaches were applied by using AQWA-LINE and AQWA-NAUT software which are capable of calculating motions by using 3D diffraction/radiation and potential flow theory. The single - DOF time and frequency domain models based on reducing number of DOF from 3 to 1 by using the quasi-static Froude-Krylov assumption [20,21,22] were used for analytical approaches. In the models, heave and pitch effects were considered by means of time varying restoring moment [20,21,22]. Basically, three analytical approaches were used in the present study. The first is Bulian's PolyFour Model, the other one is the simplified model which may be considered a simplified version of the first model [21,22] , and the other one is linear model [23]. The accuracy of numerical and analytical approaches were evaluated by comparing experimental results obtained for the sample ship form [21].

DESCRIPTION OF MODELS

ANSYS AQWA program suite was used for 6 -DOF, 5 -DOF (surge, sway, heave, pitch and roll) and 4 - DOF (surge, heave, pitch and roll) analyses. AQWA-NAUT uses frequency - domain results for added mass and damping coefficients to obtain time-history analysis results of ship motions in regular waves by calculating hydrodynamic forces acting on the ship at each time step. AQWA-LINE program module calculates harmonic ship motion amplitudes and hydrodynamic coefficients in frequency domain. The program calculates hydrodynamic forces acting on the ship by integrating hydrodynamic pressure over the wetted surface of the ship. It calculates Froude-Krylov forces due to unscattered - incident wave potential, diffraction forces due to diffracted wave potential and radiation forces due to the resultant potential of 6 - DOF motion of the ship hull. Furthermore, the potentials are handled by a boundary surface method, distributing pulsating sources on the ship wetted surface by assuming a mean equilibrium position in the calm water by means of Green's theorem [24].

In analytical approaches, models should be simplified to obtain steady state solution in frequency domain. The effect of non-hydrostatic pressure can usually be neglected for regular waves longer than half of the ship's length. Therefore, non-hydrostatic pressure referred as Smith effect is neglected. Sway and yaw motions are neglected by considering the capability of the ship to keep her course. It is assumed that the ship is able to maintain a constant speed, accordingly surge motion is neglected. On these assumptions, only roll motion is taken into account in the present analysis. The influence of roll on pitch and heave could be modeled as an explicit forcing, whereas the influence of heave and pitch on roll can be modeled as a parametric excitation by utilizing quasi-static Froude-Krylov assumption [20,21]. The essence of quasi-static assumption is balancing heave and pitch statically in a wave. In this study, non-hydrostatic effects under a wave profile are neglected and restoring moment curves are computed with standard hydrostatic calculation software. Quasi-static assumption is carried out in order to solve the model analytically without unnecessary cumbersome calculations. A detailed explanation of the procedure may be found in [21,22].

Assuming that the displacement of the ship is constant, roll motion equation is written in the following form:

$$\left(I_{xx}+\delta I_{\mathbf{xx}}\right)\ddot{\phi}+B(\dot{\phi},\phi)+\Delta GZ(\phi,t)=0 \qquad (1)$$

where $\left(I_{xx}+\delta I_{\mathbf{xx}}\right)$ is moment of inertia, ϕ is roll angle, $B(\dot{\phi},\phi)$ is damping function and $\Delta GZ(\phi,t)$ is restoring function. Eq. (1) may be re-written as;

$$\ddot{\phi}+b(\dot{\phi},\phi)+\frac{\omega_0^{\,2}}{GM_0}GZ(\phi,t)=0 \qquad (2)$$

In Eq. (2), $\Delta GZ(\phi,t)$ may be approximated by different expressions. Bulian used Eq. (3) to identify the GZ surface analytically [21].

$$GZ(\phi,t)=\sum_{j=1,3,5,7,9}\left(Q_{j0}+\sum_{n=1}^{N_h}Q_{jn}^{c}\cos\left(\frac{n.\omega_e t}{\cos(\chi)}\right)+Q_{jn}^{s}\sin\left(\frac{n.\omega_e t}{\cos(\chi)}\right)\right)\phi^{j} \tag{3}$$

$$\left\{\begin{array}{l} Q_{j0}=A_{j0} \\ Q_{jn}^{c}=A_{jn}^{c}\cos(n.\psi_{c0})+A_{jn}^{s}\sin(n.\psi_{c0}) \\ Q_{jn}^{c}=-A_{jn}^{c}\sin(n.\psi_{c0})+A_{jn}^{s}\cos(n.\psi_{c0}) \\ \psi_{c0}=2\pi\dfrac{x_{c0}}{\lambda_W} \end{array}\right\} \tag{4}$$

In the above-mentioned study, $A_j(x_c)$ values are polynomial coefficients for each wave crest position and can be expressed as Fourier series in the variable x_c with main period equal to the wave length λ_W. Nh is the maximum number of harmonic components that can be estimated from polynomial coefficients by using Nyquist sampling theorem [25]. Coefficients of the Fourier series related to $A_j(x_c)$ are calculated as follows:

$$\left\{\begin{array}{l} Q_{j0}=A_{j0} \\ Q_{jn}^{c}=A_{jn}^{c}\cos(n.\psi_{c0})+A_{jn}^{s}\sin(n.\psi_{c0}) \\ Q_{jn}^{c}=-A_{jn}^{c}\sin(n.\psi_{c0})+A_{jn}^{s}\cos(n.\psi_{c0}) \\ \psi_{c0}=2\pi\dfrac{x_{c0}}{\lambda_W} \end{array}\right\} \tag{5}$$

In the simplified model, unlike the aforementioned model, GZ surface is approximated more simply by using only polynomial coefficients of wave- crest and wave- trough GZ curves. Furthermore, the values between aft and fore of a ship are approximated by a sinusoidal function.

$$GZ(\phi,t)=\sum_{n=1}^{N}\left(m_{2n-1}+k_{2n-1}\cos(\omega_e t)\right)\phi^{2n-1} \tag{6}$$

Coefficients "*m*" and "*k*" in Eq. (6) are obtained from restoring moment curves in wave crest and wave trough conditions.

$$m_{2n-1}=\frac{c_{2n-1,trough}+c_{2n-1,crest}}{2} \tag{7}$$

$$k_{2n-1}=\frac{c_{2n-1,trough}-c_{2n-1,crest}}{2} \tag{8}$$

$$b(\phi,\dot{\phi})=2\mu\dot{\phi}+\beta\dot{\phi}|\dot{\phi}|+d\dot{\phi}^3 \tag{9}$$

In Eq. (7) and (8), "$c_{2n-1,crest}$ and $c_{2n-1,trough}$ " show the coefficients of polynomials fitted to the restoring moment curves in wave- trough and wave- crest conditions. 7th degree polynomials are used for developing the restoring moment surfaces. In this study, quadratic damping and linear damping terms are used.

Steady state solution of the equation of roll motion in longitudinal waves was carried out by means of the averaging method in order to determine bifurcations in frequency domain. The averaging method was introduced by Bulian [21] to estimate roll amplitudes in frequency domain. Bulian's PolyFour model and simplified model are solved by using the same procedure as that applied in [21]. Time domain solution of the simplified model was carried out by using the 4.5 order Runge-Kutta (Dormand-Prince) method.

1- DOF linear model is based on linear Mathiue-Hill equation and its solution Ince-Strutt diagram [23]. Resonant frequencies can be estimated easily by using Ince-Strutt diagram. But predicting roll amplitudes at these frequencies is not possible. Stability of pitchfork bifurcations at minimum and maximum frequencies, in other words , ship speeds can be predicted by using a method detailed in [22].

SAMPLE SHIP AND ENVIRONMENTAL CONDITIONS

The sample ship used throughout the analysis is of a frigate form whose experimental tests were carried out at the towing tank of DINMA [21].The experiments were carried out for 3 DOF (heave, pitch and roll) [21].The sample ship has no bilge keels and appendages. The form and main characteristics of the sample ship, named F1, is given in Fig. 1 and Tab. 1, respectively.

Tab.1. Main characteristics of the sample ship

	Type	LBP	B	T	KG
F1	Frigate	120.00 m	14.25 m	4.060 m	6.557 m

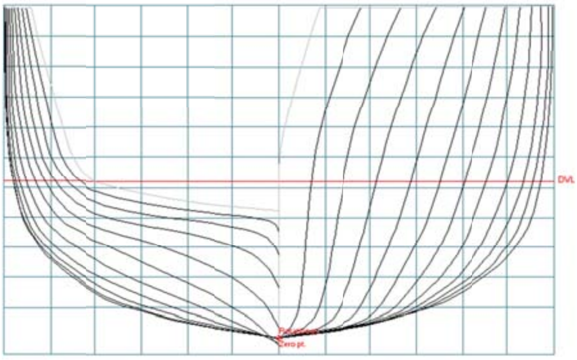

Fig. 1: Frame body plan of the sample ship

In this study, wind force was neglected in order to create results which are comparable with the experimental ones. AQWA mesh model and pressure contours of the sample ship are presented in Fig. 2.

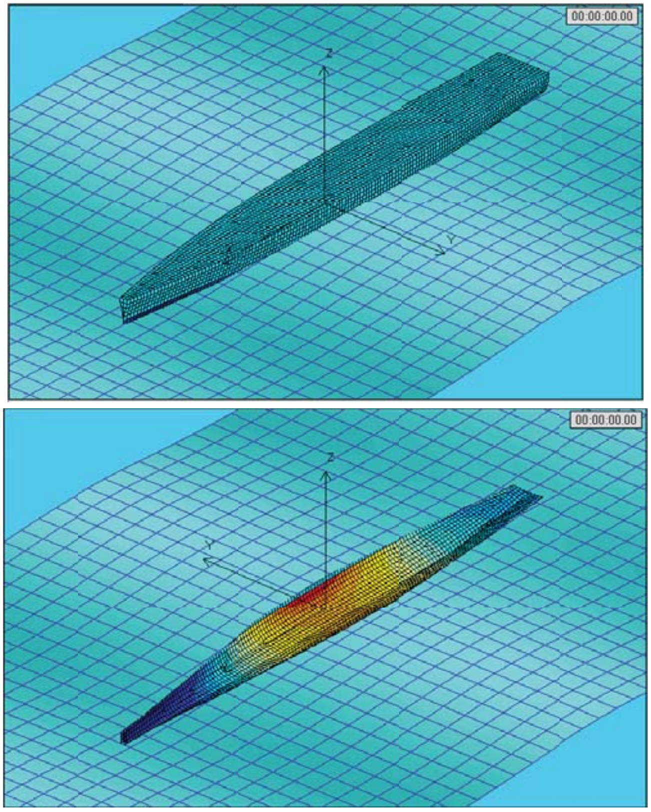

Fig. 2 AQWA mesh model and pressure contours of the sample ship.

The wave length and wave height were chosen equal to 120 m and 2.4 m, respectively. In AQWA program suitable thrust forces were chosen to include ship speed and altering wave effect. Non-dimensional linear damping coefficient is chosen equal to 0.212. Bulian's values with respect to Froude number given in Fig. 3 and 4 are used for nonlinear damping coefficient.

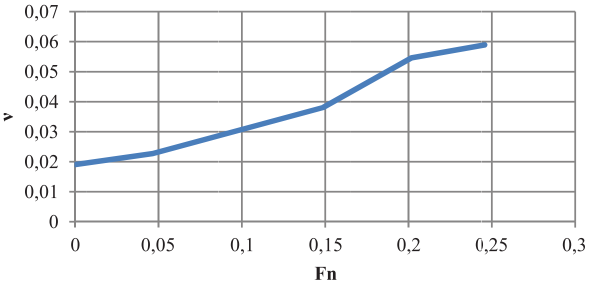

Fig. 3 Linear part of the damping term in Eq. (9)

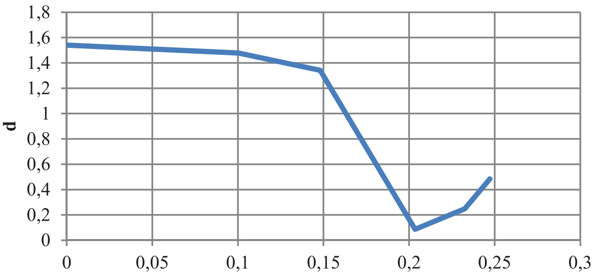

Fig. 4 Nonlinear part of the damping term in Eq. (9)

RESULTS OF THE ANALYSES

As explained above, the maximum roll amplitudes with respect to ship speed were determined by using different approaches. Roll amplitudes over ship speeds were simulated in time domain by using AQWA-NAUT™ for various thrust forces. Sway and yaw motions are neglected in 4-DOF model by considering course keeping capability of ship. Maximum roll amplitudes and mean ship speeds obtained by means of AQWA-NAUT™ (4, 5 and 6 DOF) were plotted in Fig. 5 in comparison with the experimental results obtained in DINMA [21]. It can be perceived that the results of AQWA-NAUT (4 DOF) and (5 DOF) are in good agreement with the experimental results in general, however, there is no results of AQWA-NAUT (6 DOF) where the ship has no course keeping capability at speeds higher than 3 m/s. It is observed that periodic variation of ship speed has no unfavorable effect on results.

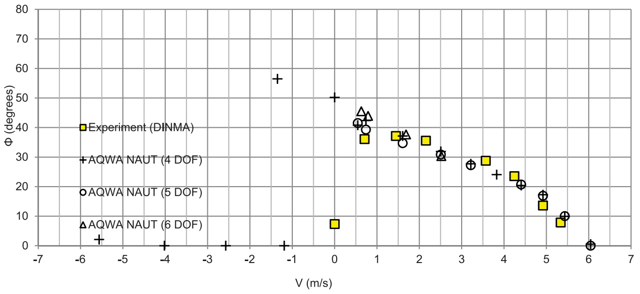

Fig. 5 Comparison of AQWA-NAUT results with the experimental results

Comparison of the analytical results with the experimental results was given in Fig. 6. Roll amplitudes from Bulian's model are approximately by 10° lower than the experimental and AQWA-NAUT results (Fig. 6). However, this fact does not suggest strong conclusive evidence that they may be disregarded in preliminary design. The simplified model was solved for both linear and nonlinear damping conditions (Fig. 6). The amplitudes acquired from the simplified model for nonlinear damping are approximately by 5° lower than the experimental results. However, when linear damping is used, the results of the simplified model are in good

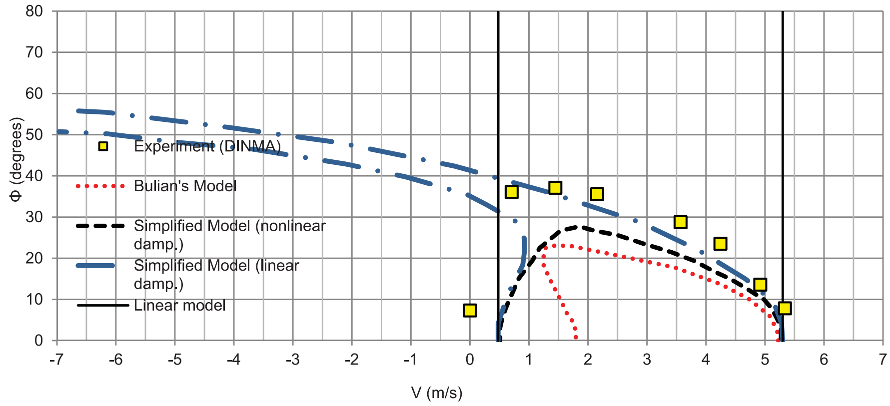

Fig. 6 Comparison of approximation method results and experimental results

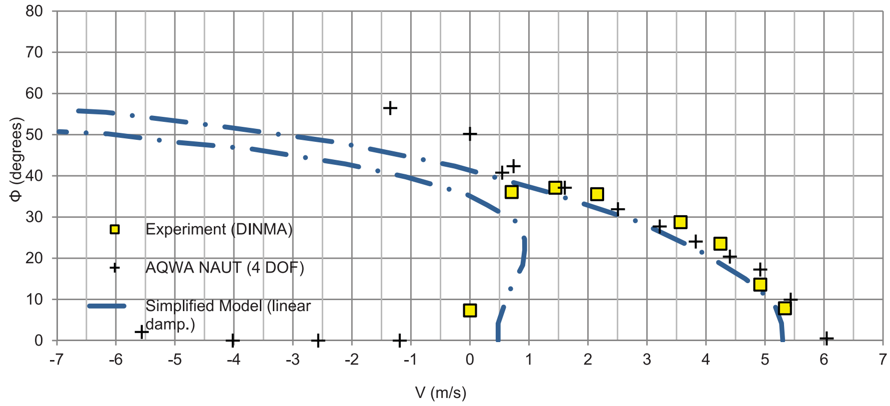

Fig. 7 Comparison of the simplified model (linear damping) and AQWA-NAUT (4 DOF) results with the experimental results

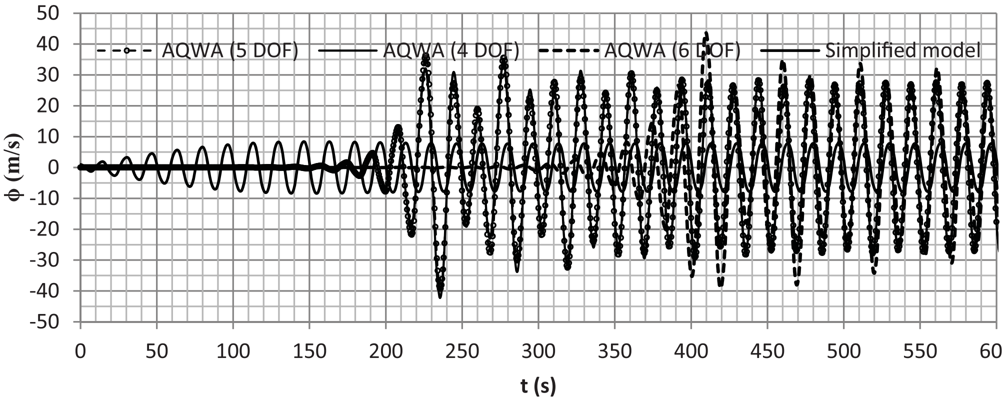

Fig. 8 Variation of roll angles in time domain at neighborhood of 0.7 m/s ship speed

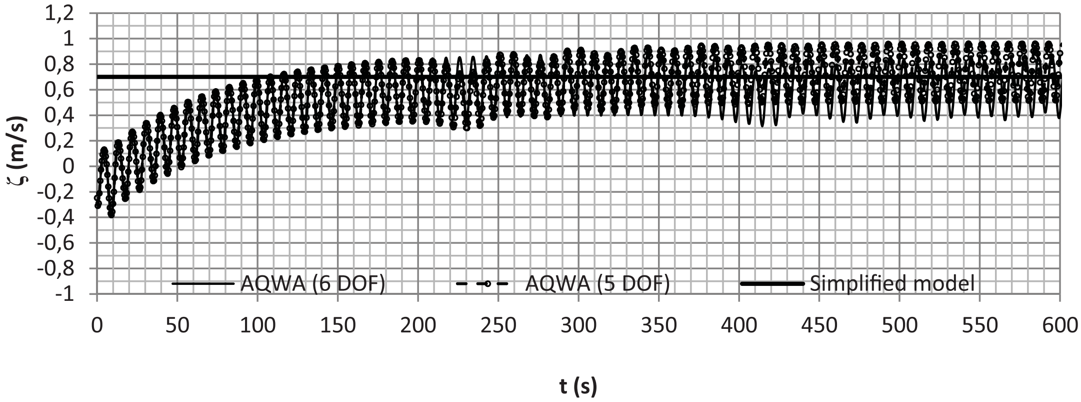

Fig. 9 Variation of ship speed in time domain

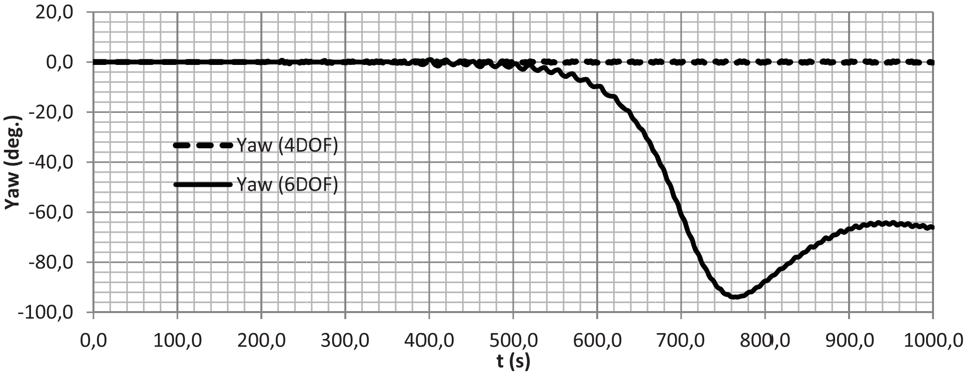

Fig. 10 Variation of yaw motion in time domain

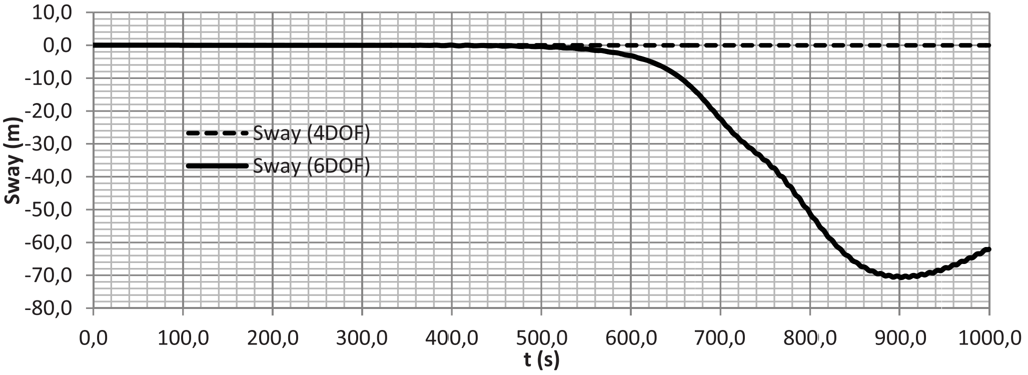

Fig. 11 Variation of sway motion in time domain

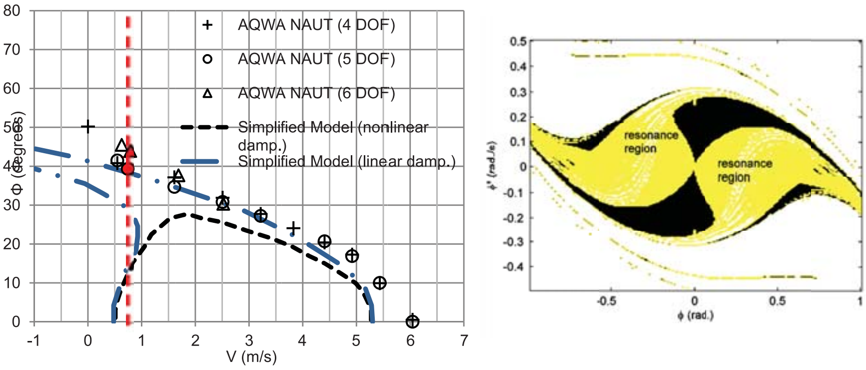

Fig. 12 Results obtained from AQWA-NAUT and the simplified model in frequency domain , and the resonance region diagram with respect to initial values

agreement with the experimental results. Results of linear model (Mathieu-Hill Eq.) are also a stable trivial solution of the simplified model. Unstable trivial solution means a boundary of speeds in which parametric roll motion starts (minimum and maximum ship speeds).

Results of AQWA-NAUT (4 DOF) and simplified model with linear damping were compared in Fig. 7. It can be perceived that results are in good agreement in a region where fold bifurcation occurs. Amplitudes obtained from AQWA-NAUT (4 DOF) were approximately by 10° higher than the amplitudes resulting from the simplified model (linear damping) at following waves. The experiments were made for only head waves therefore fold bifurcation could not be proved experimentally.

The time domain simulation procedure applied in 4- DOF approach is repeated in 5- and 6- DOF calculations with the use of AQWA-NAUT in a specific condition. The results of AQWA-NAUT calculations given in Fig. 8 are compared with the results of analytical simplified model in time domain. Variation of ship speed is presented in Fig. 9. As it can be observed from Fig. 8, the results of AQWA-NAUT calculations with 4 and 5 DOF are compatible with each other but results of calculations with 6 DOF differ a little from 4 -DOF and 5 -DOF results due to the degrees of freedom. Results of sway and yaw motion in 6- DOF simulations are shown in Fig. 10 and Fig. 11. Results of the analytical model in time domain are obtained by solving the equation numerically. It is observed that maximum roll amplitudes do not exceed 15° unlike

AQWA-NAUT results since there is fold bifurcation. The fold bifurcation means that there are three different solutions and the obtained result is the smallest one due to initial values , as shown in Fig. 12. Resonant and non-resonant regions are also plotted with respect to initial values in Fig. 12.

CONCLUSIONS

In this study, results obtained from numerical and analytical approaches were compared with experimental results for parametric roll evaluation of a sample ship. Selection of the solution method has a great influence on setting up the model. A ship sailing in longitudinal regular waves experiences 6 - DOF motions under wind and wave effects. Therefore, the model has to be set up by considering 6 - DOF motion to obtain more accurate results. For this reason, AQWA-NAUT™ program was used for the numerical analysis. The program was applied in three modes, namely: of 6 DOF, 5 DOF and 4 DOF. The results of analyses with 6 DOF, 5 DOF and 4 DOF are highly complying with the experimental results. It may be concluded that 5-DOF and 4- DOF time domain simulations are good tools for determining roll amplitudes. It should be noted that the results of 6 - DOF simulations naturally differ from 4-DOF results because of the presence of sway and yaw motions. Oscillation of ship speed is included as a second excitation term in restoring moment for roll motion in longitudinal waves, as expressed in Eq. (10). In Eq. (10), *V(t)* can be assumed a sinusoidal function (Fig.9), hence it can be said that excitation is doubled depending on amplitude of *V(t)*. However results of this study revealed that oscillation of ship speed has no unfavourable effect (Fig.7). Comparison of constant speed and oscillating speed indicates that amplitudes of about 0.3 m/s in value are not adequate to change results.

$$GZ(\phi,t)=\sum_{n=1}^{N}\left(m_{2n-1}+k_{2n-1}\cos\left(\left(\omega_w+\frac{\omega_w^{\,2}}{g}V(t)\right)t\right)\right)\phi^{2n-1} \tag{10}$$

Numerical simulations usually start with a particular initial condition that lies in a specified domain of attraction. Thus, co-existence of any other steady state solutions cannot be estimated without changing the initial condition. Numerical solutions are somewhat inadequate to give a global picture of the response curve and bifurcations involved in the phenomenon. However, an analytical approach is usually able to give such an overall view in a more practical way. In addition, analytical approaches have some practical advantages such as determining roll amplitudes and also bifurcations above the stability threshold. It is strongly believed that the quasi-static approximation maybe a good model producing the main features of the phenomenon such as loss of stability and bifurcations [21,22]. In this study, two analytical approaches were used: Bulian's PolyFour and simplified models [21,22] in which nonlinear damping term was applied. It is found that the results of Bulian's PolyFour and simplified models containing nonlinear damping term are in good agreement, comparatively. However, they are not in a close agreement with the experimental results: PolyFour model result is approximately by 10° lower and the simplified model result is approximately by 5° lower than the experimental results. The simplified model was also run with linear damping term. As seen from Fig. 4, the result of the simplified model with linear damping term complies satisfactorily with AQWA-NAUT and experimental results. Damping coefficients have great effect on results, but unfortunately they cannot be predicted accurately without full-scale experiments. Ships designed for high speed have lower block coefficients and these ship forms tend to encounter parametric roll motion more than others. In this study, a frigate ship form of block coefficient equal to 0.453, was chosen. Behaviour of other ship forms can be found in [21] and [22]. The most important factor of parametric roll motion is restoring moment term. The shape of wave crest-wave trough restoring moment curves and roll responses of container ships and frigates are almost the same as indicated in [22].

Finally, this study indicates that using the linear damping term gives comparable results and the nonlinear damping term is not demanded. The most important advantage of the analytical approach is ability of determining types of bifurcation. It is critically important to determine the type of bifurcations since they lead to jump phenomenon. It should be also stressed that subcritical bifurcations may lead to large roll amplitudes or capsizing [26,22].

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