THEORETICAL ANALYSIS OF THE FINITE AMPLITUDE WAVES INTERACTION PROBLEM FOR TWO TYPES OF PISTON

ANNA BARANOWSKA

Gdansk University of Technology, Department of Mathematical and Numerical Analysis
ul. G. Narutowicza 11/12, 80-952 Gdansk, Poland
anbar@mif.pg.gda.pl

The aim of the paper was the numerical investigations of the finite amplitude waves interaction in water. The problem was considered as an axial symmetric one. It was assumed that circular piston and ring-shaped piston were the source of two different frequency finite amplitude waves. The mathematical model was built on the basis of the Khokhlov - Zabolotskaya - Kuznetsov (KZK) equation. To solve the problem the finite-difference method was applied. The mathematical model and some results of numerical investigations are presented. The spectrum of the wave for fixed distances from the source, the pressure amplitude changes for different frequency waves as a function of distance from the source and pressure amplitude distribution for fixed distances from the piston were investigated.

INTRODUCTION

The finite waves interaction problem is one of the most important problems in nonlinear acoustics. This problem has been described in the literature for many years. It is considered in both experimental [5] and theoretical investigations [1, 2, 3, 4, 6, 8]. It is possible to find the works about practical applications of it, too.

The waves during their propagation in the same direction in water interact. The formation of the different frequency waves, another that primary one, is the result of it. The most important in practical application is existence of the difference frequency wave. The generation of this wave has application at investigation and construction of the parametric acoustic arrays (for example [7]). Narrow beam pattern with low frequency for small size of the transducer and high frequencies of primary waves are the most important characteristics of them.

The aim of this paper is the numerical analysis of the finite amplitude waves interaction. This problem can be described using KZK equation. The paper presents mathematical model and some examples of the results of numerical investigations. The circular piston and ring-shaped piston were study.

1. MATHEMATICAL MODEL

To solve the finite amplitude interaction problem we assume that the source of finite amplitude waves is placed in plane yOz and the waves are propagated in the x direction. It means that x axis corresponds to the beam axis. Moreover we assume axial symmetry of the source.

The mathematical model of the problem is built on the basis of the KZK equation:

$$\frac{\partial}{\partial \tau}\left(\frac{\partial p'}{\partial x}-\frac{\varepsilon}{\rho_o c_o^3}p'\frac{\partial p'}{\partial \tau}-\frac{b}{2\rho_o c_o^3}\frac{\partial^2 p'}{\partial \tau^2}\right)=\frac{c_o}{2}\Delta_\perp p' \tag{1}$$

where $p'=p-p_o$ denotes an acoustic pressure, variable $\tau=t-x/c_o$ is the time in the coordinate system fixed in the zero phase of the propagating wave, $\Delta_\perp$ - Laplace operator defined in the (y,z) plane, perpendicular to the x axis, ρ_o - medium density at rest, c_o - speed of sound, b - dissipation coefficient of the medium, ε - nonlinerity parameter. This equation describes the acoustic pressure changes in nonlinear and dissipative medium along the sound beam.

Because of axial symmetry of the source it is comfortably to solve the problem in cylindrical coordinates (x,r). Now the Laplace operator is defined in following form:

$$\Delta_\perp=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}$$

where $r=\sqrt{y^2+z^2}$.

The solution of Eq. (1) is look for inside a hypothetical cylinder with radius R (R>a), for $\tau\in[0,T]$, i.e. in domain

$$D=\left\{(x,r,\tau)\in R^3 : x\in[0,X],\ r\in[0,R],\ \tau\in[0,T]\right\}$$

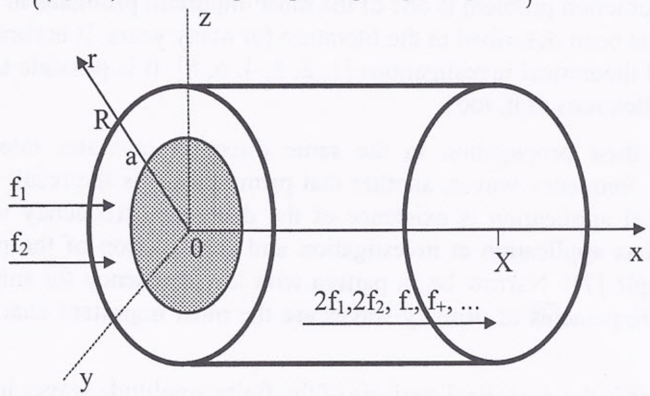


Fig.1. The geometry of the problem

To complete the problem the boundary condition is added. In general this condition is written in the form:

$$p'(x=0,r,\tau)=\begin{cases}A_1(r)\sin\omega_1\tau+A_2(r)\sin\omega_2\tau & for\ r\le a\\ 0 & for\ r>a\end{cases} \tag{2}$$

where parameters $A_1(r)$ and $A_2(r)$ denote primary wave amplitudes, angular frequencies are defined by $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ respectively.

Two sources of primary waves are considered. Firstly it is assumed that circular piston is the source of two harmonic waves (Fig. 2a). Then the waves distribution on the piston is defined by:

$$p'(x=0,r,\tau)=-p_o\sin\omega_1\tau-p_o\sin\omega_2\tau \tag{3}$$

for r≤a. Secondly we assume that ring-shaped piston is the source of two different frequency finite amplitude waves (Fig. 2b). In this situation the source consists of coaxial disc and six rings. Additionally we assume that the surface area of all source elements is the same, i.e. the radii of rings are equal $r_k=\sqrt{k}r_1$, k=2,3,...,7 where r_1 denotes the disc radius. In this situation the distribution on the source is equal:

$$p'(x=0,r,\tau)=\begin{cases}-p_o\sin\omega_1\tau & for\ r\in[0,r_1]\cup[r_2,r_3]\cup[r_4,r_5]\cup[r_6,r_7]\\ -p_o\sin\omega_2\tau & for\ r\in[r_1,r_2]\cup[r_3,r_4]\cup[r_5,r_6]\end{cases} \tag{4}$$

for r≤a.

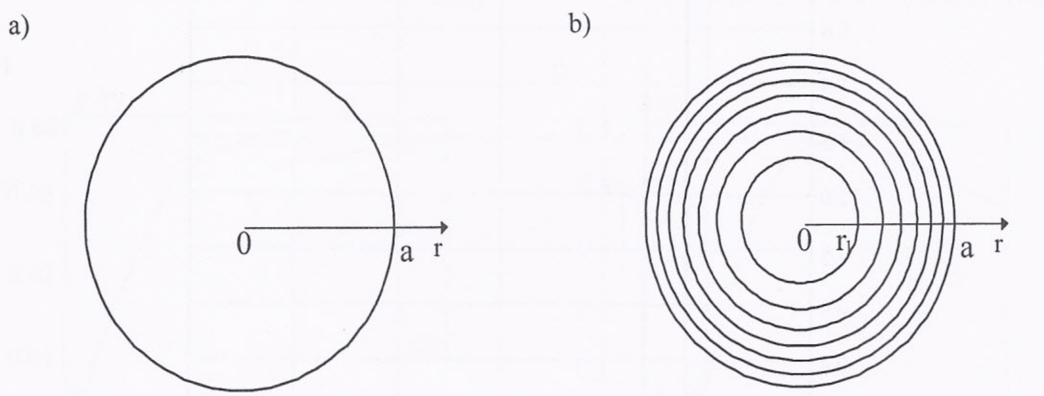


Fig.2. The primary waves source: a) circular piston, b) ring-shaped piston

2. NUMERICAL SOLUTION

The finite-difference method is used to solve the problem numerically. To solve Eq. (1) numerically the rectangular net is constructed. The net is defined in following form

$$x_n = n\Delta x, \quad r_k = k\Delta r, \quad \tau_m = m\Delta\tau$$
$$\Delta x = X / N_x, \Delta r = R / N_r, \Delta\tau = T / N_\tau \tag{5}$$

where n=0,1,..., N_x-1, k=0,1,..., N_r-1, m=1,2,..., N_τ. In this way function p'(x,r,τ) is discretized in both space and time.

The pressure changes along the sound beam are the result of computer calculations. The waveform change is equivalent with spectrum change. The harmonic analysis is very often used to investigate wave distortion. The knowledge of pressure changes allows to calculate the spectrum changes. The fast Fourier transform (FFT) is used to calculate spectrum.

3. NUMERICAL INVESTIGATIONS

The numerical investigations were carried out assuming that waves are propagated in water where speed of sound c_o=1500 m/s, medium density ρ_o=1000 kg/m^3, nonlinearity parameter ε=3.5, and dissipation coefficient b=0.004. Moreover it is assumed that the piston radius was equal a=24 mm and f_1=600 MHz, f_2=800 MHz, p_o=150 kPa.

The appearance of different frequency waves is the result of waves interaction. Figure 3 presents the spectrum on the beam axis for distance x=0.23 m from the source. In this example the distribution on the source was defined by formula (3).

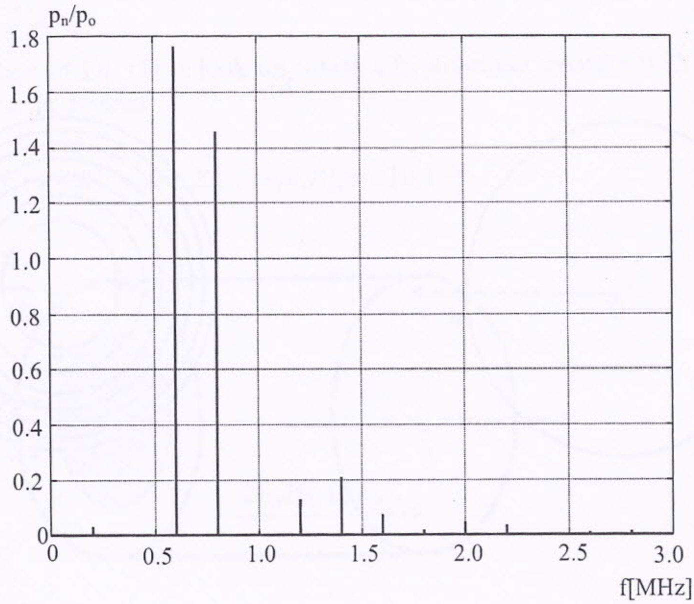


Fig.3. The spectrum on the beam axis for distance x=0.23 m from the source (circular piston)

Fig.4. Normalized on-axis pressure amplitude for different frequency waves as a function of distance from the source (circular piston): a - f_1 , b - $2f_1$, c - f_2 , d - $2f_2$, e - difference, f - sum frequency wave

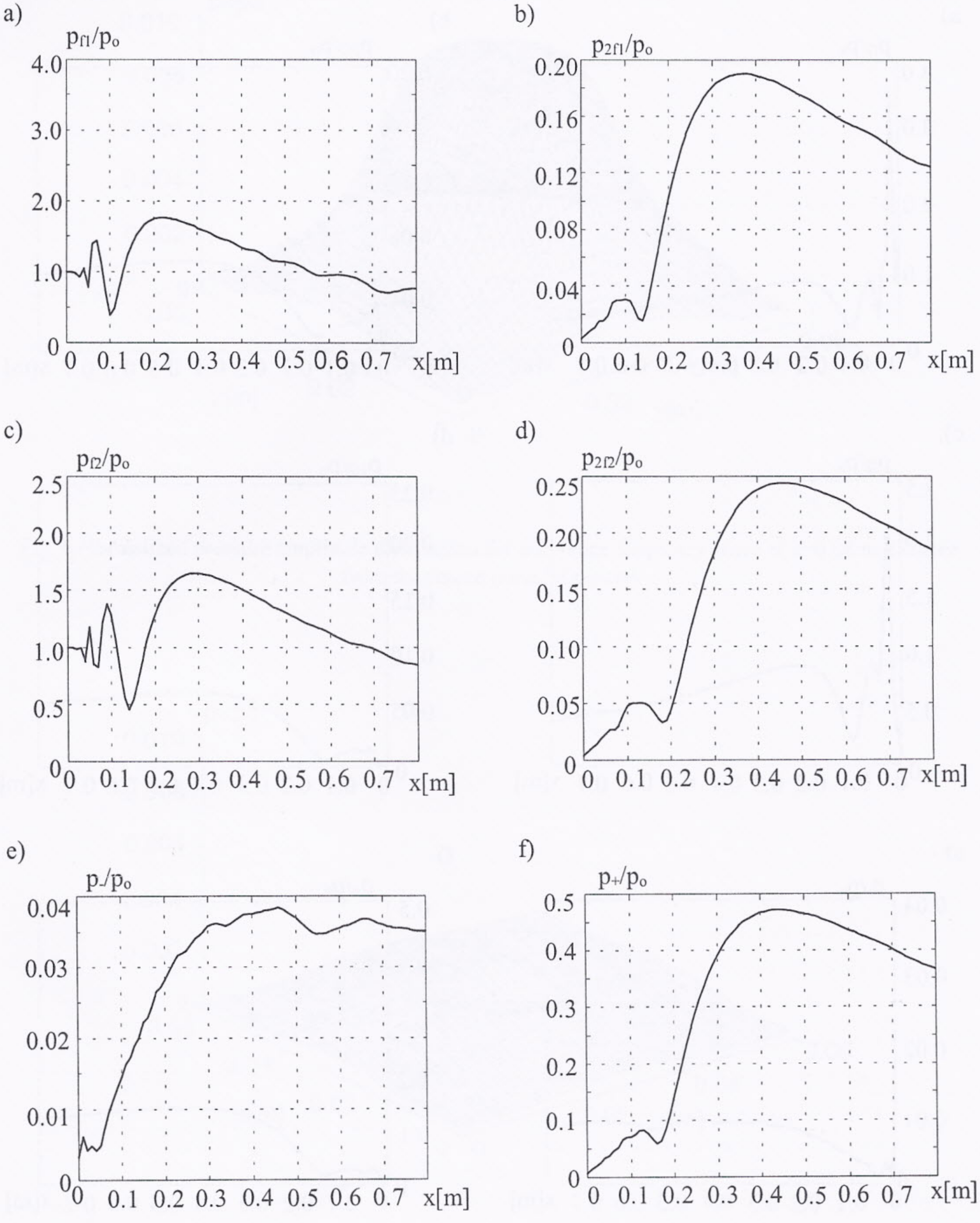
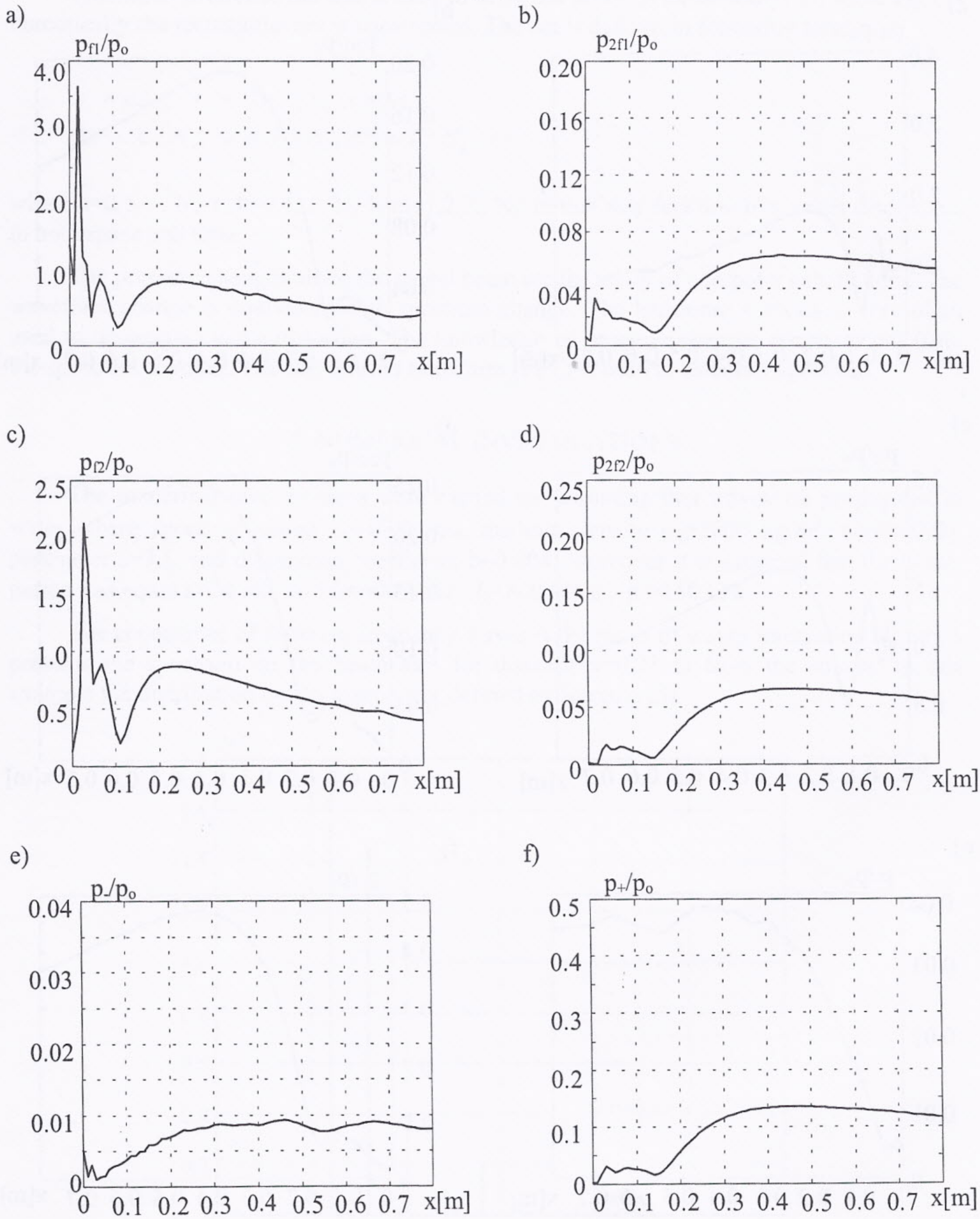


Fig.5. Normalized on-axis pressure amplitude for different frequency waves as a function of distance from the source (ring-shaped piston): a - f_1 , b - $2f_1$, c - f_2 , d - $2f_2$, e - difference, f - sum frequency wave



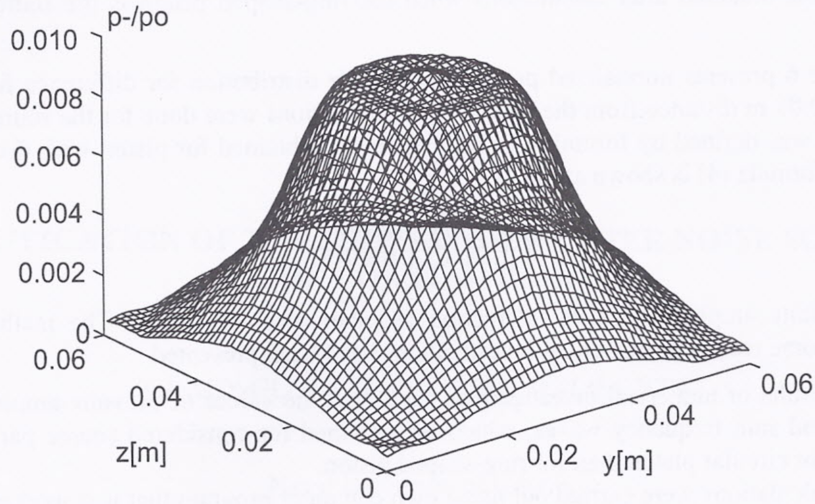


Fig. 6 Normalized pressure amplitude distribution for difference frequency wave at x=0.08 m distance from the source (circular piston)

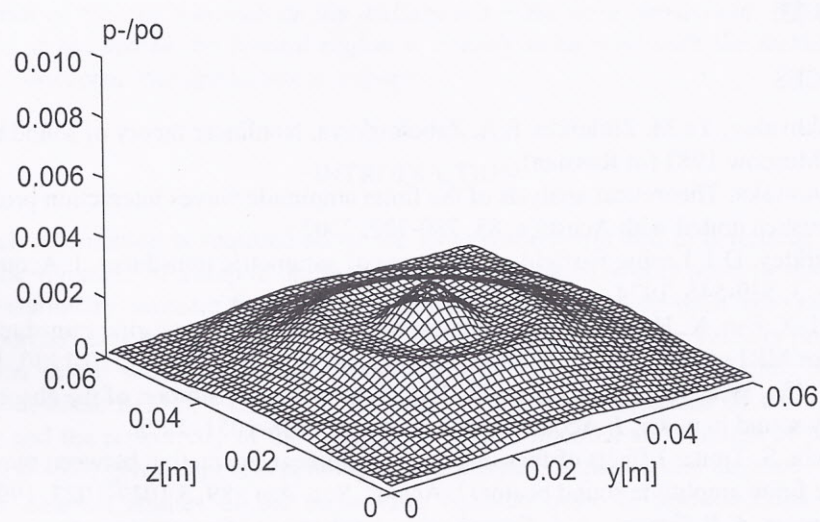


Fig. 7 Normalized pressure amplitude distribution for difference frequency wave at x=0.08 m distance from the source (ring-shaped piston)

Figure 4 shows normalized on-axis pressure amplitude for f_1, $2f_1$, f_2, $2f_2$, difference and sum frequency wave as a function of distance from the circular piston. Figure 5 presents similar results obtained after calculations when the ring-shaped piston is the source of the waves.

Figure 6 presents normalized pressure amplitude distribution for difference frequency wave at x=0.08 m distance from the source. The calculations were done for the source which distribution was defined by formula (3). Similar results obtained for piston with distribution defined by formula (4) is shown at Fig. 7.

4. CONCLUSIONS

The finite amplitude waves interaction problem was considered. The mathematical model and some results of numerical investigations have been presented.

The results of numerical investigations show that the values of pressure amplitude for difference and sum frequency waves, which are obtained for considered source parameters, are greater for circular piston than for ring-shaped piston.

The calculations were carried out using own computer program that was worked out on the basis of obtained algorithm. Proposed method can be used to analyse the waves interaction for different values of source and medium parameters. Moreover it is possible to solve the problem for different numerical parameters, too.

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