

THE INFLUENCE OF MATERIAL MODEL OF THE POLYURETHANE ELASTOMER ON THE FEM CALCULATIONS QUALITY FOR THE VARIOUS MODES OF LOADING

The paper presents research to verify the effectiveness of nine selected material models of elastomeric materials based on uniaxial tension test. Basing on the cyclic uniaxial tension test of elastomers sample, the stress-strain characteristic for the 18th load cycle was prepared. On the basis of the obtained characteristic, the values of material constants were calculated for the studied models (Neo-Hookean, Mooney with two and three constants, Signorini, Yeoh, Ogden, Arruda-Boyce, Gent and Marlow) and simulation of tensile, upsetting and bending processes was performed with the usage of the software MARC/Mentat. The effectiveness of the selected models was determined based on a comparison of results obtained in the experimental tensile test, upsetting test and bending test of an elastomeric samples with the results of numerical FEM calculations for each models. The research has shown that, for modeling of the elastomeric cylinder upsetting in the range of deformation of 62%, the best results with the comparison of the experiment were obtained by using the Yeoh model. In the bending process none of the analyzed models indicate a high convergence of results from an experiment. Analyzing the characteristics of the experimental and numerical tensile test it can be seen that in the entire range of punch movement (0 to 55 mm), models Signorini, Marlow, Ogden(N3) and Mooney(3) give the best results.

Keywords: elastomers, material tests, material models, FEM simulation, experiment

1. Introduction

Elastomers due to several of its advantages (eg. good formability, low cost, flexibility, energy absorption capacity) are increasingly used in many industries. Elastomeric tools are most commonly used with a group of tooling for forming of sheet metal parts. Scientific literature reported that approximately 60% of all aviation sheet metal parts are shaped using a rubber tools [1,2]. Elastomers and natural rubber are applied as a group of tools used in various technological forming operations including bending, punching, stamping, etc. Therefore, they are frequently used in the construction as punches and hold-down products.

Nowadays, the production market and a really well-developed competition in the aerospace industry is a huge challenging task for process engineers, who require product, which is reliability and with high quality. In order to meet the stringent requirements, while minimizing manufacturing costs, it requires knowledge of materials and their properties already at the design stage prior introduce the first production. That is reason, why more often design processes is supported by computer simulations. This allows that before you even start the first production to take into account some factors which affecting the process and avoid the costly material and operating time waste .

Effective application of an elastomeric material numerical simulation is highly dependent on the knowledge of the various

material models and their constants, which are characteristic of elastic materials and their appropriate choice of depending on the parameters during the forming operation. In order to correctly determine the material constants for elastomers, literature recommends to perform four material tests: uniaxial tension test, biaxial tension test, planar tension test or simple shear test [3]. Therefore it is really important that material models and their constant that will best describe the actual behavior of the elastomer tools in a particular process should be determined already at the design stage depending on the tool load value. Due to the fact that the elastomeric tool for forming operations include sheet metal, in most cases works in cyclic upsetting process, so it was decided that the test for determining the material constant in the analyzed study will be only one test material: uniaxial tension test. The advantage is that it can be performed on a standard testing machine, without the use specialized equipment and complicated samples. The increasing interest in the use of elastomers for the production and the difficulties associated with the determination of material constants in suitable experimental trials have become motivated to undertake research on this topic.

The purpose of this study and the research is to analyze the effectiveness of the nine selected elastomeric materials models in the modeling of the upsetting and bending processes, for which the material constants were determined on the basis of one material test – uniaxial tensile test.

* RZESZÓW UNIVERSITY OF TECHNOLOGY, 12 POWSTAŃCÓW WARSZAWY AV., 35-959 RZESZÓW, POLAND

** PRATT&WHITNEY AEROPower RZESZÓW, 120 HETMAŃSKA, 35-078 RZESZÓW, POLAND

[#] Corresponding author: stan_kut@prz.edu.pl

2. Elastomer material models

One of the main features that can distinguish elastomeric materials from other materials is the ability to carry large deformations, which is not directly proportional to the given load and it indicates on the unique nature of the elastomer. Scientific literature shows many material models used in determining the elastomers characteristics [1-5]. The study was done in terms of designing in engineering practice, therefore models implemented in FEM software were used: Neo-Hookean, Mooney (2) (with two constants) Mooney (3) (with three constants), Signorini, Yeoh, Ogden (with the number of components equals $N = 1, 2$ and 3), Arruda-Boyce, Gent and Marlow [6-9].

Neo-Hookean model is one of the simplest models to describe the materials hyperelastic. It is used mostly for materials, which are subjected to tensile test. The strain energy model Neo-Hookean takes the form:

$$W = C_{10} (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) \quad (1)$$

where: $\lambda_i = 1 + \varepsilon_i$

$$\varepsilon_i = \left(\frac{\Delta L_i}{L_i} \right) - \text{contractual strain,}$$

C_{10} – material constant,
 ΔL_i – change in length,
 L_i – original length.

The assumption incompressibility can be written as: $\lambda_1 \lambda_2 \lambda_3 = 1$

The equations for the stress σ and shear stress τ take the form of:

$$\sigma = \frac{\partial W}{\partial \lambda} = \sigma(\varepsilon); \tau = \frac{\partial W}{\partial \lambda} = G\gamma \quad (2)$$

where:

G – shear modulus,
 γ – pure shear strain.

However, in the case of biaxial stress, Neo-Hookean model is inefficient. Therefore, this type of stress Mooney model is used, also known as the Mooney-Rivlin. This model uses in its formula two Mooney (2) or three Mooney (3) material constants, determined from experimental studies. The basic premise the Mooney model is the assumption incompressible, isotropy and the validity of Hooke's law in the state of pure shear. The strain energy function for this notation is presented in the form:

$$W = C_1 (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) + C_2 \quad (3)$$

The strain energy function for pure shear can be written as:

$$W = (C_1 + C_2) \left(\lambda_1^2 + \frac{1}{\lambda_1^2} - 2 \right) = (C_1 + C_2) \gamma^2 \quad (4)$$

$$\tau = \frac{dW}{d\gamma} = 2(C_1 + C_2) \gamma \quad (5)$$

Hence shear modulus takes the form: $G = 2(C_1 + C_2)$

$$\sigma = 2 \left(\lambda - \frac{1}{\lambda^2} \right) \left(C_1 + \frac{C_2}{\lambda} \right) \quad (6)$$

where: C_1, C_2 – material constants.

Using the tensor formula strain invariants can be written as:

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad (7)$$

$$I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \quad (8)$$

$$I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2 \quad (9)$$

where: I_1, I_2, I_3 – strain invariants.

From the assumption incompressibility: $I_3 = 0$ and hence: $W = W(I_1, I_2)$.

The strain energy function model in Mooney notation with two parameters on the basis of above equation takes the form:

$$W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) \quad (10)$$

The strain-energy function in Mooney-Rivlin notation with three parameters can be written as:

$$W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) + C_{11} (I_1 - 3)(I_2 - 3) \quad (11)$$

where: C_{10}, C_{01}, C_{11} – material constants.

Signorini model is a modification of the model Mooney, which the form of the strain energy function for Signorini model with three material constants takes the form:

$$W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) + C_{20} (I_1 - 3)^2 \quad (12)$$

Yeoh model is more effective for a much greater range of deformation. The character of the strain energy function for Yeoh model can be represented as:

$$W = C_{10} (I_1 - 3) + C_{20} (I_1 - 3)^2 + C_{30} (I_1 - 3)^3 \quad (13)$$

where C_{20}, C_{30} – material constants.

Ogden model is often used to model the rubber elements with low compressibility, which can be seen a non-linear stress-strain dependence. In comparison with these other models, Ogden model can be used in several variants, depending on the number of ingredients in function [3,6]:

$$W = \sum_{n=1}^N \frac{\mu_n}{\alpha_n} \left[J^{\frac{-\alpha_n}{3}} (\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n}) - 3 \right] + 4,5K \left[J^{1/3} - 1 \right]^2 \quad (14)$$

where: μ_n, α_n – material constants; $J = \lambda_1 \lambda_2 \lambda_3$.

Arruda-Boyce model is one of the constituent micro-mechanical models for hyperelastic materials, often used to describe elastomeric products. The strain energy form in this model is based on the molecular structure of the elastomer, that provided by the eight-chain model to simulate a non-Gaussian the behavior of individual chains of the network. The parameters: n, k, θ, N (where: n – chain density, k – Boltzmann constant, θ – temperature, N – the number of statistical links of length "l" in the chain between chemical crosslinks) define certain limitations extensibility of the chain, related with an the molecular chains orientation. Most of the models in order to describe the deformation of the rubber material takes the form of strain energy function, which is obtained by adjustment the experimental

data from one state deformation to another, which describe the deformations is not accurately. That is why Arruda-Boyce model takes into account this disadvantage and it becomes unique material model for the data of a standard tensile test, which ensure acceptable accuracy for several types of deformation. The model is constructed of an eight-chain networks [7]. In the cube of dimension α_0 (Fig. 1) with unstretched network including eight chain, their length equals $r_0 = \sqrt{NL}$, while a fully extended chain has an approximate length of NL . The chain vector from the center of the cube to corner take the form of:

$$C_1 = \frac{\alpha_0}{2} \lambda_1 i + \frac{\alpha_0}{2} \lambda_2 j + \frac{\alpha_0}{2} \lambda_3 k \quad (15)$$

Using geometrical considerations, the chain vector length can be written as:

$$r_{chain} = \frac{1}{\sqrt{3}} \sqrt{NL} (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^{1/2} \quad (16)$$

and

$$\lambda_{chain} = \frac{r_{chain}}{r_0} = \frac{1}{\sqrt{3}} (I_1)^{1/2} \quad (17)$$

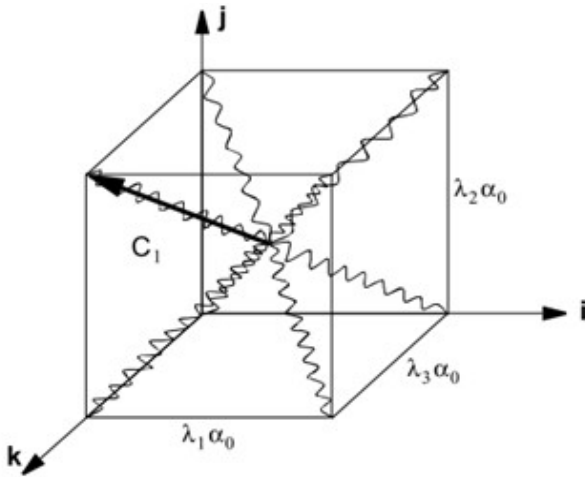


Fig. 1. Eight- chain network in stretched configuration

Taking into account the statistical mechanics aspects it can be assumed that the work of deformation is proportional to the entropy change in unstretched chains to the stretched state. It can be determined as a function of the chain length as:

$$W = nk\theta N \left(\frac{r_{chain}}{NL} \beta + \ln \frac{\beta}{\sinh \beta} \right) - \theta \hat{C} \quad (18)$$

where: n – chain density; \hat{C} – constant; β – the constant and β is an inverse Langevin function correctly accounts for the limiting chain extensibility and is defined as:

$$\beta = L^{-1} \left(\frac{r_{chain}}{NL} \right) \quad (19)$$

where Langevin formula can be written as:

$$\mathfrak{J}(\beta) = \coth(\beta) - \frac{1}{\beta}$$

After considering the above equations, the model Arruda-Boyce is defined as:

$$W = nk\theta \left(\frac{1}{2} (\hat{I}_1 - 3) + \frac{1}{20N} (\hat{I}_1^2 - 9) + \frac{11}{1050N^2} (\hat{I}_1^3 - 27) + \frac{19}{7000N^3} (\hat{I}_1^4 - 81) + \frac{519}{673750N^4} (\hat{I}_1^5 - 243) \right) \quad (20)$$

Moreover, given the notion of limiting chain extensibility, Gent [8] proposed constitutive equation below:

$$W = \frac{-EI_m}{6} \log \left(1 - \frac{I_1^*}{I_m} \right) \quad (21)$$

where: $I_1^* = \hat{I}_1 - 3$.

The constant EI_m is not depend on of the molecular length and degree of molecular crosslinking. Gent model is a useful and attractive due to its simplicity of determination and the fact that reflects the fundamental behavior of extensible molecules network in the whole range of possible variations.

If the strain energy density of the incompressible hyperelastic material is a function of only the first strain invariant, it can be fully defined by a single material test, which is a uniaxial tension test. Strain energy is defined in this case as a general first invariant Marlow constitutive model. The stress-strain behavior of the model in the basic deformation modes is reasonable when the uniaxial behavior is existed for analyzed material. The advantage of this model is easy to determine the function and / or other variables analyzed, extended by the compressibility of the material. These features make this model often used in the case of the availability of data for a single strain state. Therefore Marlow model will accurately reflect the stress-strain characteristics, which is used for the real determination. Due to that, the model defines the behavior of the material on the basis of a material test, at the same time it can not replicate exactly the stress-strain behavior under different conditions of deformation, but obtained for the case of a satisfactory approximation. It should be noted, however, that the model based on one material test is not as accurate as in the case of multi-parameter models, which take into account data from all four tests (uniaxial, planar, equibiaxial). The function of strain energy for Marlow model takes the form:

$$W(I) = \int_0^{\lambda_T(\hat{I})-1} T(\varepsilon) d\varepsilon \quad (22)$$

where:

$T(\varepsilon)$ – nominal uniaxial traction,

\hat{I} – taking account the value of the first invariant

and: $\lambda_T(\hat{I})^3 - \hat{I} \lambda_T(\hat{I}) + 2 = 0$

$\lambda_T(\hat{I})$ – uniaxial stretch [9].

3. Uniaxial tension test

Dumbbell samples to cyclic uniaxial tension test were prepared from an elastomer with a hardness of 90 ShA, 5 mm thick and 10 mm wide. Experiment was performed on ZWICK/ROELL Z030 testing machine (Fig. 2). Measuring base of extensometer was 50 mm.



Fig. 2. The uniaxial tension test of elastomer sample

Stress-strain characteristics were created after 18 load cycles of elastomer sample (Fig. 3). In doing this Mullins effect is ignored. This characteristics obtained as a result of the experiment inserted into the program MARC / Mentat, which material constants were determined for nine of the analyzed material models: (Neo-Hookean, Mooney (2) (with two constant) Mooney (3) (with three constants), Signorini, Yeoh, Ogden (with the number of components equals $N = 1, 2$ and 3), Arruda-Boyce, Gent and Marlow). The material constants for the model Marlow has not been determined because there is no curve fitting. The calculated material constants for the analyzed models are presented in tables (Tab. 1-3).

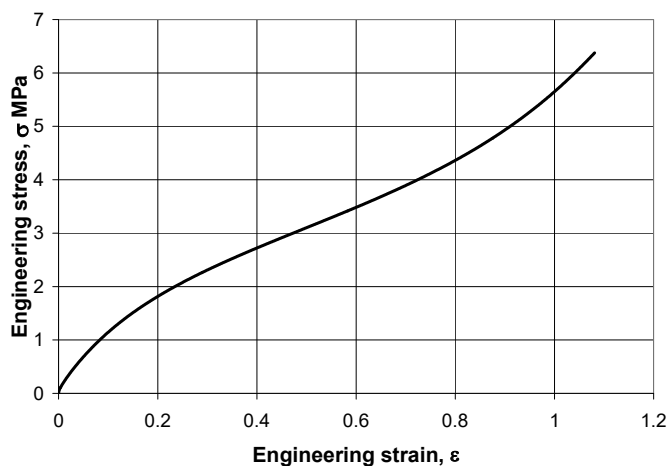


Fig. 3. Stress-strain curve for 18th loading cycle

TABLE 1

The material constants determined from the uniaxial tensile test for phenomenological models

No.	Model name	Material constants				
		C_{10}	C_{01}	C_{11}	C_{20}	C_{30}
I	NEO-HOOKEAN	1,568	—	—	—	—
II	MOONEY (2)	0,787	1,226	—	—	—
III	MOONEY (3)	-2,89	5,523	0,779	—	—
IV	SIGNORINI	-1,877	4,414	—	0,326	—
V	YEOH	1,958	—	—	-0,484	0,135

TABLE 2

The material constants determined from the uniaxial tensile test for principal stretch models

No.	Model name	Number of components N	Material constants	
			Modulus μ_n	Exponent α_n
VI	OGDEN	1	-1,7794	-5,1831
		2	-1,184	3,706
			-2,246	-7,103
		3	96,7923	-0,12575
			-0,14822	3,09525
			-5,46016	-4,42401

TABLE 3

The material constants determined from the uniaxial tensile test for micromechanical models

Lp	Model name	Material constants	
VII	ARRUDA-BOYCE	NK Θ	N
		3,171	725068
VIII	GENT	E	I M
		9,515	3799470

4. Experimental results of upsetting and bending tests

In order to obtain experimental results necessary to verify the results of numerical calculations, three tests was carried out: cyclic upsetting tests, cyclic bending tests and cyclic tensile tests. The tests for prepared samples were performed on the testing machine Zwick/Z030 ROELL. The research samples for upsetting tests were cylindrical with diameter $d = 11.8$ mm and a height $h_0 = 19$ mm (Fig. 4a). In order to eliminate the impact of sliding friction and stabilizing the contact conditions, the sandpaper was used between the contact surfaces of the sample and the upsetting tool (Fig. 4a).

Parallelepipedic samples for cyclic bending tests were made from the same elastomer material with the dimensions: width 20 mm, height 19.5 mm, length 100 mm, which was performed a cyclic three point bending test. (Fig. 4b). The distance between the supports was 70 mm.

Dumbbell samples for tensile tests were made from the same elastomeric material as in previous tests with the dimensions: width 10 mm, height 5 mm (Fig. 2). The samples were subjected to cyclic tensile tests, as described in section 3.

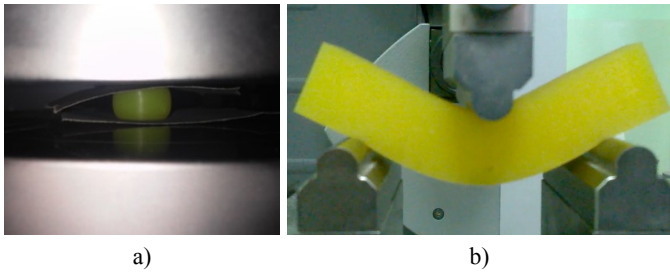


Fig. 4. A view of the elastomeric samples: a) during upsetting process b) during bending process

For the same load cycle as in the process of uniaxial tension done force characteristics, which was then used to analyze the convergence of the results obtained using the selected material model of the results which were obtained by the experiment.

5. Numerical modeling of experimental upsetting, bending and tensile test

In engineering applications during the designing of stamping process using the flexible tools, FEM is widely used. Therefore, in this case to verify the theoretical predictions this method was applied. Numerical simulation upsetting process was performed using MSC MARC/Mentat 2014 applied to analyze nonlinear and contact issues. The numerical model (Fig. 5a) was constructed based on the experimental model. Due to the usage of the sandpaper, between the sample surface and a tool – the “glue” contact has been applied. The sample model was performed the discretization on 32832 hex8 finite elements of type 84 [10]. The sample in the simulation was upset by three-quarters of its height as in the experiment. The deformation during the upsetting of elastomeric sample is calculated from:

$$\varepsilon_h = \left(1 - \frac{h_1}{h_0}\right) 100\% = \frac{s}{h_0} 100\% \quad (23)$$

The result was $\varepsilon_h = 65\%$, where: h_1 – height of the sample after deformation, h_0 – the initial sample height, s – movement of the tool upsetting.

The numerical model of bending process was created based on an experimental test of the three-point bending process for elastomer sample with a hardness of 90 ShA. Due to the symmetry plane (along and across the sample) the numerical model was constructed for the 1/4 sample. The model of parallelepipedic sample was discretisation on 17420 hex8 finite elements of type 84 [10] (Fig 5b). Friction coefficient equal to 0.25 was established in contact plane between the sample and supports. For such a constructed model simulation of experimental bending test was performed.

The numerical model of tensile test was created based on an experimental uniaxial tensile test for elastomeric sample. Due to the symmetry planes along and across the sample the numerical model was constructed for the 1/4 sample. The number of elements hex8 type 84 in the model has been applied.

Then on the basis of numerical calculations of upsetting, bending and tensile tests, force characteristics were prepared for all nine tested material models.

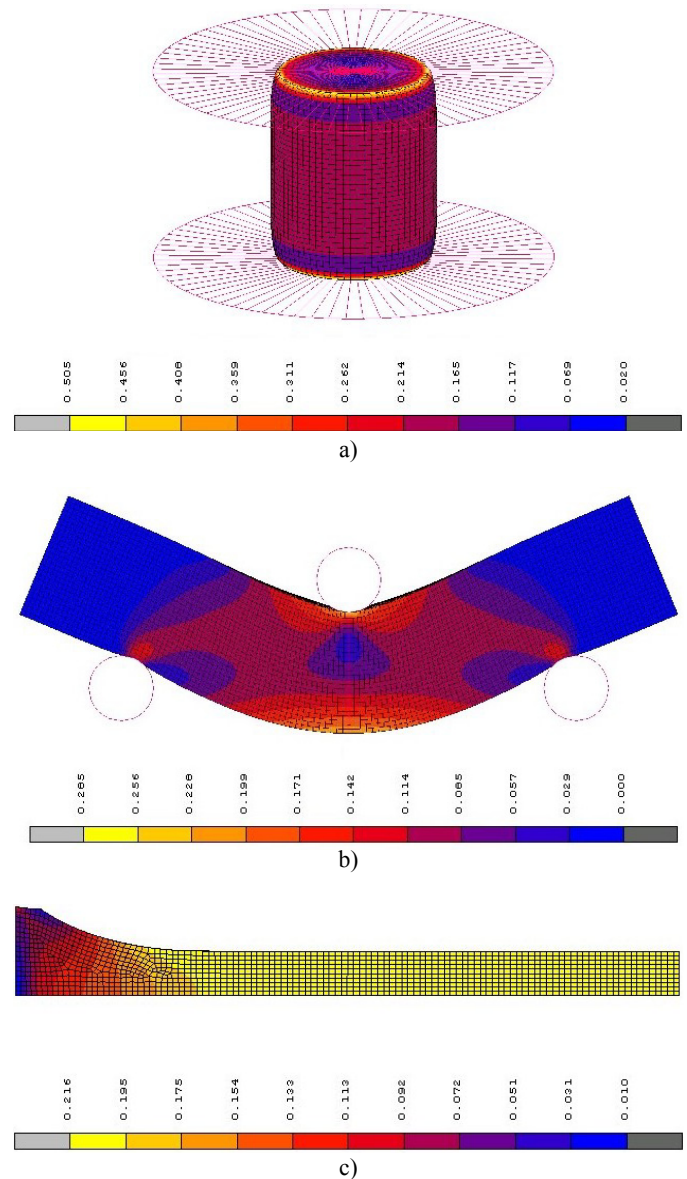


Fig. 5. Exemplary result of FEM calculations – total equivalent strain, a) upsetting process, b) bending process, c) uniaxial tensile test

6. Analysis of convergence of numerical simulation with the experiment

In order to analyze the convergence of the results with numerical calculations performed an experiment, the graphs were created (upsetting force as a function of engineering strain, bending force as a function of punch displacement, uniaxial tensile force as a function of punch displacement) for nine various material models as well as experimental data (Fig. 6-8).

Analyzing upsetting test (Fig. 6), efficacy of models are carried out in two categories. The first concerns the material models that are characterized by an almost complete convergence results in the specified range of deformation. As $\varepsilon_h = \text{range } (0 \text{ to } \dots)$

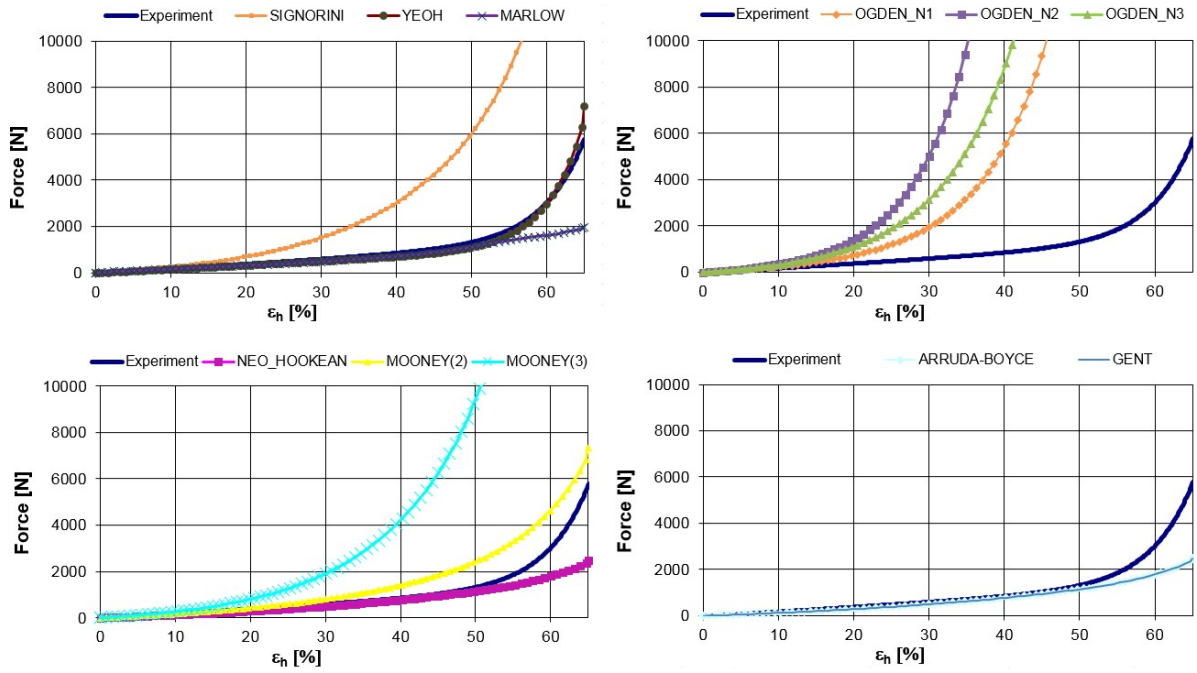


Fig. 6. Experimental and calculated characteristic of the upsetting force as a function of engineering strain for nine selected material models

17%) was the most preferred Mooney model (2) to give in this area an almost complete convergence of the results.

Then, this condition is fulfilled in order: Marlow $\epsilon_h = (0\div 14)\%$, Yeoh $\epsilon_h = (0\div 12)\%$, Neo Hookean $\epsilon_h = (0\div 6)\%$ and Gent $\epsilon_h = (0\div 5)\%$. Other models of this condition are met with little value $\epsilon_h < 2\%$. The second category concerns the comparison of the model with the greatest convergence as far as possible deformations. The second condition is met best by the consecutively models: Yeoh $\epsilon_h = (0\div 62)\%$, Neo Hooken, Arrude Boyce

and Gent $\epsilon_h = (0\div 45)\%$. Other models, especially models Ogden, Singniorini and Mooney (3), have shown a large discrepancy simulation results with the performed experiment.

When analyzing the bending test (Fig. 7) it can be concluded that the appropriate selection of the elastomeric material model in the numerical simulation of the process has an impact on the results for the simulation results: this is important information for modeling of real processes, in which the elastomer tool is loaded mainly to bending.

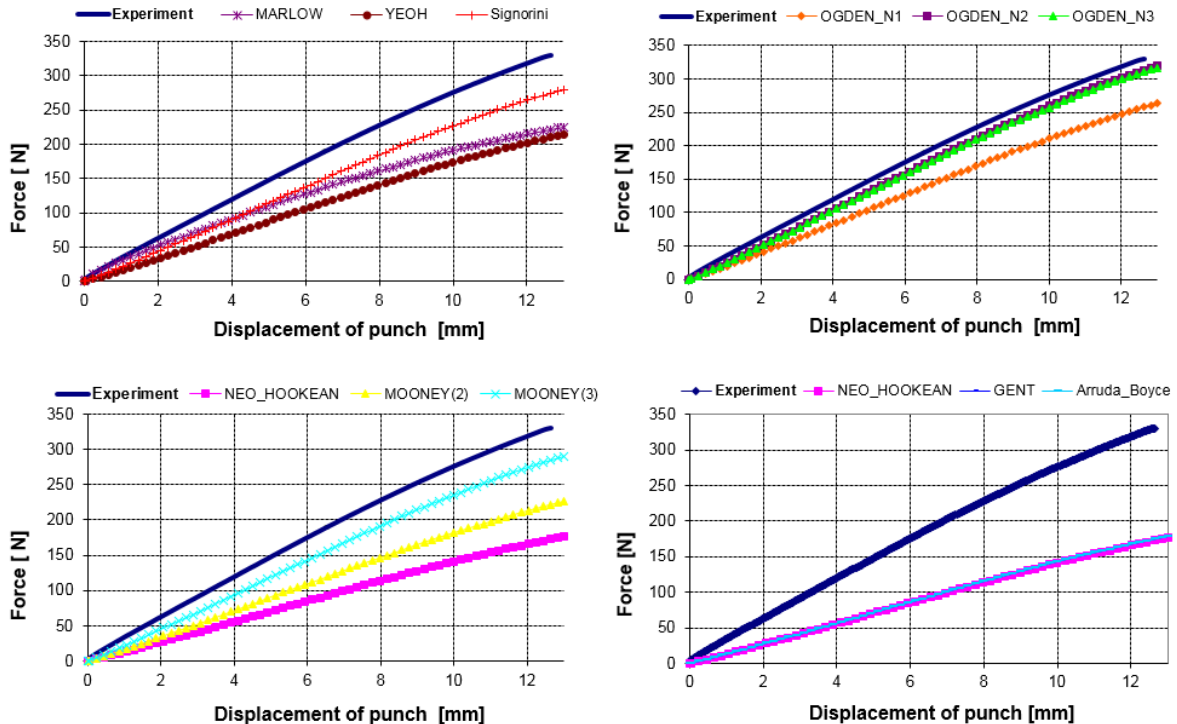


Fig. 7. Experimental and calculated characteristic of the bending force as a function of punch displacement for nine selected material models

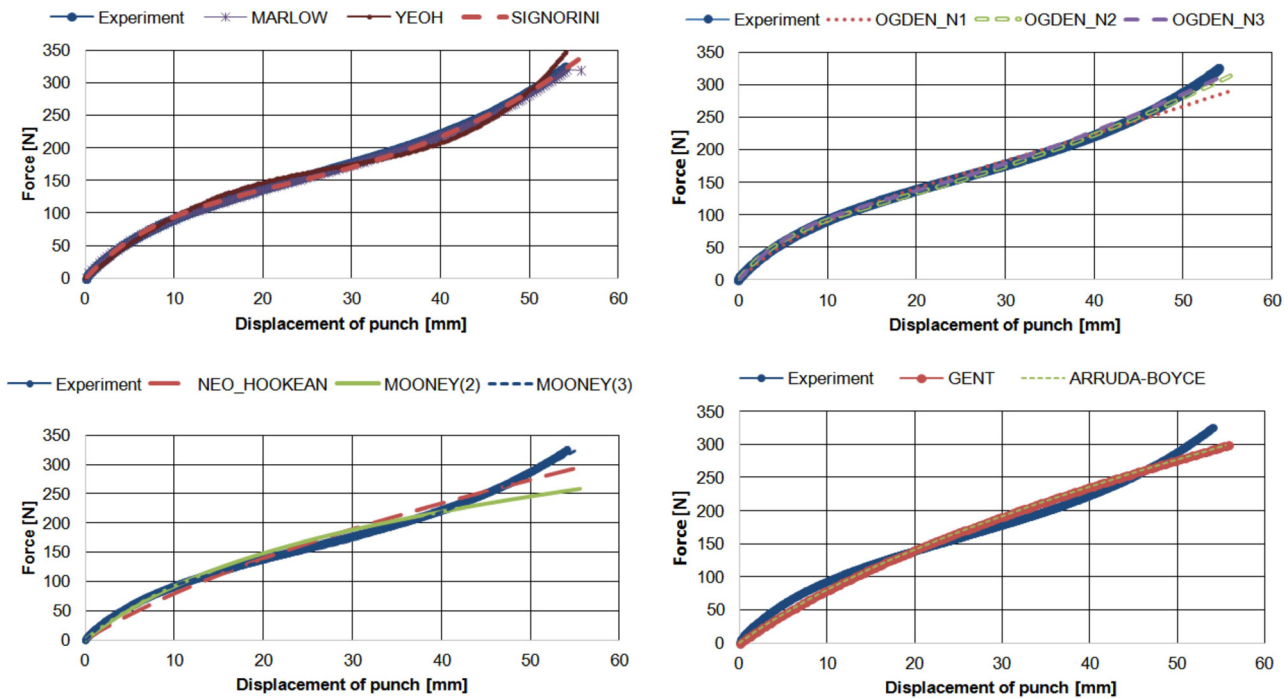


Fig. 8. Experimental and calculated characteristic of the tensile test force as a function of punch displacement for nine selected material models

None of the analyzed models indicate a high convergence of results from an bending experiment. When comparing the FEM results in the entire range of punch movement (0 to 13 mm), model Ogden N2 was closest to the experimental results of the bending process, but unfortunately, showing significant differences between the both results.

Analyzing the characteristics of the experimental and numerical tensile test it can be seen that much more material models have good convergence in this test as compared to the compression test and the bending test results. This is caused by the fact that the material constants are determined in uniaxial tension test, therefore all the models should show a high similarity with the experimental results of the tension test.

When comparing the FEM results in the entire range of punch movement (0 to 55 mm), models Signorini, Marlow, Ogden(N3) and Mooney(3) were closest to the experimental results (Fig. 8). Arruda-Boyce and Gent have the lowest convergence over the range of punch movement. Model Mooney(2) has shown the largest differences between the both results for large values of punch displacement.

7. Conclusions

The results of the comparative analysis of numerical modeling and experimental upsetting process, bending process and tension test can show that the convergence in the calculations is significantly affected by selection of the appropriate material model and depends on the degree of material's deformation.

In the upsetting process the research has shown that the convergence of the results is highly dependable on the degree of

the upset, expressed amount of deformation ε_h . Therefore, when modeling the technological issues with the usage of the flexible tools, it is necessary to take the material model into account, whose convergence with the experiment is satisfactory at least for the distortion occurring in the actual process.

The research has shown that, for modeling of the elastomeric cylinder upsetting in the range of deformation of 62%, the best results with the comparison of the experiment were obtained by using the Yeoh model. Based on the calculations performed it can be concluded that setting the constants in the elastomers material model, based solely on the one material test, (e.g. uniaxial tension test) can give satisfactory results. But the key is to choose the appropriate model material for the actual amount and type of deformation.

In the bending process none of the analyzed models indicate a high convergence of results from an experiment. When comparing the FEM results in the entire range of punch movement (0 to 13 mm), model Ogden N2 was closest to the experimental results of the bending process, but unfortunately, showing significant differences between the both results. Marlow model only for small punch displacements (0 to 1 mm) showed the most similar results compared to the other analyzed material models. The results of other models differ significantly from the values obtained during the experimental bending process of elastomeric sample.

The research has shown that for modeling the bending process, regardless of the material model has been applied to the calculation process, the material constants determined in only one test is insufficient to achieve a satisfactory convergence of the numerical and experimental results.

Analyzing the characteristics of the experimental and numerical tensile test it can be seen that much more material

models has good convergence in this test as compared to the compression test and the bending test results. When comparing the FEM results in the entire range of punch movement (0 to 55 mm), models Signorini, Marlow, Ogden(N3) and Mooney(3) give the best results.

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