TLM-method for Computer Modeling
of Transformers' Windings Frequency Response

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The diagnostics of power transformers is a very fast developing branch, due to increasing average age of assets and changes in asset management strategies, nowadays companies introduce asset management based on technical condition. One of important methods used for diagnostics of a transformer's active part is Frequency Response Analysis (FRA). It allows determination of mechanical condition of windings, their displacements, deformations and electric faults, as well as some problems with internal leads and connections, core and bushings. For the aim of windings impedance modeling the transmission-line models are applied. The idea of TLM was presented in [1]. While the length of transformer wires is comparable to wave length at high frequency, such approach is reasonable. The paper presents examples of models created for simple windings.

1. Introduction

The FRA method is based on the analysis of transfer function of windings. The winding can be described by a set of local capacitances, self and mutual inductances and resistances. Every change in winding geometry leads to change of these parameters, therefore the transfer function's shape is also changed. The analysis of FRA results is based on comparison of data presented usually as sine signal damping along frequency spectrum in logarithmic scale. This can be compared to results recorded for given transformer in time intervals, between phases, between twin or sister units or with help of computer models. The first method is optimal, but for most of old transformers there is no fingerprint data available. The next two approaches are usually applied in industrial practice, however they are quite uncertain and may lead to misinterpretations. Each transformer can have differences in FRA curve compared between phases or, if compared to other units, due to constructional differences [2]. Helpful results can be obtained from controlled deformations, but this method cannot be applied in mass scale and generalized [3].

All above lead to introduction of computer models, which may be useful for interpreting changes in FRA charts. Models are usually based on real physical dimensions and properties of transformers. However, it is still difficult to construct

models having frequency response identical to real units and allowing to simulate various defects with similar effect on the FRA curve as real deformation in winding. There are various methods used for transformer modeling, based on lumped or distributed parameters models, FEM methods etc. [3-5].
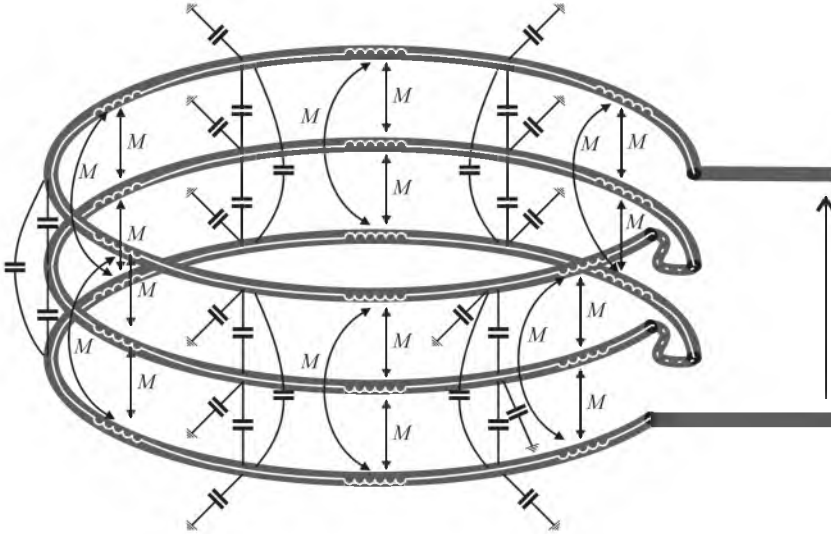

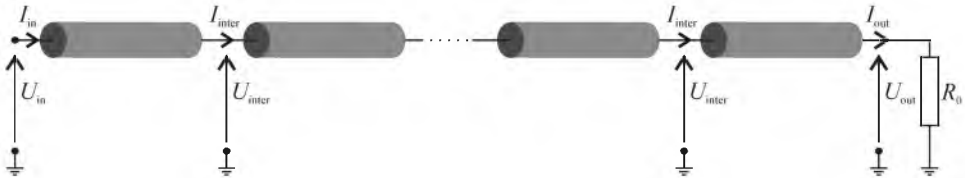Fig. 1. Transmission line model of a simple winding

Fig. 2. Voltages and currents in the transmission-line system

2. Transmission-line models

TLM of a simple winding is shown in Fig.1. It consists of three wires. Each of them is placed at one plane and is replaced by one transmission line. The telegraph equations read

$$\frac{\mathrm{d}U}{\mathrm{d}x} = -Z \cdot I, \quad Z = R + \mathrm{j}\omega L \left[\frac{\Omega}{\mathrm{m}}\right]$$

$$\frac{\mathrm{d}I}{\mathrm{d}x} = -Y \cdot U, \quad Y = \mathrm{j}\omega C \left[\frac{\mathrm{S}}{\mathrm{m}}\right].$$

(1)

Z and *Y* mean the matrices of own and mutual impedances and admittances per unit

length for the whole winding. As long as (1) presents the system of coupled differential equation, the analytical solution is difficult or even not possible. The method proposed by de Gersem et all in [1] is based on decomposition of Z and Y to diagonal form

$$Z = Q_u \cdot Z_d \cdot Q_i^{-1} , \quad Y = Q_i \cdot Y_d \cdot Q_u^{-1} \tag{2}$$

Authors of [1] propose calculation of Q_i from the decomposition of $Z \cdot Y$, while Q_u is obtained as $Q_u = Z \cdot Q_i$. The consequence of this assumption is always unitary matrix for Z_d, because $Z_d = Q_u^{-1} \cdot Z \cdot Q_i = [1]$. Our proposal is to calculate both, Q_i and Q_u utilizing eigendecomposition of:

$$\begin{aligned} Z \cdot Y &= Q_u \cdot Z_d \cdot Y_d \cdot Q_u^{-1} \\ Y \cdot Z &= Q_i \cdot Y_d \cdot Z_d \cdot Q_i^{-1} . \end{aligned} \tag{3}$$

When taking into account uncoupled model, owing matrices Z and Y, which are from the beginning diagonal, results in no corrections introduced to the both matrices.

This decomposition allows for decoupling of telegraph equations

$$\frac{dU}{dx} = -Q_u \cdot Z_d \cdot Q_i^{-1} \cdot I \quad \rightarrow \quad Q_u^{-1} \cdot \frac{dU}{dx} = -Z_d \cdot Q_i^{-1} \cdot i \quad \rightarrow$$

$$\frac{dU_m}{dx} = -Z_d \cdot I_m ,$$

$$\frac{dI}{dx} = -Q_i \cdot Y_d \cdot Q_u^{-1} \cdot U \quad \rightarrow \quad Q_i^{-1} \cdot \frac{dI}{dx} = -Y_d \cdot Q_u^{-1} \cdot u \quad \rightarrow \tag{4}$$

$$\frac{dI_m}{dx} = -Y_d \cdot U_m .$$

The solution of equations (4) occurs in the usual manner and the chain matrix for one of transmission lines is

$$A_{m,k} = \begin{bmatrix} \mathrm{ch}\beta_k l & Z_{c,k} \cdot \mathrm{sh}\beta_k l \\ \dfrac{1}{Z_{c,k}} \mathrm{sh}\beta_k l & \mathrm{ch}\beta_k l \end{bmatrix} = \begin{bmatrix} A_{m,uu,k} & A_{m,ui,k} \\ A_{m,iu,k} & A_{m,ii,k} \end{bmatrix} \tag{5}$$

with: $\beta_k = \sqrt{Z_{dk,k} \cdot Y_{dk,k}}$, $\quad Z_{c,k} = \sqrt{\dfrac{Z_{dk,k}}{Y_{dk,k}}}$.

The single wires should now to be interconnected, building the whole winding. The matrix for the whole system is assumed to be

$$\begin{bmatrix} U_m \\ I_m \end{bmatrix} = \begin{bmatrix} A_{m,uu} & A_{m,ui} \\ A_{m,iu} & A_{m,ii} \end{bmatrix} \begin{bmatrix} U_m(0) \\ I_m(0) \end{bmatrix} , \tag{6}$$

where: $A_{m,xx}$ are diagonal matrices containing appropriate terms taken from single element matrix (5) and $U_m(0)$, $I_m(0)$ denote values at the beginning of wires. Substituting the decomposed voltages and currents by its originals

$$\begin{bmatrix} \boldsymbol{Q}_{\mathrm{u}}^{-1} \cdot \boldsymbol{U} \\ \boldsymbol{Q}_{\mathrm{i}}^{-1} \cdot \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\mathrm{m,uu}} & \boldsymbol{A}_{\mathrm{m,ui}} \\ \boldsymbol{A}_{\mathrm{m,iu}} & \boldsymbol{A}_{\mathrm{m,ii}} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}_{\mathrm{u}}^{-1} \cdot \boldsymbol{U}(0) \\ \boldsymbol{Q}_{\mathrm{i}}^{-1} \cdot \boldsymbol{I}(0) \end{bmatrix} \tag{7}$$

the equations system for the whole system is obtained:

$$\begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Q}_{\mathrm{u}} \cdot \boldsymbol{A}_{\mathrm{m,uu}} \cdot \boldsymbol{Q}_{\mathrm{u}}^{-1} & \boldsymbol{Q}_{\mathrm{u}} \cdot \boldsymbol{A}_{\mathrm{m,ui}} \cdot \boldsymbol{Q}_{\mathrm{i}}^{-1} \\ \boldsymbol{Q}_{\mathrm{i}} \cdot \boldsymbol{A}_{\mathrm{m,iu}} \cdot \boldsymbol{Q}_{\mathrm{u}}^{-1} & \boldsymbol{Q}_{\mathrm{i}} \cdot \boldsymbol{A}_{\mathrm{m,ii}} \cdot \boldsymbol{Q}_{\mathrm{i}}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}(0) \\ \boldsymbol{I}(0) \end{bmatrix},$$

$$\begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\mathrm{uu}} & \boldsymbol{A}_{\mathrm{ui}} \\ \boldsymbol{A}_{\mathrm{iu}} & \boldsymbol{A}_{\mathrm{ii}} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}(0) \\ \boldsymbol{I}(0) \end{bmatrix}. \tag{8}$$

According to Fig. 2 the voltages and currents are subdivided on

$$\begin{bmatrix} U_{\mathrm{in}} \\ \boldsymbol{U}_{\mathrm{inter}} \\ I_{\mathrm{in}} \\ \boldsymbol{I}_{\mathrm{inter}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\mathrm{uu}} & \boldsymbol{A}_{\mathrm{ui}} \\ \boldsymbol{A}_{\mathrm{iu}} & \boldsymbol{A}_{\mathrm{ii}} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{\mathrm{inter}} \\ U_{\mathrm{out}} \\ \boldsymbol{I}_{\mathrm{inter}} \\ I_{\mathrm{out}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\mathrm{uu}} & \boldsymbol{A}_{\mathrm{ui}} \\ \boldsymbol{A}_{\mathrm{iu}} & \boldsymbol{A}_{\mathrm{ii}} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{\mathrm{inter}} \\ U_{\mathrm{out}} \\ \boldsymbol{I}_{\mathrm{inter}} \\ \dfrac{U_{\mathrm{out}}}{R_0} \end{bmatrix}. \tag{9}$$

For the calculation of transfer function only U_{in} and U_{out} are necessary. For this aim the equations (9) are transformed to

$$\begin{bmatrix} U_{\mathrm{in}} \\ \boldsymbol{U}_{\mathrm{inter}} \\ I_{\mathrm{in}} \\ \boldsymbol{I}_{\mathrm{inter}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\mathrm{uu,1}} & \boldsymbol{A}_{\mathrm{uu,1,n}} & \boldsymbol{A}_{\mathrm{ui,1}} & \boldsymbol{A}_{\mathrm{ui,1,n}} \\ \hat{\boldsymbol{A}}_{\mathrm{uu}} & \boldsymbol{A}_{\mathrm{uu,n}} & \hat{\boldsymbol{A}}_{\mathrm{ui}} & \boldsymbol{A}_{\mathrm{ui,n}} \\ \boldsymbol{A}_{\mathrm{iu,1}} & \boldsymbol{A}_{\mathrm{iu,1,n}} & \boldsymbol{A}_{\mathrm{ii,1}} & \boldsymbol{A}_{\mathrm{ii,1,n}} \\ \hat{\boldsymbol{A}}_{\mathrm{iu}} & \boldsymbol{A}_{\mathrm{iu,n}} & \hat{\boldsymbol{A}}_{\mathrm{ii}} & \boldsymbol{A}_{\mathrm{ii,n}} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{\mathrm{inter}} \\ U_{\mathrm{out}} \\ \boldsymbol{I}_{\mathrm{inter}} \\ \dfrac{U_{\mathrm{out}}}{R_0} \end{bmatrix} \tag{10}$$

and finally to the system of linear equations to be solved:

Fig. 3. Structure of the matrices in equations system (10), (11). n – number of wires
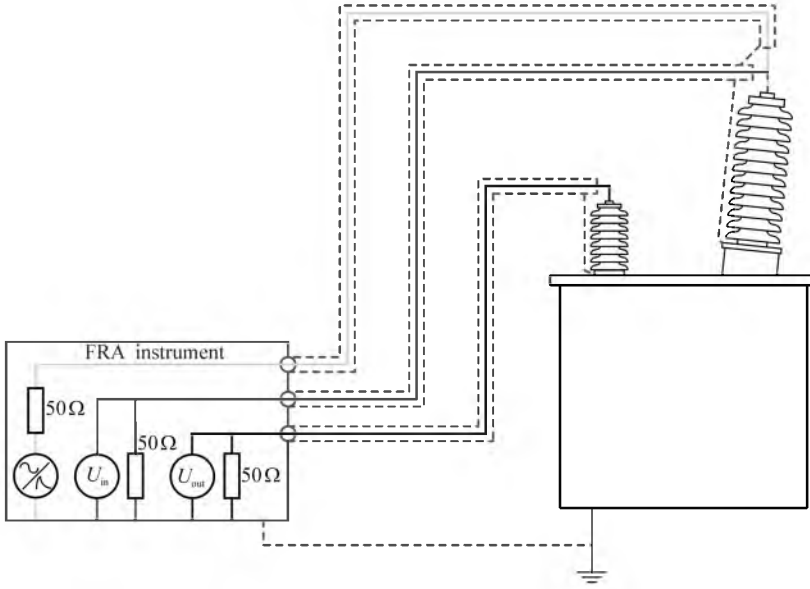
Fig. 4. FRA measurement system

$$
\begin{bmatrix} 0 \\ 0 \\ I_{\text{in}} \\ 0 \end{bmatrix} =
\begin{bmatrix}
A_{\text{uu},1} & A_{\text{uu},1,n}+\dfrac{A_{\text{ui},1,n}}{R_0} & A_{\text{ui},1} & -1 \\[2mm]
\hat{A}_{\text{uu}}-1 & A_{\text{uu},n}+\dfrac{A_{\text{ui},n}}{R_0} & \hat{A}_{\text{ui}} & 0 \\[2mm]
A_{\text{iu},1} & A_{\text{iu},1,n}+\dfrac{A_{\text{ii},1,n}}{R_0} & A_{\text{ii},1} & 0 \\[2mm]
\hat{A}_{\text{iu}} & A_{\text{iu},n}+\dfrac{A_{\text{ii},n}}{R_0} & \hat{A}_{\text{ii}}-1 & 0
\end{bmatrix}
\begin{bmatrix} U_{\text{inter}} \\ U_{\text{out}} \\ I_{\text{inter}} \\ U_{\text{in}} \end{bmatrix}.
\tag{11}
$$

Structure of the matrix in (10) and (11) is shown in Fig. 3.

3. FRA Measurements

At this stage of FRA development results of measurements are given as damping of windings to sine signal in frequency range, according to formula:

$$
FRA[\text{dB}]=20\log\left|K_{\text{u}}\left(f\right)\right|=20\log\frac{\left|U_{\text{out}}\right|}{\left|U_{\text{in}}\right|}
\tag{12}
$$

where: U_{in} – voltage signal applied to winding, U_{out} – voltage signal measured.

The measurement can be taken either on two ends of the winding (with secondary winding open or shortened and grounded) or between windings of the same phase. The typical measuring system is shown in Fig. 4.

4. Exemplary solution

For the testing of described algorithms a simple coil consisting of three wires is assumed. The coil diameter was 23cm, and its electric parameters were calculated using approximate formulas. The following matrices were received:

$$R = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix} m\Omega, \quad C = \begin{bmatrix} 1000 & -700 & -300 \\ -700 & 1000 & -700 \\ -300 & -700 & 1000 \end{bmatrix} pF,$$

$$L = \begin{bmatrix} 0,1 & 0,09 & 0,05 \\ 0,09 & 0,1 & 0,09 \\ 0,05 & 0,09 & 0,1 \end{bmatrix} mH .$$

(13)
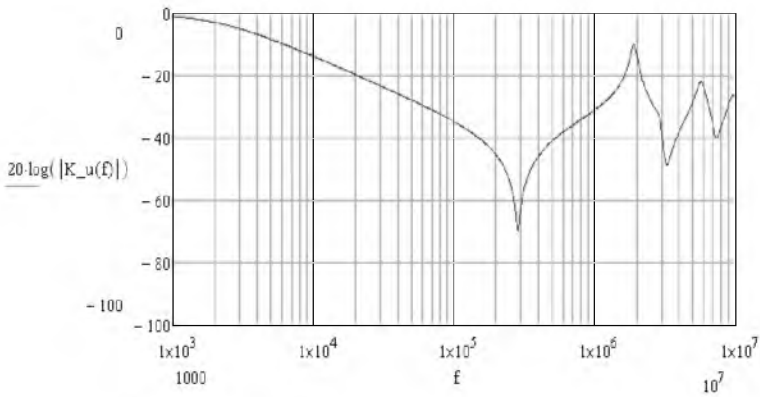
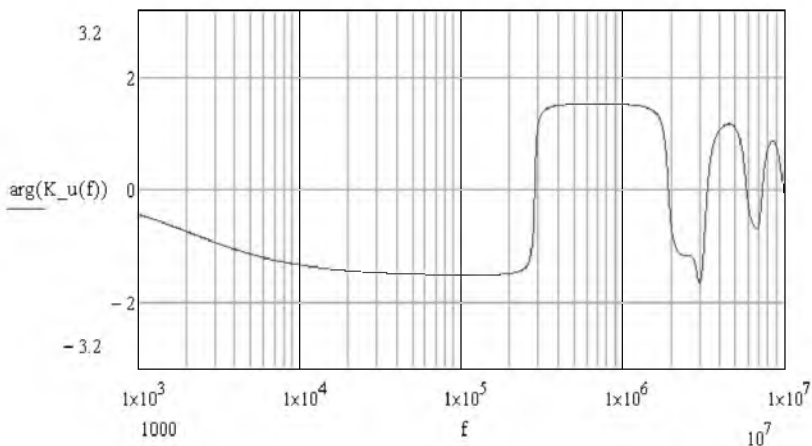Fig. 5. Module of transfer function (in dB) versus frequency

Fig. 6. Phase of transfer function (radians) versus frequency

Due to the skin-effect the resistance of wires is assumed to be linear function increasing with frequency. The input resistance of FRA-instruments is 50 Ω. Test solution was carried out utilizing Mathcad build-in functions for eigenvector evaluation in Schur' decomposition (3). The solution of linear equations (11) was also done by Mathcad. The obtained transfer function in shown in Figs. 5, 6. It reveals characteristic resonances by 200 kHz and 2 MHz, and less by higher frequencies.

5. Solution for real winding (18 wires)

In the following the TLM-solution for one disc of real 800 kVA transformer consisting of 18 wires is shown. The R, L, C values were derived from ANSOFT Maxwell solving electromagnetic field model for R, L and electric model for capacitances C. The current density inside wires by feeding frequency of 1 kHz shows Fig. 7. One can observe distinct skin-effect increasing the resistance of wires. Similar calculations were done for the frequency range of 100 Hz-10 MHz.
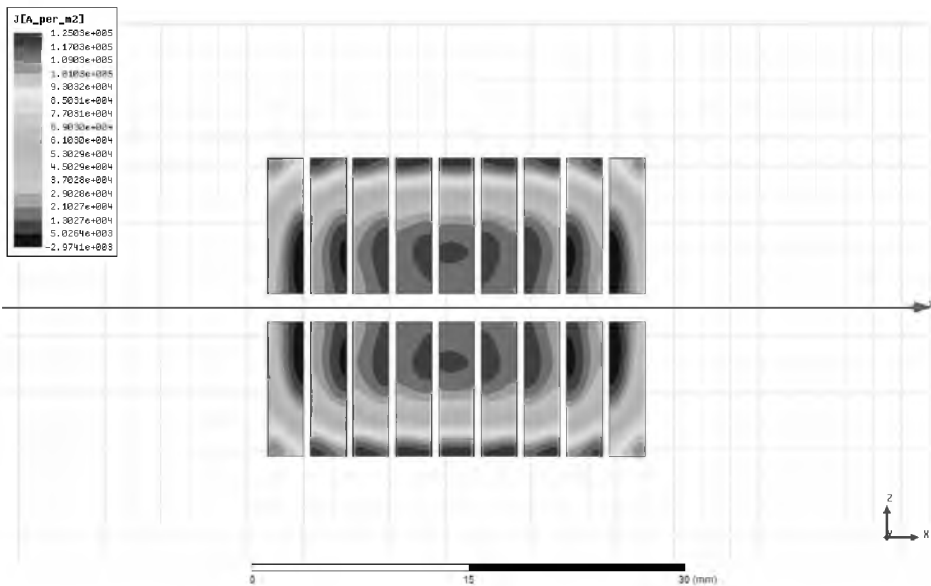

Fig. 7. Current density inside wires by 1kHz

The calculated transfer function was compared to this measured directly on the winding. Screen-copy from FRA-instrument with the simulation results drawn within is shown on the Fig. 8.
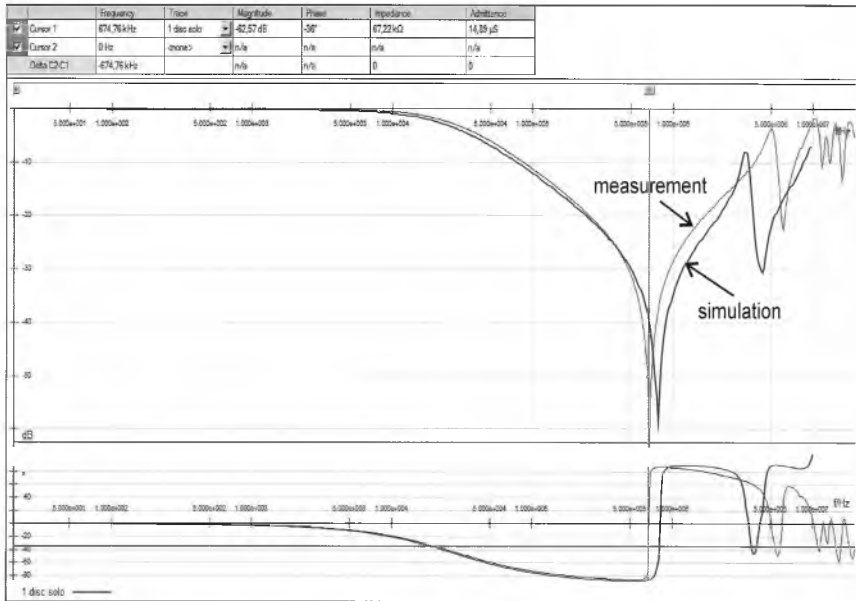
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Fig. 8. Module and phase of transfer function measured and calculated using TLM

6. Summary and future work

FRA method is still developing, at present measuring technique is described and the first standard in this topic will be published soon. However, interpretation of results is mostly based on the experience of diagnostic staff and rich library of exemplary cases for comparison. Therefore application of various computer models is very useful, as it allows simulation of different faults in transformer's active part and shown their influence of frequency response spectrum. FRA method is usually applied to power transformers. The described modeling method can be applied to the analysis of such transformers, especially with a large number of turns. Then, the determination of the resistances, capacitances, inductances and mutual inductances can be done using the finite element method. In many cases, the circular-cylindrical symmetry allowing for 2D analysis can be used. However, since the aim of future work is to analyze the deformation of the windings, it will need the full 3D analysis.

References

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