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Terrain surface depressions as a function of cohesion as well as unconfined compressive strength and indirect tensile strength of rocks in terms of modelling the deformation of rock mass in construction in mining areas

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Abstract: The results of approximation of the settlement function of two known theories, i.e. the Knothe-Budryk theory and the Chudek-Stefański theory, have been presented. They concern the effects of under-ground mining on the surface and rock mass. The depression function, determined in these theories with the use of integral formulas, has been approximated to the algebraic form, in such a manner so that it was not necessary to take advantage of the integral calculus, at the same time taking into account the mean geomechanical values of the rock mass. Empirical data obtained from 34 types of rocks acquired from 16 drilling and research holes of the same rock mass allowed creating a correlation between the radius of the main influences and cohesion, unconfined compressive and indirect tensile strength of rocks, as well as the mean volumetric weight of rock mass. The obtained mathematical dependencies allow calculating the depression of land surface on the basis of the known geometry of excavations as well as their depth of deposition, and the already mentioned rock mass parameters. The differences between the results obtained from integral formulas and the obtained approximation formulas have been compared. The presented solutions are used in modelling land surface de-formations due to underground exploitation of deposits resulting from the needs of conducting analyses concerning construction in mining areas, without the need to use an integral calculus.

Keywords: numerical modelling, integral and approximate formulas, Knothe-Budryk theory, Chudek-Stefański theory, construction in mining areas

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Introduction

In engineering practice, geomechanical parameters such as: cohesion (c), water saturation strength (R_{cn}), unconfined compressive strength (R_c) indirect tensile strength (R_r), and Young's modulus of elasticity (E), are commonly used to assess rock masses, and their values are provided in the unit of pressure, with MPa being used in practice. Correlation relations between the discussed parameters lead to mean values (Borecki, 1980; Chudek & Stefański, 1987; Knothe, 1984; Kwiatek 1998; Paleczek, 2007; 2013; 2020a; 2020b; 2021). Such mean values could characterize a rock mass in a given region. In works (Paleczek, 2013; 2020a; 2020b; 2021), they have been presented as follows: R_{cn} = 11.11 MPa, R_r = 2.00 MPa, c = 3.33 MPa, R_c = 25.00 MPa, E = 4630.00 MPa. Mean volumetric weight of rocks: γ= 0.02 MPa/m.

The cited values entered into proper empirical and approximating formulas give the same approximate result (Figs. 1 and 2). The value of the theory parameter referred to as the radius of the range of main influences r has been determined in the Knothe-Budryk theory with formulas (1), while in the Chudek-Stefański theory with formulas (2) (Knothe, 1984; Chudek & Stefański, 1987; Borecki, 1980; Paleczek, 2007). The formulas (3) constitute the proposed approximation solution in the function of five variables, which have been determined on the basis of empirical data analysis (Paleczek, 2007). At this point, it can also be mentioned that the value of the parameter $tg(\beta)$ is also used to determine protective pillars (Borecki, 1980).

$$r(H,\beta) = \frac{H}{tg(\beta)} \Rightarrow tg(\beta) = \frac{H}{r} \tag{1}$$

$$r(H,R_r,\gamma) = \sqrt{\frac{H \cdot R_r}{\gamma}} \Rightarrow tg(\beta) = \sqrt{\frac{H \cdot \gamma}{R_r}} \tag{2}$$

$$\begin{gathered} r(H,\gamma,c,R_c,R_r) = \frac{\sqrt{3}}{15}\sqrt{\frac{H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}} \\ \Rightarrow tg(\beta) = 5 \cdot \sqrt{3} \cdot \sqrt{\frac{H \cdot \gamma}{15 \cdot c + 2 \cdot R_c + 25 \cdot R_r}} \end{gathered} \tag{3}$$

1. Formulas used for approximation

In the Knothe-Budryk theory, the depression for the case of an infinite half-plane was determined by the formula (4), and in the spatial task by the formula (5) while maintaining a simplification consisting in the fact that the parameters of the initial compression of the rock mass resulting from drainage w_w as well as the value of the mining periphery d have not been taken into account in the considerations – details on interpreting such simplifications are discussed in the book (Paleczek, 2013):

$$w(x) = \frac{a \cdot g}{r} \cdot \int_{-\infty}^{x} e^{\frac{-\pi \cdot \lambda^2}{r^2}} d\lambda \tag{4}$$

$$w(x,y) = \int_{X_1}^{X_2} \int_{Y_1}^{Y_2} \frac{a \cdot g}{r^2} \cdot e^{\frac{-\pi \cdot [(\xi - y)^2 + (\eta - x)^2]}{r^2}} d\xi d\eta \tag{5}$$

Symbols in formulas (4) and (5):

$w(x), w(x,y)$ – respectively the values of vertical displacements (settlements, depressions) at a point with current coordinates x for the task of "infinite half-plane", while with coordinates x, y in the spatial task, while X_1, X_2, Y_1, Y_2 constitute coordinates of the diagonal of a rectangle that constitutes a single, elementary mining field;

a – mining coefficient (coefficient of filling the post-mining void);

g – thickness of the mined layer;

r – the radius of the range of main influences from the relationship defined by formulas (1), (2), (3), in which the angle β is the angle of the range of main influences, while H constitutes the depth of the mined layer.

After entering the relationship (3) to the formula (4), and at the assumption that $N = \pi$ it was possible to obtain a depression $w(x)$ as a function of eight variables, i.e. $w(x, a, g, H, \gamma, c, R_c, R_r)$, as shown by the formula (6)

$$w(x) = \frac{15 \cdot a \cdot g}{\sqrt{\frac{3H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}}} \cdot \int_{-\infty}^{x} e^{\frac{-15^2 \cdot N \cdot \gamma \cdot \lambda^2}{3 \cdot H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}} d\lambda \tag{6}$$

After entering the relationship (3) to (5), and at the assumption that $N = \pi$, it was possible to obtain a formula for the depression $w(x,y)$ of a single, rectangular mining field, with the coordinates of its diagonal respectively X_1, X_2, Y_1, Y_2 in a function of nine variables, i.e. $w(x, y, a, g, H, \gamma, c, R_c, R_r)$, as shown by the formula (7).

$$w(x,y) = \int_{X_1}^{X_2} \int_{Y_1}^{Y_2} \frac{15^2 \cdot a \cdot g \cdot \gamma}{3 \cdot H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)} \cdot e^{\frac{-15^2 \cdot \gamma \cdot N \cdot [(\xi - y)^2 + (\eta - x)^2]}{3 \cdot H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}} d\xi d\eta \tag{7}$$

Trying to find a solution closed to the equation determined by formula (6), using approximation methods, its first approximation has been obtained. It has been defined with the formula (8) – in the calculations a negative value of the parameter a was assumed, so that the resultant value of settlement was a negative number; an empirical value of the parameter has been $v = 4.25$, see (Paleczek, 2007; 2013; 2020a; 2020b; 2021).

$$w(x) = a \cdot g \cdot \left(1 - \frac{1}{1 + e^{\frac{15 \cdot v \cdot x}{\sqrt{\frac{3 \cdot H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}}}}}\right) \tag{8}$$

It is worth noting that the maximum differences in the results obtained from formula (8) in relation to the integral formula (6) do not exceed the value of ±0.04 m (Paleczek, 2020a; 2020b; 2021). These differences clearly result from the applied approximation function model.

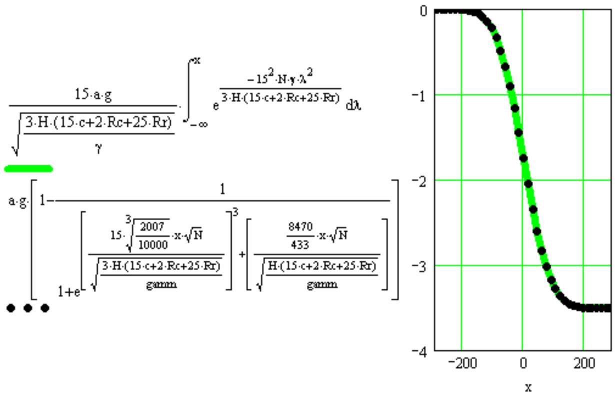
In order to increase the accuracy of the calculations, formula (6) was approximated to the form determined by the formula (9).

$$w(x) = a \cdot g \cdot \left(1 - \frac{1}{1 + e^{\left(\frac{15 \cdot \sqrt[3]{\frac{2007}{10000}} \cdot x \cdot \sqrt{N}}{\sqrt{\frac{3 \cdot H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}}}\right)^3 + \left(\frac{\frac{8470}{433} \cdot x \cdot \sqrt{N}}{\sqrt{\frac{H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}}}\right)}}\right) \tag{9}$$

The maximum differences in results obtained from formulas (6) and (9) do not exceed the values ±0.6 mm (Fig. 2), which for engineering applications, in the tasks of modelling post-mining depressions of the land surface seems to be an acceptable value, because this type of inaccuracies consists of many factors which include, for example, the following: rock mass heterogeneity, the degree of recognizing the rock mass through data from drilling and research holes, the accuracy of laboratory test results when determining the values of geomechanical parameters of rocks from collected, stressed-relived rock mass samples (Paleczek, 2007; 2020a; 2020b; 2021). Figure 1 shows a collective graph of functions resulting respectively from formulas (6) and (9) as well as (6) and (8), while Figure 2 shows the maximum differences in the results obtained respectively from formulas (6) and (9), according to numerical data, as in Figure 1. Whereas, Figure 2 shows values obtained from the formula (10).

$$Q(x) = \left(\frac{15 \cdot a \cdot g}{\sqrt{\frac{3H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}}} \cdot \int_{-\infty}^{x} e^{\frac{-15^2 \cdot N \cdot \gamma \cdot \lambda^2}{3 \cdot H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}} d\lambda\right) - \left(a \cdot g \cdot \left(1 - \frac{1}{1 + e^{\left(\frac{15 \cdot \sqrt[3]{\frac{2007}{10000}} \cdot x \cdot \sqrt{N}}{\sqrt{\frac{3 \cdot H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}}}\right)^3 + \left(\frac{\frac{8470}{433} \cdot x \cdot \sqrt{N}}{\sqrt{\frac{H \cdot (15 \cdot c + 2 \cdot R_c + 25 \cdot R_r)}{\gamma}}}\right)}}\right)\right) \tag{10}$$

a)



b)

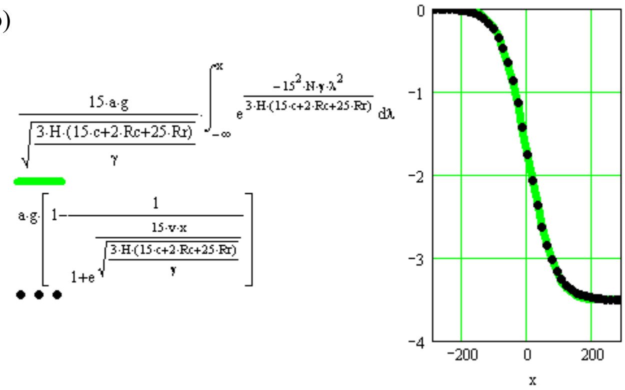


Fig. 1. a) Graphs of functions defined by formulas (6) – marking with a solid line and (9) – with a dotted line; b) graphs of functions defined by formulas (6) – marking with a solid line and (8) – with a dotted line, assuming illustrative values of variables: a = –0.9, ν = 4.25, g = 3.9 m, H = 300 m, c = 3.33 MPa, R_c = 25 MPa, R_r = 2.00 MPa; negative values on axis x determine the area above the undisturbed soil, positive values on axis x determine the area above the post-mining void, values on the vertical axis are the depression values [m]; the case of the infinite half-plane has been simplified here, because the mining periphery d and preliminary settlements resulting from the compression of the rock mass as a result of drainage w_w, (Paleczek, 2007; 2013; 2020a; 2020b) have not been taken into account

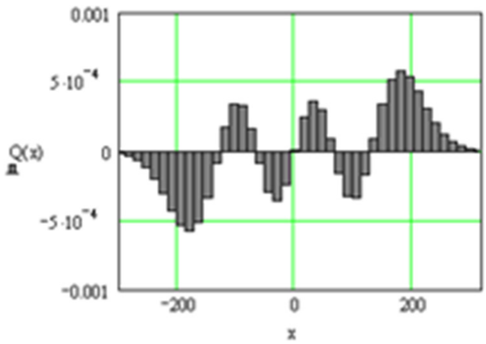


Fig. 2. The graph of the function determined by the formula (10) indicates that the differences between the results obtained from the approximation formula (9) and the integral formula (6) do not exceed the value of $\pm 6 \cdot 10^{-4}$ m = ±0.6 mm

Conclusions

The mathematical formulas obtained from the approximation allow calculating land surface depressions expressed in the theories cited in this paper without the need of using integral calculus. The differences between the results obtained from approximation formulas in relation to integral formulas do not exceed the values of a few tenths of a millimetre, which may be useful for calculating estimated values when analyzing the modelling of land surface depressions resulting from mining bedded deposits. When developing the proposed solutions, efforts have been made to enable the use of commonly available databases of geomechanical parameters of rock mass components for engineering calculations concerning construction in mining areas.

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