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COMPUTATIONAL AND STATISTICAL ASPECTS OF DETERMINING SHIP'S POSITION

ABSTRACT

In its mathematical essence, the task of determining ship's position coordinates, is to minimize appropriately defined goal function. This paper proposes to use the method of conjugate gradient for this purpose. The reason is that calculations may be performed in some seconds time because Microsoft and Apache implemented the conjugate gradient method as a tool called the Solver and embedded this tool in their widely offered and popular spreadsheets, namely Excel and the Open Office Calc, respectively. Further in this paper it is shown how to precisely assess errors of ship's position coordinates with the Monte Carlo method that employs the Solver.

Keywords:
determining ship's position, the Least Square method, conjugate gradient, the Monte Carlo method.

INTRODUCTION

Let us begin with recalling the basics. In its mathematical essence, the task of determining ship's position coordinates ϕ_o, λ_o, is to employ the Least Square method that consists in minimizing the following goal function [Kopacz et al., 2007].

$$S(\varphi_o, \lambda_o) = \sum_{i=1}^{n} \left\{ (\varphi_o - \varphi_i)^2 + \left[(\lambda_o - \lambda_i) \cdot \cos\left(\frac{\varphi_o + \varphi_i}{2} \right) \right]^2 - d_i^{\,2} \right\}^2 , \qquad (1a)$$

where:

n — the number of reference stations each of which produces one summand,

ϕ_i, λ_i — coordinates positions of particular reference stations, $i = 1,2,...,i,...n$,

$d_i = d_{ai} + \varepsilon$ — measured distances to these reference stations,

$i = 1,2,...,i,...n$, d_{ai} — actual distances,

ε — an error of measurement.

This error is assumed to be the random variable with expected value equal to zero and the standard deviation equal to σ in all measurements. This is a strong assumption that is often violated in practice where standard deviations σ_i are unequal. If so, it is recommended to minimize the following weighted goal function

$$S(\varphi_o, \lambda_o) = \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \left\{ (\varphi_o - \varphi_i)^2 + \left[(\lambda_o - \lambda_i) \cdot \cos\left(\frac{\varphi_o + \varphi_i}{2} \right) \right]^2 - d_i^{\,2} \right\}^2 . \quad (1b)$$

It is a pity that values σ_i are unknown. And there is no simple remedy for this.

This paper proposes to use the method of conjugate gradient [Fletcher, Reeves, 1964] to minimize (1). The reason is that all the very long calculations needed will be performed in some seconds time. It is possible because Microsoft and Apache implemented the conjugate gradient method as a tool called the *Solver* and embedded this tool in their widely offered environments, namely *Excel* and *Open Office Calc*, respectively.

Further in this paper it is shown how to precisely assess errors of ship's position coordinates with the Monte Carlo method that employs the Solver. The main aim of this paper is to assess how number of reference stations impacts errors of determining of ship's position.

INPUT DATA

Input data related to reference stations were gathered in Table 1.

Tab. 1. A list of reference stations

i	Location of reference station	Distance to d_i [Nm]	Coordinates φ_i	Coordinates λ_i
1	Hel	6,3894	54° 36,004'	18° 48,768'
2	Gdynia	7,3418	54° 32,018'	18° 32,839'
3	Gdańsk	6,2890	54° 23,986'	18° 41,784'
4	Górki Zachodnie	10,1622	54°22,233'N	18° 46,733'E

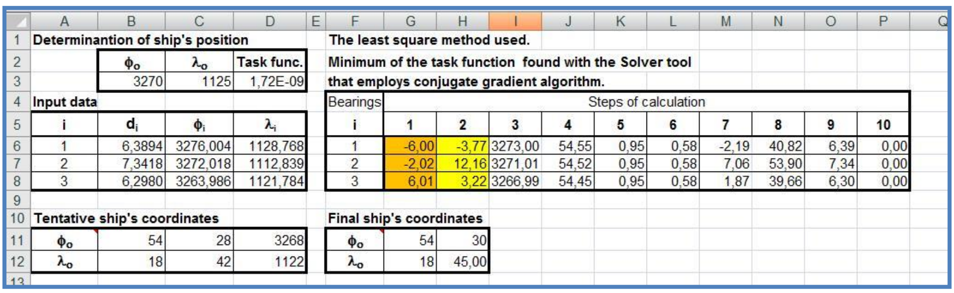
Tab. 2. Actual ship's coordinates

Coordinates $\varphi_o^{(0)}$	Coordinates $\lambda_o^{(0)}$
54° 30,000'	18° 45,000'

USING THE SOLVER

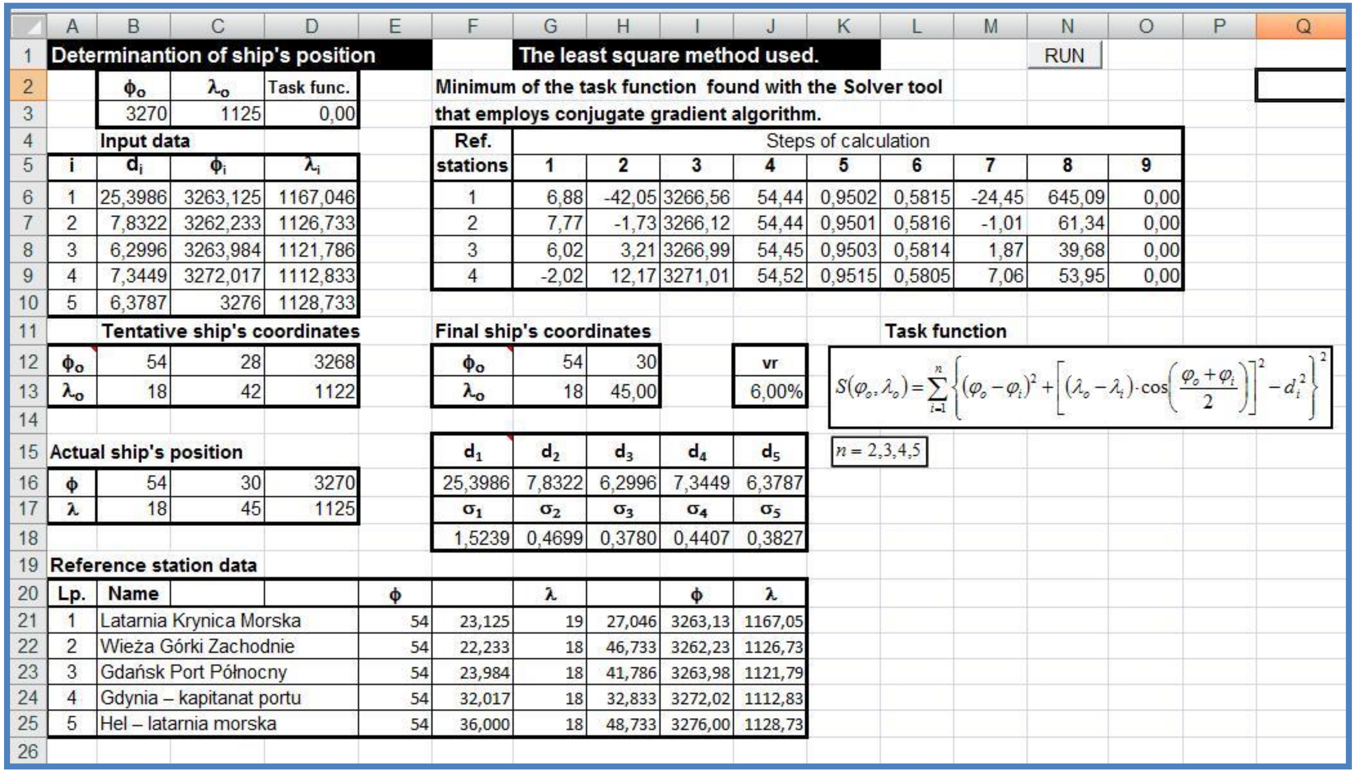
Figure 1a shows how the worksheet is arranged of for single use i.e. not for Monte Carlo simulations. Figure 1b shows haw the worksheet is arranged for multiple use, i.e. for the Monte Carlo simulation. However for its dimension the figure is located at the end of this paper. Figures are intended to enable readers to acquaint with the worksheet. They may turn out insufficient to prepare fully working worksheet by Excel novices.

Figures 2a to 2c show what particular cells of Excel worksheet contain. Figures 3a and 3b instruct how to set up the Solver.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Determinantion of ship's position					The least square method used.										
2		ϕ_o	λ_o	Task func.		Minimum of the task function found with the Solver tool										
3		3270	1125	1,72E-09		that employs conjugate gradient algorithm.										
4	Input data					Bearings	Steps of calculation									
5	i	d_i	ϕ_i	λ_i		i	1	2	3	4	5	6	7	8	9	10
6	1	6,3894	3276,004	1128,768		1	-6,00	-3,77	3273,00	54,55	0,95	0,58	-2,19	40,82	6,39	0,00
7	2	7,3418	3272,018	1112,839		2	-2,02	12,16	3271,01	54,52	0,95	0,58	7,06	53,90	7,34	0,00
8	3	6,2980	3263,986	1121,784		3	6,01	3,22	3266,99	54,45	0,95	0,58	1,87	39,66	6,30	0,00
9																
10	Tentative ship's coordinates					Final ship's coordinates										
11	ϕ_o	54	28	3268		ϕ_o	54	30								
12	λ_o	18	42	1122		λ_o	18	45,00								

Fig. 1a. Arrangement of the worksheet for single use



Determinantion of ship's position — **The least square method used.** — RUN

ϕ_o	λ_o	Task func.
3270	1125	0,00

Minimum of the task function found with the Solver tool that employs conjugate gradient algorithm.

Input data

i	d_i	ϕ_i	λ_i
1	25,3986	3263,125	1167,046
2	7,8322	3262,233	1126,733
3	6,2996	3263,984	1121,786
4	7,3449	3272,017	1112,833
5	6,3787	3276	1128,733

Ref. stations	Steps of calculation 1	2	3	4	5	6	7	8	9
1	6,88	-42,05	3266,56	54,44	0,9502	0,5815	-24,45	645,09	0,00
2	7,77	-1,73	3266,12	54,44	0,9501	0,5816	-1,01	61,34	0,00
3	6,02	3,21	3266,99	54,45	0,9503	0,5814	1,87	39,68	0,00
4	-2,02	12,17	3271,01	54,52	0,9515	0,5805	7,06	53,95	0,00

Tentative ship's coordinates

ϕ_o	54	28	3268
λ_o	18	42	1122

Final ship's coordinates

ϕ_o	54	30
λ_o	18	45,00

vr
6,00%

Task function

$$S(\varphi_o,\lambda_o)=\sum_{i=1}^{n}\left\{(\varphi_o-\varphi_i)^2+\left[(\lambda_o-\lambda_i)\cdot\cos\left(\frac{\varphi_o+\varphi_i}{2}\right)\right]^2-d_i^2\right\}^2$$

$n = 2,3,4,5$

Actual ship's position

ϕ	54	30	3270
λ	18	45	1125

d_1	d_2	d_3	d_4	d_5
25,3986	7,8322	6,2996	7,3449	6,3787
σ_1	σ_2	σ_3	σ_4	σ_5
1,5239	0,4699	0,3780	0,4407	0,3827

Reference station data

Lp.	Name	ϕ		λ		ϕ	λ
1	Latarnia Krynica Morska	54	23,125	19	27,046	3263,13	1167,05
2	Wieża Górki Zachodnie	54	22,233	18	46,733	3262,23	1126,73
3	Gdańsk Port Północny	54	23,984	18	41,786	3263,98	1121,79
4	Gdynia – kapitanat portu	54	32,017	18	32,833	3272,02	1112,83
5	Hel – latarnia morska	54	36,000	18	48,733	3276,00	1128,73

Fig. 1b. A general arrangement of Excel worksheet for Monte Carlo simulations

	A	B	C	D
1	**Determinantion of sh**			
2		**ϕo**	**λo**	**Task func.**
3		3269,99999849389	1124,99999031043	=SUMA.KWADRATÓW
4	**Input data**			
5	**i**	**di**	**ϕi**	**λi**
6	1	6,3894	=54*60+36,004	=18*60+48,768
7	2	7,3418	=54*60+32,018	=18*60+32,839
8	3	6,298	=54*60+23,986	=18*60+41,784
9				
10	**Tentative ship's coo**			
11	**ϕo**	54	28	=60*B11+C11
12	**λo**	18	42	=60*B12+C12
13				

Fig. 2a. The content of cells that comprise input data range

F	G	H	I	J	K
The least square					
Minimum of the ta					
that employs conj					
Bearings					Steps c
i	**1**	**2**	**3**	**4**	**5**
1	=B$3-C6	=C$3-D6	=(B$3+C6)/2	=I6/60	=2*PI()*J6/360
2	=B$3-C7	=C$3-D7	=(B$3+C7)/2	=I7/60	=2*PI()*J7/360
3	=B$3-C8	=C$3-D8	=(B$3+C8)/2	=I8/60	=2*PI()*J8/360
Final ship's coord					
ϕo	=ZAOKR.DO.CA	=MOD(B3;60)			
λo	=ZAOKR.DO.CA	=MOD(C3;60)			

Fig. 2b. The content of cells that comprise the left segment calculation range

L	M	N	O	P
of calculation				
6	**7**	**8**	**9**	**10**
=COS(K6)	=H6*L6	=G6*G6+M6*M6	=PIERWIASTEK(N6)	=B6-O6
=COS(K7)	=H7*L7	=G7*G7+M7*M7	=PIERWIASTEK(N7)	=B7-O7
=COS(K8)	=H8*L8	=G8*G8+M8*M8	=PIERWIASTEK(N8)	=B8-O8

Fig. 2c. The content of cells that comprise the right segment calculation range

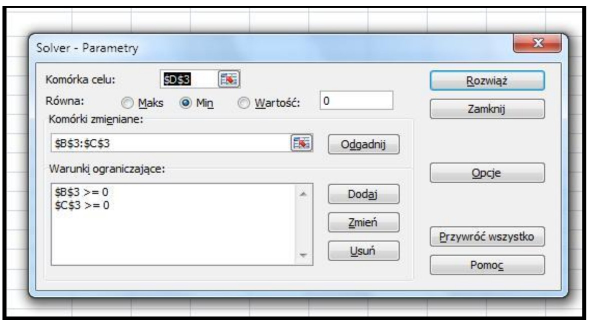


Fig. 3a. Setting the Solver

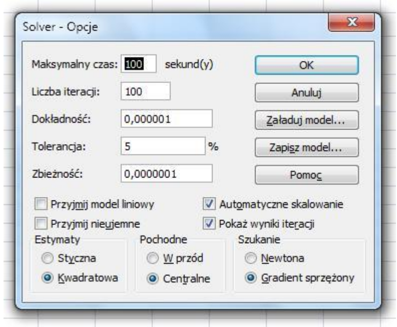


Fig. 3b. Setting Solver options

Figure 4 gives an overview of the goal function having a minimum that has to be localized to determine ship's position. The minimum is situated in a deep valley with steep slopes. It may cause the method to diverge when guesses are taken too far from the solution. A symptom of bad convergence is when a value displayed in the B3 cell is greater than 1E-5. Figure 1a exemplifies a case of good convergence.

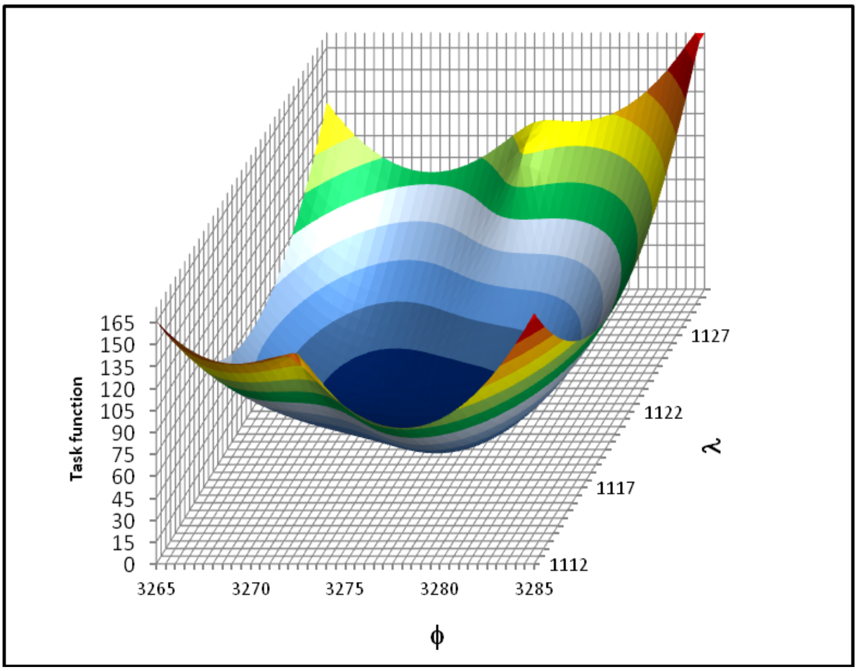


Fig. 4. An overview of the goal function

AN ERROR ASSESSMENT OF SHIP'S POSITION

It is assumed that positions of reference stations are purely deterministic variables i.e. are free of error. In contrast distances to reference stations are assumed to be random variables that follow the Normal distribution. Let us remember that Normal distribution has two parameters: the location parameter and the scale parameter. The location parameters of distance distributions are set equal to exact distances. Two variants of calculating the scale parameter are considered. The scale parameter is equated with σ introduced in Section 1.

Variant I:

The scale parameter was assumed to be a fraction of the scale parameter. This fraction is named variability ratio (vr). In the other words

$$scale\ \ parameter = vr \cdot location\ \ parameter\,, \tag{2a}$$

where $vr = 0.15\%, 0.30\%, ..., 0.75\%$ were taken in accordance with [IALA 2015].

This variant violates the assumption about σ of Section 1, but reflects what we face in common practice.

Variant II:

The scale parameter is constant

$$scale\ \ parameter = 0.05 \cdot sea\ mile\,. \tag{2b}$$

Variant II fulfill the σ — related assumption already considered in Variant I.

Below two VBA (Visual Basic for Application) procedures are presented. The first named Proc01 accomplish the Monte Carlo method. The second named Solver makes Solver to find a minimum of the goal function being repeatedly called by Proc01. Instructions that comprise Proc01 are commented very detailedly. These comments are addressed to novices in practice of the Monte Carlo method.

NEW CONCEPT OF RADAR POSITIONING

```
Sub Proc01()
Dim i As Integer, j As Integer, k As Integer, v As Integer
Dim d0(5) As Single, sd(5) As Single: 'Matrices that contain location
and scale parameters.
```

Dim Ss As Single: 'Adder of uniformly distributed random numbers. Used to obtain normal random numbers.
Randomize Timer: 'This statement causes that seeds for internal generator of uniformly distributed random numbers are formed on the basis of data obtained from computer's internal clock.
Application.ScreenUpdating = False: *'This statement „freezes" a screen until all the whole Monte Carlo procedure will be completed. It considerably shortens realization of this procedure.*
Let d0(1) = Cells(13, 10): Let d0(2) = Cells(13, 11): Let d0(3) = Cells(13, 12): Let d0(4) = Cells(13, 13): Let d0(5) = Cells(13, 14): 'Reading location parameters from the worksheet.
Let sd(1) = Cells(15, 10): Let sd(2) = Cells(15, 11): Let sd(3) = Cells(15, 12): Let sd(4) = Cells(15, 13): Let sd(5) = Cells(15, 14): Reading scale parameters from the worksheet.
Range("B21:C1020").Select: Selection.ClearContents: *'Clears block of cells that is a container of Monte Carlo results.* *For v = 1 To 1000:* 'Looping over subsequent simulations.
Application.StatusBar = v: *Shows simulation number I the status bar.*
Let Cells(v + 20, 1) = v: *'Writes in subsequent simulation number to the worksheet.*
For i = 1 To 5: *'Looping over reference stations.*
Let Ss = -6: *'Sets an initial value to the adder (see relevant comment in declarations)*
For j = 1 To 12: *'Looping over summands that form Normal random number.*
Let Ss = Ss + Rnd(1): *'Subsequent uniformly distributed random number added.*
Next j
'Now Ss contains the N(0,1) normal random number i.e. with location parameter equal to zero and scale parameter equal to one.
*Let Cells(5 + i, 2).Value = sd(i) * Ss + d0(i):* *'Converting N(0,1) into Normal random numbers having location parameters equal to actual distances and scale parameters equal to fractions stated above in the main text.*
Next i
Let Cells(3, 2) = Cells(16, 8).Value: Cells(3, 3) = Cells(17, 8).Value: *Setting "guesses" i.e. values from which Solver will start looking for a minimum of the goal function.*
SOLVER: *'Calling the Solver procedure.*
'Solver placed guesses with results
Let Cells(v + 20, 2).Value = Cells(3, 2).Value : *Results are transferred to the container of Monte Carlo results.*
Let Cells(v + 20, 3).Value = Cells(3, 3).Value
Next v
'The Monte Carlo procedure is ended.
'The set of instructions below sorts results in ascending order.
Range("B21:C1020").Select

```
ActiveWorkbook.Worksheets("Monte_Carlo_3").Sort.SortFields.Clear
ActiveWorkbook.Worksheets("Monte_Carlo_3").Sort.SortFields.Add Key:=Range( _
    "B21:B1020"), SortOn:=xlSortOnValues, Order:=xlAscending, DataOption:= _
    xlSortNormal
  With ActiveWorkbook.Worksheets("Monte_Carlo_3").Sort
    .SetRange Range("B21:C1020")
    .Header = xlGuess
    .MatchCase = False
    .Orientation = xlTopToBottom
    .SortMethod = xlPinYin
    .Apply
  End With
Application.ScreenUpdating = True
End Sub

Sub SOLVER()
Dim wynik As Long
SolverOptions MaxTime:=100, Iterations:=200, Precision:=0.00000001, _
AssumeLinear:=False, StepThru:=False, Estimates:=2, Derivatives:=2, _
SearchOption:=2, IntTolerance:=5, Scaling:=True, Convergence:=0.0001, _
AssumeNonNeg:=False
SolverOk SetCell:="$D$3", MaxMinVal:=2, ValueOf:="1", ByChange:="$B$3:$C$3"
Let wynik = SolverSolve(True)
End Sub
```

Figures 5a and 5b and relates to Variant I. Figure 6 relates to Variant II.

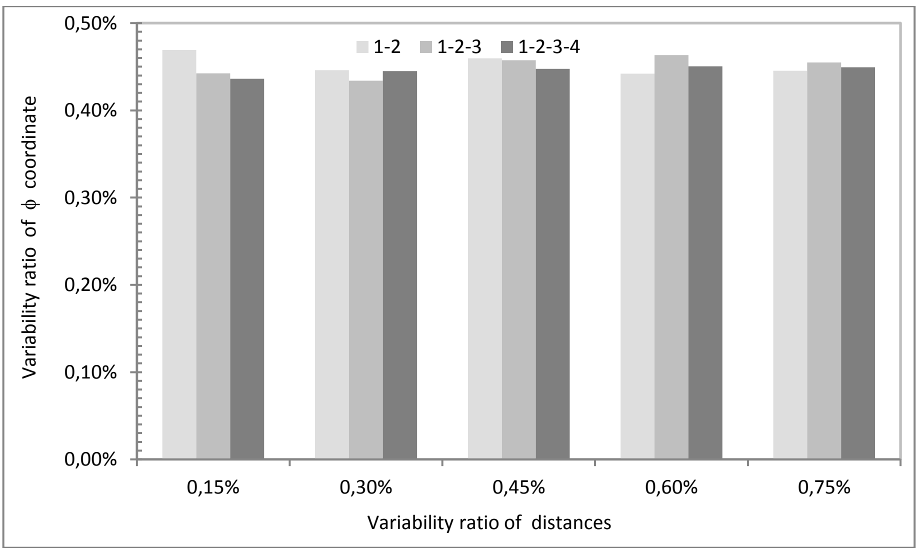


Fig. 5a. The variability ratio of distances versus variability ratio of ϕ coordinate

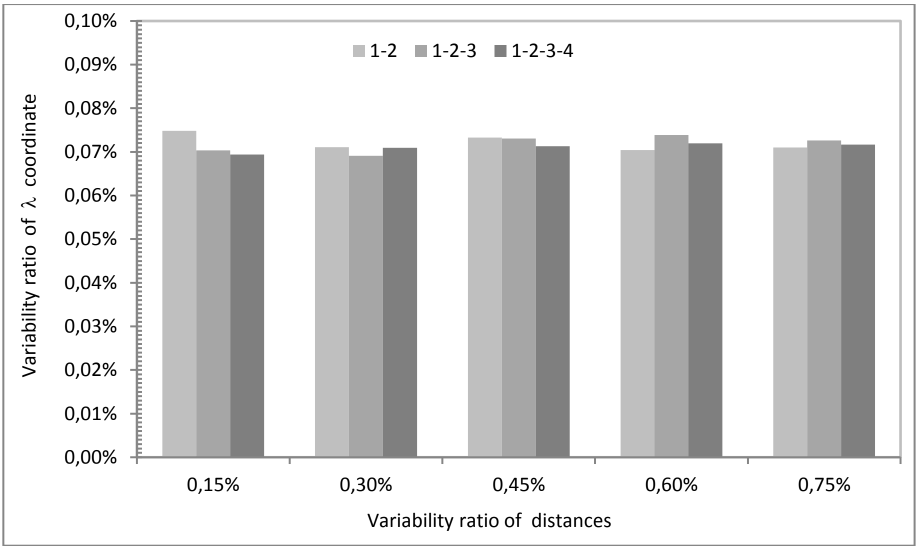


Fig. 5b. The variability ratio of distances versus variability ratio of λ coordinate

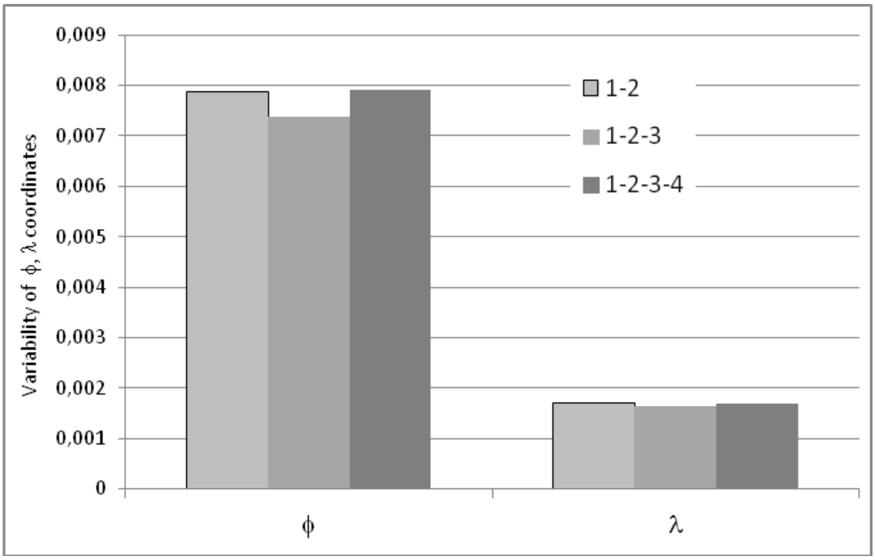


Fig. 6. The variability ratio of ϕ and λ coordinates

Table 3a–3d and 4a, 4b contain statistical characteristics of distributions of ship's positions i.e. φ_o and λ_o coordinates. In these tables the following notation was used:

Tab. 3a. Notation used in tables 3b–3d

Symbol	Meaning	Symbol	Meaning
α_1	Mean value	γ_1	Skewnes
σ	Standard deviation	γ_2	Kurtosis

Tab. 3b. Statistical characteristics of distributions of ship's positions; reference stations taken into account: 1-2

			Variability ratio of distances to reference stations				
			0,15%	**0,30%**	**0,45%**	**0,60%**	**0,75%**
Ship's coordinates	ϕ_o	α_1	30.000	30.004	29.9983	30.0006	30.0029
		σ	0.1408	0.1339	0.1379	0.1326	0.1336
		γ_1	0.0321	–0.0586	0.0818	0.0221	0.1660
		γ_2	–0.0427	0.0831	–0.1275	0.1084	–0.2274
	λ_o	α_1	44.999	44.999	45.0003	44.9998	44.9992
		σ	0.0337	0.0320	0.03297	0.0317	0.0319
		γ_1	–0.0985	–0.0087	–0.1449	–0.0890	–0.0737
		γ_2	0.0135	0.1088	–0.0691	0.1550	–0.2070

Tab. 3c. Reference stations taken into account: 1-2-3

			Variability ratio of distances to reference stations				
			0,15%	**0,30%**	**0,45%**	**0,60%**	**0,75%**
Ship's coordinates	ϕ_o	α_1	29.9947	30.0017	30.0017	29.9945	30.0027
		σ	0.1327	0.1303	0.1372	0.1390	0.1365
		γ_1	0.0688	–0.0144	0.0234	0.1317	0.0693
		γ_2	–0.271	–0.2358	–0.2010	0.1110	–0.0019
	λ_o	α_1	45.0012	44.9995	44.9973	45.0012	44.9992
		σ	0.03167	0.0311	0.0329	0.0332	0.0327
		γ_1	–0.1277	–0.0404	–0.0848	–0.2021	–0.1365
		γ_2	–0.1616	–0.1275	–0.1666	0.1812	0.0645

Tab. 3d. Reference stations taken into account: 1-2-3-4

			Variability ratio of distances to reference stations				
			0,15%	**0,30%**	**0,45%**	**0,60%**	**0,75%**
Ship's coordinates	ϕ_o	α_1	29.9981	29.9998	29.9998	30.0050	30.0036
		σ	0.1381	0.1353	0.1342	0.1352	0.1348
		γ_1	–0.0006	0.0732	0.0595	0.1085	0.1285
		γ_2	0.0733	0.02500	–0.1889	–0.2612	0.1732
	λ_o	α_1	45.0004	45.0000	45.0003	44.9987	44.9991
		σ	0.0312	0.0319	0.0321	0.0324	0.0323
		γ_1	–0.0634	–0.1392	–0.1193	–0.1688	–0.1986
		γ_2	0.0967	0.0816	–0.1346	–0.2041	0.2700

Tab. 4a. The coefficient of correlation between ϕ_o and λ_o coordinates, Variant I

Ref.	**Variability ratio of distances to reference stations**				
stations	**0,15%**	**0,30%**	**0.45%**	**0.60%**	**0.75%**
1-2	–0,996	–0,998	–0,997	–0,995	–0,995
1-2-3	–0,998	–0,997	–0,998	–0,999	–0,997
1-2-3-4	–0,995	–0,999	–0,997	–0,996	–0,997

Tab. 4b. The coefficient of correlation ρ between ϕ_o and λ_o coordinates, Variant II

	Ref. stations		
	1-2	1-2-3	1-2-3-4
ρ	+0.676	+0.635	+0,668

CONCLUSIONS

1. A general, qualitative conclusion derived from Figures 5a, 5b and Figure 6 is rather surprising. When distance errors are so small as [2] requires them to be, then number of reference stations has no noticeable impact on position accuracy. The differences observed on the figures are statistically insufficient and there is no trend observed. This conclusion holds both to Variant I and II.

2. As one can read from Figures 5a, 5b and Figure 6 the Variability Ratio of position coordinates in Variant I are much greater than in Variant II. One may find as a punishment for violating σ related assumption.
3. Measures of skewnes and kurtosis (see Tables 3a–3c) are very small. It means that position coordinates are random variables that follow (like distance errors) the Normal distribution.
4. The second surprising result is that when one passes from Variant I to Variant II correlation coefficient not only significantly changes its value but even changes its sign!

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STRESZCZENIE

Matematyczna istota wyznaczania współrzędnych pozycji okrętu to minimalizacja odpowiednio zdefiniowanej funkcji celu. Artykuł proponuje wykorzystanie do tego metody gradientu sprzężonego. Dzięki niej obliczenia mogą być wykonane w kilka sekund, ponieważ Microsoft i Apache zaimplementowały metodę gradientu sprzężonego jako narzędzie

nazwane Solver i umieściły je w swych szeroko oferowanych i popularnych arkuszach kalkulacyjnych: Excelu i Open Office Calc. W artykule pokazano także, jak precyzyjnie określić błędy oszacowania współrzędnych pozycji okrętu metodą Monte Carlo.