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## The measurement test for the identification of current - flux linkage characteristics in synchronous reluctance motors

### Abstract

In the paper a new measurement procedure of identification of synchronous reluctance motors parameters is presented. It is based on extension of traditional current decay test, used both in case of induction and synchronous motors [1, 3]. The main aim of the method is to enable to obtain the current-flux linkage characteristic necessary in case of Hamiltonian model of the machine (eq.1). In case of 3-phase machine without neutral wire, being the most popular in practical applications [3], the method is based on measurement of currents in two independent phases (Fig.4, eq.5). The result of its application in a form of trajectories presenting traces of the chosen flux linkage in a current space is shown in Fig.8b.

**Keywords:** Synchronous Reluctance Motor, Hamiltonian equations, Current-Flux linkage characteristic.

### 1. Introduction

In the analysis of electromechanical energy converters various measurement methods are used for identification of their model parameters. These methods can be divided into two main groups:

- steady state analysis - no-load tests and methods based on neglecting of transient components in dq reference frame. These methods enable the evaluation of idealized model equivalent circuit parameters [1, 2, 12];
- dynamic analysis - short-circuit, SSFR (stand-still frequency response), current decay [11, 13].

The paper focuses on the extension of the current decay test which is usually performed separately for direct (d) and quadrature (q) axes in case of synchronous machines [2, 7]. In the proposed method two currents are automatically and separately controlled which allows for evaluation of the whole current-flux characteristic [4, 5]. In case of synchronous reluctance motors (SynRM, [11]) and inset permanent magnet synchronous motors (IPMSM [10]) it allows for precise description of saturation and cross-coupling phenomenon.

Similar measurement method was described in [11]. However, it was based on manually preset initial current values and allowed for evaluation of static and dynamic inductances (eq.2,4). Alternative measurement algorithm described in reference [12] allows for the determination of the current-flux linkage characteristic, like the proposed one, but is restricted to dq reference frame and incapable of including eg. effects of slotting.

### 2. Lagrangian and Hamiltonian models of electromechanical energy converters

Mathematical models which are used in electromechanics employ either Lagrangian or Hamiltonian form of equations, depending on the choice of state variables [6, 8].

The matrix equation describing the Hamiltonian model, using flux linkages as state variables, has the following form:

$$\frac{d}{dt} \Psi = \mathbf{u} - \mathbf{R} \mathbf{i}(\varphi, \Psi) \quad (1)$$

where  $\mathbf{i}(\varphi, \Psi)$  is the function that defines currents in terms of rotor angular position  $\varphi$  and generalised electric momenta (flux linkages)  $\Psi = [\psi_1, \dots, \psi_N]$ ;  $\mathbf{i} = [i_1, \dots, i_N]$  - generalised electric velocities (currents);  $\mathbf{u}$  - generalised external electric forces (voltages);  $\mathbf{R}$  - resistance matrix [4].

The Lagrangian equation for the same device has a form:

$$\frac{d}{dt} \mathbf{i} = (\mathbf{L}_d(\varphi, \mathbf{i}))^{-1} \left( \mathbf{u} - \left( \omega \frac{\partial L_n(\varphi, \mathbf{i})}{\partial \varphi} + \mathbf{R} \right) \mathbf{i} \right) \quad (2)$$

where:

$$\mathbf{L}_d(\varphi, \mathbf{i}) = \frac{\partial \Psi(\varphi, \mathbf{i})}{\partial \mathbf{i}} = \begin{bmatrix} \frac{\partial \psi_1}{\partial i_1} & \dots & \frac{\partial \psi_1}{\partial i_N} \\ \dots & \dots & \dots \\ \frac{\partial \psi_N}{\partial i_1} & \dots & \frac{\partial \psi_N}{\partial i_N} \end{bmatrix} \quad (3)$$

$$\Psi(\varphi, \mathbf{i}) = L_n(\varphi, \mathbf{i}) \quad (4)$$

where  $N$  is a number of independent flux linkages and currents;  $\omega$  - mechanical angular velocity;  $\mathbf{L}_d$  - dynamic inductance matrix;  $L_n$  - nonlinear, static inductance matrix [8, 9].

In control systems these equations are typically transformed to some artificial frame of reference in order to simplify the solution [4, 7, 8, 9]. The evaluation of parameters in both cases is equivalent and therefore the method presented in the paper is focused on phase variables [8].

The main difference between Lagrangian and Hamiltonian form of equations is the continuity of current - flux linkage characteristic which is necessary in their successful implementation. In case of Lagrangian equations current-flux characteristic must be smooth ( $C^1$  continuity) because it is necessary to evaluate both static and dynamic inductance matrices (eq.2). It makes it necessary to implement special algorithms to evaluate elements of these matrices as the data are in discrete form [10, 11]. Otherwise the obtained results may lead to instability of simulations. In case of Hamiltonian formalism it is only necessary to have simple continuity ( $C^0$  continuity) of current-flux characteristic [5, 12]. In the paper only the derivation of data for Hamiltonian model is analyzed.

### 3. Application to SynRM

In case of 3-phase SynRM, shown in Fig. 1, the Hamiltonian equation (1) has the following form:

$$\frac{d}{dt} \begin{bmatrix} \psi_{AC} \\ \psi_{BC} \end{bmatrix} = \begin{bmatrix} e_{AC} \\ e_{BC} \end{bmatrix} - \begin{bmatrix} r_A + r_C & r_C \\ r_C & r_B + r_C \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} \quad (5)$$

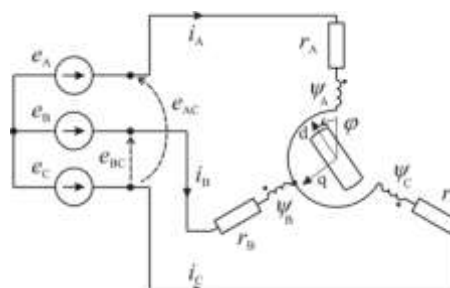


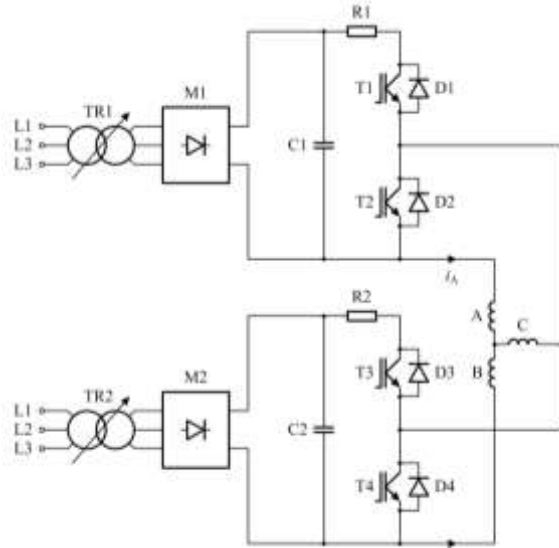
Fig. 1. Schematic representations of the synchronous reluctance motor (SynRM)

where:

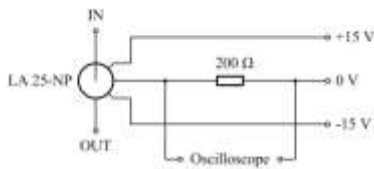
- flux linkages  $\psi_{AC}, \psi_{BC}$  - line-to-line flux linkages being linear combinations of phase flux linkages  $\psi_{AC} = \psi_A - \psi_C$ ,  $\psi_{BC} = \psi_B - \psi_C$  are generalised electric momenta  $\psi_1, \psi_2$ ,

- voltages  $e_{AC}, e_{BC}$  – line-to-line voltages being linear combinations of phase voltages  $e_{AC} = e_A - e_C, e_{BC} = e_B - e_C$  are generalised external electric forces  $u_1, u_2$ ,
- currents  $i_A, i_B$  – being simultaneously the loop and phase currents are generalised electric velocities  $i_1, i_2$ ,
- the resistance matrix is symmetric but non-diagonal.

a)



b)



c)



Fig. 2. Two-phase current decay circuit : a) schematic diagram with b) current-voltage measurement ratio 0,4 V - 1 A and c) general view

The measurement circuit is shown in Fig. 2. Its supply consists of two 3-phase separating transformers (TR1, TR2) with variable output voltage. Secondary windings of the transformers supply the diode bridges (M1, M2) creating two separate DC buses with capacitors C1 and C2. To each bus a two-transistor leg (IGBTs with integrated diodes) was connected through a power resistor (R1 and R2). Phase windings of the SynRM motor were connected to these legs in a way shown in Fig. 2. Current measurement is performed using two LEM current transducers (LA 25-NP) with additional resistors for connecting oscilloscope voltage probes (Fig. 2b, Fig. 4).

Measurement procedure for two-phase current decay is as follows:

- setting the initial currents point  $i_p = [i_A(t=0), i_B(t=0)]$  using appropriate voltage control (eg. variable transformer output voltage, PWM) for blocked rotor ( $\varphi = const$ ),
- short-circuiting the machine windings using the converter transistors at time instant  $t = 0$ ,
- recording the currents for a sufficient time period  $t_p$ , at which the currents can be regarded as zero  $i(t_p) \cong 0$  (Fig. 3).

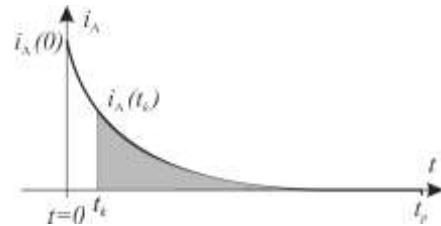


Fig. 3. Idealized result of current decay test for one current

### 4. Results of measurements

Machine which was used in measurement was a prototype SynRM using a stator of an induction motor RSg 80-4A ( $P_n=0.37$  kW,  $I_n=2.2$  A,  $2p=4$  – number of poles), with the rotor lamination stack 72 mm long [4].

Control of the measurement system was implemented using a microprocessor. It enabled a sequential implementation of the proposed procedure shown in Fig. 4. Exemplary results are presented for a case where the procedure is repeated  $M=4$  times.

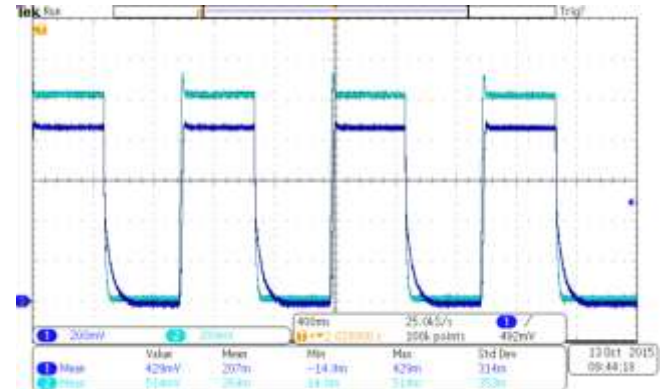


Fig. 4. Exemplary results of current decay test using its sequential implementation

It makes it possible to use an averaging procedure which decreases the influence of disturbances (eg. white noise) [15]. In case of phase A current  $i_A$  it has the following form:

$$i_A(t_j) = 1/M (\sum_{m=1}^M i_A(t_j^m)) \tag{6}$$

Simplified graphical representation of discrete data used in the averaging procedure is shown in Fig. 5.

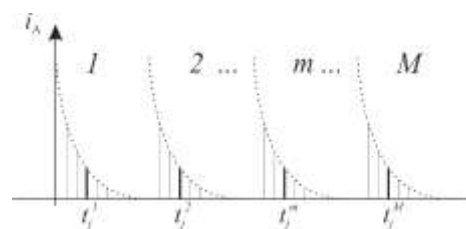


Fig. 5. Simplified representation of measured discrete data

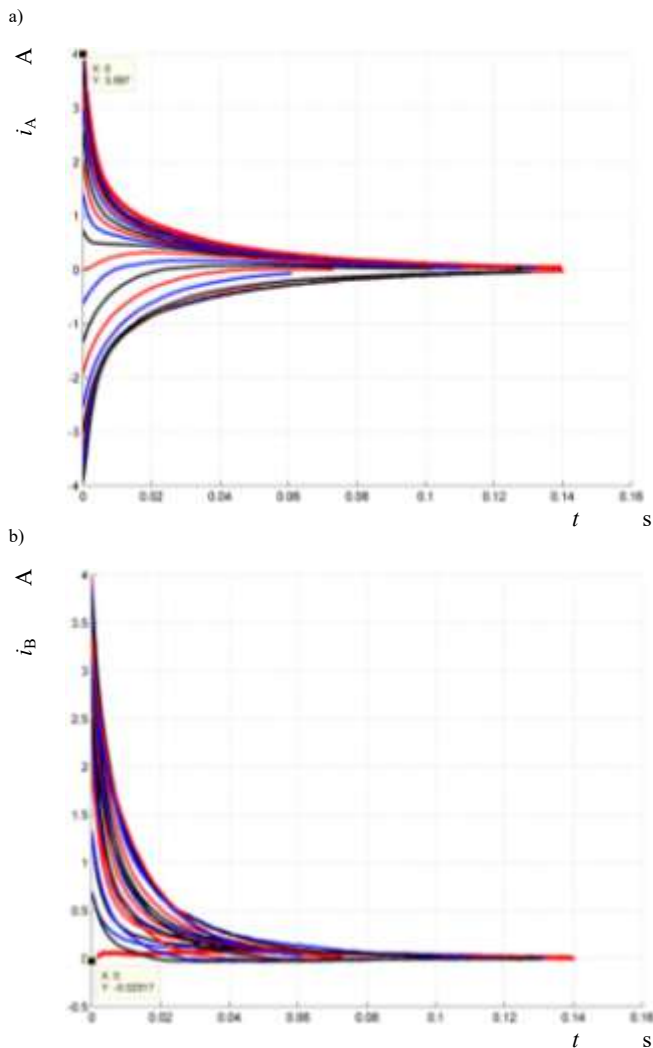


Fig. 6. Exemplary results of current decay test showing two averaged phase currents a)  $i_A(t)$ , b)  $i_B(t)$  plots

Form of eq. 6 takes into account the discrete form of obtained data which are available only at time instances  $t_j$  depending on oscilloscope sampling rate. In the presented example it was equal to 25 kS/s (Fig. 4). Results of averaging are shown in Fig. 6.

Equations (5) for short-circuit conditions ( $e=0$ ) can be integrated which enables the evaluation of flux linkages at any time instant  $t_k$  ( $t_p \geq t_k \geq 0$ ):

$$\Psi(i(t_p)) - \Psi(i(t_k)) = - \int_{t_k}^{t_p} R(T) i(t) dt \quad (7)$$

Assuming the short time of measurement we can at this stage of analysis neglect the change of resistance related to temperature  $T$  which leads to the following relationship:

$$\Psi(i(t_p)) - \Psi(i(t_k)) = -R \int_{t_k}^{t_p} i(t) dt \quad (8)$$

Assuming that for SynRM machine (without magnetic flux residual) for sufficiently large  $t_p$ : ( $t_p \rightarrow \infty$ ) both currents  $i(t_p) \cong 0$  and flux linkages  $\Psi(t_p) \cong 0$  vanish, one obtains:

$$\Psi(t_k) = \begin{bmatrix} \psi_{AC}(t_k) \\ \psi_{BC}(t_k) \end{bmatrix} = R \int_{t_k}^{t_p} \begin{bmatrix} i_A(t) \\ i_B(t) \end{bmatrix} dt \quad (9)$$

Integral in eq.8 describes the surfaces under the time curves  $i_A(t)$  (Fig.3 - shaded surface) and  $i_B(t)$ . Integration in eq. 8 due to discrete form of data obtained during measurements can be performed using a trapezoid formula (Fig.7) [14].

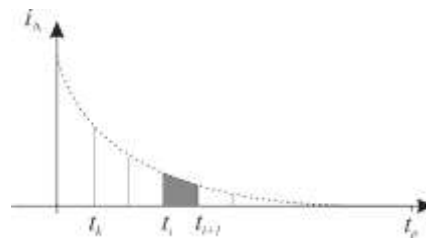


Fig. 7. Interpretation of trapezoid integral formula

Using the data obtained from averaging procedure (eq.6), one obtains the following formula for integral of  $i_A(t)$ :

$$\int_{t_k}^{t_p} i_A(t) dt = \sum_{t=t_k}^{t_p} (i_A(t_i) + i_A(t_{i+1}))(t_{i+1} - t_i) / 2 \quad (10)$$

The obtained results are shown in Fig.8.

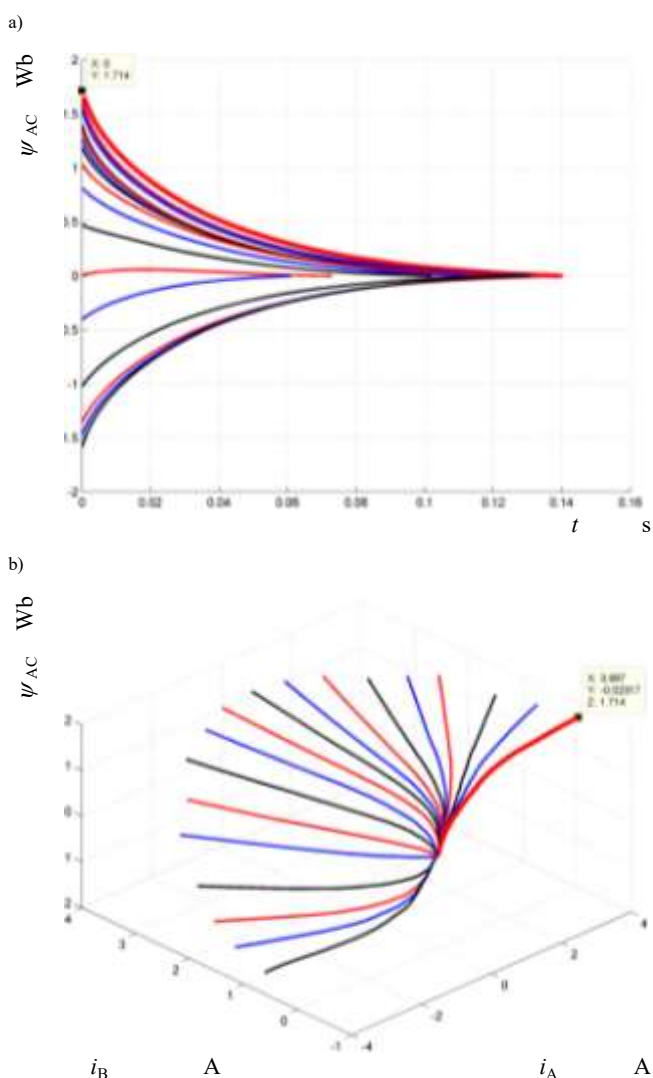


Fig. 8. Exemplary results of current decay test showing one flux-linkage as a function of a) time and b) two phase currents

Application of a special topological algorithm makes it possible to create an approximate, continuous ( $C^0$  continuity) surface based on the data shown in Fig. 8 [5].

### 5. Conclusions

Obtained results prove that the proposed method makes it possible to obtain the current-flux linkage characteristic which is necessary in case of Hamiltonian model of SynRM [4, 5].

Range of applications of the proposed method is defined by the following conditions:

- substantial influence of saturation phenomenon on the machine parameters,
- no eddy-currents (no starting cage or solid iron).  
These conditions are met by [3]:
- synchronous reluctance motors (SynRM),
- switched reluctance motors (SRM, MCSRSM),
- inset permanent magnet machines (IPMSM).

The difference in case of IPMSM machine is that at no-current conditions the flux linkage is not zero due to the presence of permanent magnet and additional test is necessary to evaluate the PM fluxes [10].

The following additional conclusions could be drawn:

- the current build-up time should be as short as possible to avoid the temperature problem,
- sequential procedure should be optimized, because while it improves the quality of results it makes the heating problem more complicated.

The problem of windings' temperature estimation based on DC currents and voltages needs further investigation as it directly influences the obtained results (eq.7). In the paper this effect has not been analyzed.

The future goal is to implement the proposed method in control system of the inverter supplying the motor. It would make it possible to perform the self-tuning of the whole drive [3].

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