Journal of Polish Safety and Reliability Association
Summer Safety and Reliability Seminars, Volume 9, Number 1, 2018

Kołowrocki Krzysztof

Kuligowska Ewa

Soszyńska-Budny Joanna
Gdynia Maritime University, Gdynia, Poland

Optimization of critical infrastructure operation and safety with considering climate-weather change impact – minimizing operation cost

Keywords

critical infrastructure, safety, operation process, climate-weather change, operation cost, optimization

Abstract

The general model of a critical infrastructure changing its safety structure, its components safety parameters and its operation cost during the variable operation process and linear programming are applied to optimize the critical infrastructure operation process in order to get the critical infrastructure operation cost optimal value. The optimization problem allowing to find the optimal values of the transient probabilities of the critical infrastructure operation process at the particular operation states that minimize the critical infrastructure operation cost mean value in the safety states subset not worse than a critical safety state is presented. The optimization of operation cost of the critical infrastructure is proposed with considering climate-weather change process influence on the system safety.

1. Introduction

To tie the investigation of the critical infrastructure safety together with the investigations of its operation and climate-weather change impact, the semi-Markov process can be used to describe this critical infrastructure operation processes related to the climate-weather change [Klabjan, Adelman, 2006], [Kolowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, Soszyńska-Budny, 2012a-b], [Kołowrocki, Soszyńska-Budny, 2014]. The models of these two processes, under the assumption on the critical infrastructure structure multistate model [Xue, 229], [Xue, Yang, 1995] can be used to construct the general safety model of the multistate critical infrastructure changing its safety structure and its components safety parameters during variable operation process related to the climate-weather change [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, Soszyńska-Budny, 2012a-b]. Further, using this general model, it is possible to find the critical infrastructure main safety characteristics such as the critical infrastructure safety function, the critical infrastructure mean lifetimes in critical infrastructure safety subsets and risk function [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, Soszyńska-Budny, 2012a-b] and its operation cost related to climate-weather change. Having these characteristics it is possible to optimize the critical infrastructure operation process related to the climate-weather change to get their optimal values [Kuo, Prasad, 2000], [Kuo, Zuo, 2003], [Tang et al., 2007], [Vercellis, 2009]. To this, the linear programming [Klabjan, Adelman, 2006], can be applied for minimizing the critical infrastructure operation cost.

2. Critical infrastructure operation process related to climate-weather change process

We consider the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t),\ t \in <0,\infty)$, in a various way at this process states $zc_{bl},\ b = 1,2,...,\nu,\ l = 1,2,...,w.$ We assume that the changes of the states of operation process related to the climate-weather change process $ZC(t),\ t \in <0,\infty)$, at the critical infrastructure operating area have an influence on the critical infrastructure safety structure and on the safety of the critical infrastructure assets A_i, $i=1,2,...,n,$ as well [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017].

We assume, as in [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017], that the critical infrastructure during its operation process is taking $\nu, \nu \in N,$ different operation states $z_1, z_2, \ldots, z_\nu$. We define the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty),$ with discrete operation states from the set $\{z_1, z_2, \ldots, z_\nu\}$. Further, we assume that we have either calculated analytically or evaluated approximately by experts the vector

$$[p_b]_{1x\nu} = [p_1, p_2, ..., p_\nu] \quad (1)$$

of limit values of transient probabilities (OPC1)

$$p_b(t) = P(Z(t) = z_b),\ t \in <0,+\infty),\ b=1,2,...,\nu,$$

of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b=1,2,...,\nu.$

Moreover, as in [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017], we assume that the climate-weather change process $C(t)$, $t \in <0,+\infty),$ at the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. We assume that we have either calculated analytically or evaluated approximately by experts the vector

$$[q_l]_{1xl} = [q_1, q_2, ..., q_w] \quad (2)$$

of limit values of transient probabilities (C-WCPC1)

$$q_l(t) = P(C(t) = c_l),\ t \in <0,+\infty),\ l=1,2,...,w,$$

of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $l=1,2,...,w.$

Under the above assumptions about the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty),$ and the climate-weather change process $C(t)$, we introduce the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to climate-weather change marked by

$$ZC(t),\ t \in <0,+\infty),$$

and we assume that it can take $\nu w, \nu, w \in N,$ different operation states related to the climate-weather change

$$zc_{11}, zc_{12}, \ldots, zc_{\nu w},$$

We assume that the critical infrastructure operation process related to climate-weather change $ZC(t)$, at the moment $t \in <0,+\infty),$ is at the state zc_{bl}, $b=1,2,...,\nu,$ $l=1,2,...,w,$ if and only if at that moment, the operation process $Z(t)$ is at the operation states z_b, $b=1,2,...,\nu,$ and the climate-weather change process $C(t)$ is at the climate-weather state c_l, $l=1,2,...,w,$ what we express as follows:

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$
$$t \in <0,+\infty),\ b=1,2,...,\nu,\ l=1,2,...,w.$$

Further, the transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b=1,2,...,\nu,$ $l=1,2,...,w,$ can be defined by

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}),\ t \in <0,+\infty),$$
$$b=1,2,...,\nu,\ l=1,2,...,w.$$

In the case when the processess $Z(t)$ and $C(t)$ are independent the expession for the transient probabilities can be expressed in the following way

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}) = P(Z(t) = z_b \cap C(t) = c_l)$$
$$= P(Z(t) = z_b) \cdot P(C(t) = c_l) = p_b(t) \cdot q_l(t),$$
$$t \in <0,+\infty),\ b=1,2,...,\nu,\ l=1,2,...,w,$$

where $p_b(t)$, $b=1,2,...,\nu,$ are the transient probabilities of the operation process $Z(t)$ at the particular operation startes z_b, $b=1,2,...,\nu,$ and $q_l(t)$, $l=1,2,...,w,$ are the trsansient probabilities of the climate-weather change process $C(t)$ at the particular clmate-weather startes c_l, $b=1,2,...,w.$

Hence the limit values of the transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b=1,2,...,\nu,$ $l=1,2,...,w,$

$$pq_{bl} = \lim_{t\to\infty} pq_{bl}(t),\ b=1,2,...,\nu,\ l=1,2,...,w, \quad (3)$$

can be found from [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017]

$$pq_{bl} = p_b q_l, \ b = 1,2,...,\nu, \ l = 1,2,...,w, \quad (4)$$

where p_b, $b = 1,2,...,\nu$, are the limit trsansient probabilities of the operation process $Z(t)$ at the particular operation startes z_b, $b = 1,2,...,\nu$, and q_l, $l = 1,2,...,w$, are the limit trsansient probabilities of the climate-weather change process $C(t)$ at the particular clmate-weather startes c_l, $b = 1,2,...,w$.

Other interesting characteristics of the critical infrastructure operation process $ZC_{bl}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}$, $b = 1,2,...,\nu$, $l = 1,2,...,w$, at the particular operation states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, during the fixed sufficiently large critical infrastructure operation time θ. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta, \ b = 1,2,...,\nu, \ l = 1,2,...,w, \quad (5)$$

where pq_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are defined by (3) and given by (4) in the case the processes $Z(t)$ and $C(t)$ are independent.

3. Critical infrastructures operation cost related to climate-weather change

We may introduce the instantaneous operation cost of the critical infrastructure impacted by the operation process $ZC_{bl}(t)$, $t \in <0,\infty)$, related to the climate-weather change process in the form of vector

$$\boldsymbol{K}^4(t,\cdot) = [1, \ \boldsymbol{K}^4(t,1), ..., \ \boldsymbol{K}^4(t,z)], \ t \in <0,\infty),$$

with the coordinates given by

$$\boldsymbol{K}^4(t,u) \cong \sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl}[\boldsymbol{K}^4(t,u)]^{(bl)} \text{ for } t \in <0,\infty), \ u = 1,2,...,z, \quad (6)$$

where pq_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are the limit transient probabilities at the states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, of the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change defined by (3) and

$$[\boldsymbol{K}^4(t,u)]^{(bl)}, \ u = 1,2,...,z, \ b = 1,2,...,\nu, \ l = 1,2,...,w,$$

are the coordinates of the critical infrastructure conditional instantaneous operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u = 1,2,...,z$, impacted by the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change process at the states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, defined in the form of the vector

$$[\boldsymbol{K}^4(t,\cdot)]^{(bl)} = [1, \ [\boldsymbol{K}^4(t,1)]^{(bl)}, ..., \ [\boldsymbol{K}^4(t,z)]^{(bl)}],$$
$$t \in <0,\infty), \ b = 1,2,...,\nu, \ l = 1,2,...,w.$$

The dependency (6) can also be clearly expressed in the linear equation for the mean value of the critical infrastructure total unconditional operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u = 1,2,...,z$,

$$\bar{\boldsymbol{K}}^4(u) \cong \sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl}[\bar{K}^4(u)]^{(bl)}, \ u = 1,2,...,z, \quad (7)$$

where pq_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are the limit transient probabilities at the states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, of the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change and

$$[\bar{\boldsymbol{K}}^4(u)]^{(bl)}, \ u = 1,2,...,z, \ b = 1,2,...,\nu, \ l = 1,2,...,w,$$

are the mean values of the critical infrastructure total conditional instantaneous operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u = 1,2,...,z$, at the operation states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, defined by

$$[\bar{\boldsymbol{K}}^4(u)]^{(bl)} = \int_0^{[\mu^4(u)]^{(bl)}} [\boldsymbol{K}^4(t,u)]^{(bl)} dt, \ u = 1,2,...,z, \ b = 1,2,...,\nu, \ l = 1,2,...,w, \quad (8)$$

where $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are the mean values of the critical infrastructure conditional lifetimes $[T^4(u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, $l = 1,2,...,w$, in the safety state subset $\{u,u+1,...,z\}$ at the critical infrastructure operating process related to the climate-weather change state zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, given by [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017]

$$[\boldsymbol{\mu}^4(u)]^{(bl)} = \int_0^{\infty} [\boldsymbol{S}^4(t,u)]^{(bl)} dt, \ u = 1,2,...,z, \ b = 1,2,...,\nu, \ l = 1,2,...,w, \quad (9)$$

and

$$[\boldsymbol{S}^4(t,u)]^{(bl)},\ u = 1,2,\ldots,z,\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,$$

are the coordinates of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, conditional safety functions [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017]

$$[\boldsymbol{S}^4(t,\cdot)]^{(bl)} = [1,\ [\boldsymbol{S}^4(t,1)]^{(bl)},\ \ldots,\ [\boldsymbol{S}^4(t,z)]^{(bl)}],$$
$$t \in <0,\infty),\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,$$

The mean values of the critical infrastructure total conditional instantaneous operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the operation states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, can be alternatively defined for the critical infrastructure fixed operation time θ by

$$[\overline{\boldsymbol{K}}^4(u)]^{(bl)} = \int_0^{\hat{M}\hat{N}} [\boldsymbol{K}^4(t,u)]^{(bl)} dt,\ u = 1,2,\ldots,z,$$
$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,$$

where $\hat{M}\hat{N}_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are the mean values of the total sojourn times $\hat{\theta}\hat{C}_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the particular operation states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, during the fixed sufficiently large critical infrastructure operation time θ, determined by (5).

4. Critical infrastructures operation cost related to climate-weather change minimization

From the linear equation (3) we can see that the mean value of the critical infrastructure total unconditional operation cost $\overline{\boldsymbol{K}}^4(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure operation process at the operation states given by (3) and the mean values of the critical infrastructure total conditional operation costs $[\overline{\boldsymbol{K}}^4(u)]^{(bl)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the critical infrastructure operating process related to the climate-weather change process zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, given by (8).

Therefore, the critical infrastructure total unconditional operation cost optimization approach based on the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}q_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the limit transient probabilities pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure operation process at the operation states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, to minimize the mean value $\overline{\boldsymbol{K}}^4(u)$, $u = 1,2,\ldots,z$, of the critical infrastructure total unconditional operation cost in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, under the assumption that the mean values $[\overline{\boldsymbol{K}}^4(u)]^{(bl)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the operation states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are fixed. As a special case of the above formulated the critical infrastructure total unconditional operation cost optimization problem, if r, $r = 1,2,\ldots,z$, is a critical infrastructure critical safety state, we want to find the optimal values $\dot{p}q_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure operation process limit transient probabilities pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the operation states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, to minimize the mean value $\overline{\boldsymbol{K}}^4(r)$ of the critical infrastructure total unconditional operation cost in the safety state subset $\{r,r+1,\ldots,z\}$, under the assumption that the mean values $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure total conditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$ at the operation states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\overline{\boldsymbol{K}}^4(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\overline{K}^4(r)]^{(bl)}, \tag{10}$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\breve{p}q_{bl} \le pq_{bl} \le \hat{p}q_{bl},\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \tag{11}$$

$$\sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} = 1, \tag{12}$$

where

$$[\overline{\boldsymbol{K}}^4(r)]^{(bl)},\ [\overline{\boldsymbol{K}}^4(r)]^{(bl)} \geq 0,\ b = 1,2,...,v,\ l = 1,2,...,w, \quad (13)$$

are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r, r+1,...,z\}$ and

$$\breve{p}q_{bl},\ 0 \leq \breve{p}q_{bl} \leq 1 \text{ and } \hat{p}q_{bl},\ 0 \leq \hat{p}q_{bl} \leq 1,\ \breve{p}q_{bl} \leq \hat{p}q_{bl},\ b = 1,2,...,v,\ l = 1,2,...,w, \quad (14)$$

are lower and upper bounds of the transient probabilities pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, respectively.

Now, we can obtain the optimal solution of the formulated by (10)-(14) the linear programming problem, i.e. we can find the optimal values $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, of the transient probabilities pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, that maximize the objective function given by (6.10).

First, we arrange the critical infrastructure total unconditional operation costs $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $b = 1,2,...,v$, $l = 1,2,...,w$, in non-decreasing order

$$[\overline{K}^4(r)]^{(bl_1)} \leq [\overline{K}^4(r)]^{(bl_2)} \leq . . . \leq [\overline{K}^4(r)]^{(bl_{vw})},$$

where $bl_i \in \{1,2,...,vw\}$ for $i = 1,2,...,vw$.

Next, we substitute

$$x_i = p_{bl_i},\ \breve{x}_i = \breve{p}_{bl_i},\ \hat{x}_i = \hat{p}_{bl_i} \text{ for } i = 1,2,...,vw, \quad (15)$$

and we maximize with respect to x_i, $i = 1,2,...,vw$, the linear form (6.10) that after this transformation takes the form

$$\overline{K}^4(r) = \sum_{i=1}^{vw} x_i [\overline{K}^4(r)]^{bl_i} \quad (16)$$

for a fixed $r \in \{1,2,...,z\}$ with the following bound constraints

$$\breve{x}_i \leq x_i \leq \hat{x}_i,\ i = 1,2,...,vw, \quad (17)$$

$$\sum_{i=1}^{vw} x_i = 1, \quad (18)$$

where

$$[\overline{K}^4(r)]^{(bl_i)},\ [\overline{K}^4(r)]^{(bl_i)} \geq 0,\ i = 1,2,...,vw,$$

are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r, r+1,...,z\}$ at the operation states zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, arranged in non-decreasing order and

$$\breve{x}_i,\ 0 \leq \breve{x}_i \leq 1 \text{ and } \hat{x}_i,\ 0 \leq \hat{x}_i \leq 1,\ \breve{x}_i \leq \hat{x}_i,\ i = 1,2,...,vw, \quad (19)$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,...,vw$, respectively.

To find the optimal values of x_i, $i = 1,2,...,vw$, we define

$$\breve{x} = \sum_{i=1}^{vw} \breve{x}_i,\ \hat{y} = 1 - \breve{x} \quad (20)$$

and

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \text{ for } I = 1,2,...,vw. \quad (21)$$

Next, we find the largest value $I \in \{0,1,...,vw\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \quad (22)$$

and we fix the optimal solution that maximize (16) in the following way:

i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 1,2,...,vw; \quad (23)$$

ii) if $0 < I < vw$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,...,I,\ \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1} \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,...,vw; \quad (24)$$

iii) if $I = vw$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,...,vw. \quad (25)$$

Finally, after making the inverse to (15) substitution, we get the optimal limit transient probabilities

$$\dot{p}q_{bl_i} = \dot{x}_i \text{ for } i = 1,2,...,vw, \quad (26)$$

that minimize the mean value of the critical infrastructure unconditional operation cost in the safety state subset $\{r, r+1,...,z\}$, defined by the linear form (6.10), giving its minimum value in the following form

$$\dot{\bar{\boldsymbol{K}}}^4(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl}[\bar{K}^4(r)]^{(bl)} \quad (27)$$

for a fixed $r \in \{1,2,...,z\}$.

Thus, considering (6), the coordinates of the optimal instantaneous critical infrastructure operation cost in the form of the vector

$$\dot{\boldsymbol{K}}^4(t,\cdot) = [1,\ \dot{\boldsymbol{K}}^4(t,1),\ ...,\ \dot{\boldsymbol{K}}^4(t,z)\ ],\ t \in <0,\infty),$$

are given by

$$\dot{\boldsymbol{K}}^4(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl}[\boldsymbol{K}^4(t,u)]^{(bl)} \text{ for } t \in <0,\infty),$$
$$u = 1,2,...,z, \quad (28)$$

where $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are the optimal limit transient probabilities at the states zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$,of the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change defined by (6.26) and

$$[\boldsymbol{K}^4(t,u)]^{(bl)},\ u = 1,2,...,z,\ b = 1,2,...,v,\ l = 1,2,...,w,$$

are the coordinates of the critical infrastructure conditional instantaneous operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u = 1,2,...,z$, impacted by the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change process at the states zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, defined in the form of the vector

$$[\boldsymbol{K}^4(t,\cdot)]^{(bl)} = [1,\ [\boldsymbol{K}^4(t,1)]^{(bl)},\ ...,\ [\boldsymbol{K}^4(t,z)]^{(bl)}\ ],$$
$$t \in <0,\infty),\ b = 1,2,...,v,\ l = 1,2,...,w.$$

Replacing in (27) r by u, we get the expressions for the optimal mean values of the critical infrastructure unconditional operation costs in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z$, giving its minimum value in the following form

$$\dot{\bar{\boldsymbol{K}}}^4(u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl}[\bar{K}^4(u)]^{(bl)},\ \{u,u+1,...,z\},$$
$$u = 1,2,...,z. \quad (29)$$

The optimal solutions for the mean values of the critical infrastructure unconditional operation costs in the particular safety states are

$$\dot{\bar{\bar{K}}}^4(u) = \dot{\bar{K}}(u) - \dot{\bar{K}}(u+1),\ u = 1,2,...,z-1,$$
$$\dot{\bar{\bar{K}}}^4(z) = \dot{\bar{K}}(z), \quad (30)$$

where $\dot{\bar{\boldsymbol{K}}}^4(u)$, $u = 1,2,...,z$, are given by (29).

Moreover, if we define the corresponding critical operation cost function by

$$\boldsymbol{K}^4(t) = \boldsymbol{K}^4(t,r)\ \ t \in <0,\infty), \quad (31)$$

and the moment ζ when the critical infrastructure operation cost exceeds a permitted level κ, by

$$\zeta = \boldsymbol{K}^{4^{-1}}(\kappa), \quad (32)$$

where $\boldsymbol{K}^4(t,r)$ is given by (6.6) for $u = r$ and $\boldsymbol{K}^{4^{-1}}(\kappa)$ is the inverse function of the critical operation cost function $\boldsymbol{K}^4(t)$ given by (31), then the corresponding optimal critical operation cost function is given by

$$\dot{\boldsymbol{K}}^4(t) = \dot{\boldsymbol{K}}^4(t,r)\ \ t \in <0,\infty), \quad (33)$$

then the optimal moment $\dot{\zeta}$ when the critical infrastructure operation cost exceeds a permitted level κ, is given by

$$\dot{\zeta} = \dot{\boldsymbol{K}}^{4^{-1}}(\kappa), \quad (34)$$

where $\dot{\boldsymbol{K}}^4(t,r)$ is given by for $u = r$ and $\dot{\boldsymbol{K}}^{4^{-1}}(\kappa)$, if it exists, is the inverse function of the optimal critical operation cost function $\dot{\boldsymbol{K}}^4(t)$ given by (30)

5. Cost analysis of critical infrastructure operation impacted by climate-weather change

We consider the multistate critical infrastructure consisted of n components in its operation process $ZC(t)$, $t \in <0,\infty)$, related to climate-weather change and we assume that the operation costs of its single basic components at the operation state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, during the critical infrastructure operation time θ, $\theta \geq 0$, amount

$$[K_i\ (\theta)]^{(bl)},\ b = 1,2,...,v,\ l = 1,2,...,w,\ b = 1,2,...,v,$$
$$i = 1,2,...,n.$$

First, we suppose that the critical infrastructure is non-repairable and during the operation time θ, $\theta \geq 0$, it has not exceeded the critical safety state r.

In this case, the total cost of the non-repairable critical infrastructure during the operation time θ, $\theta \geq 0$, is given by

$$K(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n} [K_i(\theta)]^{(bl)}, \ \theta \geq 0, \tag{35}$$

were pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, are transient probabilities defined by (3).

Further, we consider another case and we additionally assume that the critical infrastructure is repairable after exceeding the critical safety state r and its renovation time is ignored and the cost of its single renovation is constant and equal to K_{ign}.

In this case, the total operation cost of the repairable critical infrastructure with ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$K_{ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n} [K_i(\theta)]^{(bl)} + K_{ign} H(\theta, r),$$
$$\theta \geq 0, \tag{36}$$

were pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, are transient probabilities defined by (3) and $H(\theta, r)$ is the mean value of the number of exceeding the critical safety state r by the critical infrastructure operating at the variable conditions during the operation time θ defined by (3.58) in [Kołowrocki, Soszyńska-Budny, 2011] and in [Guze, Kołowrocki, EU-CIRCLE Report D4.4-GMU, 2017].

Now, we assume that the critical infrastructure is repairable after exceeding the critical safety state r and its renewal time is non-ignored and have distribution function with the mean value $\mu_0(r)$ and the standard deviation $\sigma_0(r)$ and the cost of the critical infrastructure single renovation is K_{n-ign}.

In this case, the total operation cost of the repairable critical infrastructure with not ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$K_{n-ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n} [K_i(\theta)]^{(bl)} + K_{n-ign} \overline{\overline{H}}(\theta, r),$$
$$\theta \geq 0, \tag{37}$$

were pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, are transient probabilities defined by (3) and $\overline{\overline{H}}(\theta, r)$ is the mean value of the number of renovations of the critical infrastructure operating at the variable conditions during the operation time θ defined by (3.92)[Kołowrocki, Soszyńska-Budny, 2011] and in [Guze, Kołowrocki, EU-CIRCLE Report D4.4-GMU, 2017].

The particular expressions for the mean values $H(\theta, r)$ and $\overline{\overline{H}}(\theta, r)$ for the repairable critical infrastructure with ignored and non-ignored renovation times existing in the formulae (6.45) and (6.46), respectively defined by (3.58) and (3.92) in [Kołowrocki, Soszyńska-Budny, 2011], are determined in Chapter 3 [Kołowrocki, Soszyńska-Budny, 2011] for typical multistate repairable critical infrastructure operating at the variable operation conditions and in and in [Guze, Kołowrocki, EU-CIRCLE Report D4.4-GMU, 2017].

After the optimization of the critical infrastructure operation process related to climate-weather change, the critical infrastructure operation total costs given by (35)-(37) assume their optimal values.

The total optimal cost of the non-repairable critical infrastructure during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization is given by

$$\dot{K}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} [K_i(\theta)]^{(bl)}, \ \theta \geq 0, \tag{38}$$

were $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are optimal transient probabilities defined by (26).

The optimal total operation cost of the repairable critical infrastructure with ignored its renovation time during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization amounts

$$\dot{K}_{ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} [K_i(\theta)]^{(bl)} + K_{ign} \dot{H}(\theta, r),$$
$$\theta \geq 0, \tag{39}$$

were $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are optimal transient probabilities defined by (26) and $\dot{H}(\theta, r)$ is the mean value of the optimal number of exceeding the critical safety state r by the critical infrastructure operating at the variable conditions during the operation time θ defined by (29) [Kołowrocki, Soszyńska-Budny, 2011].

The total optimal operation cost of the repairable critical infrastructure with non-ignored its renovation time during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization amounts

$$K_{n-ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n} [K_i(\theta)]^{(bl)} + K_{n-ign} \overline{\overline{H}}(\theta, r),$$
$$\theta \geq 0, \tag{40}$$

were $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are optimal transient probabilities defined by (26) and $\dot{\bar{\bar{H}}}(\theta,r)$ is the mean value of the optimal number of renovations of the critical infrastructure operating at the variable operation conditions during the operation time θ defined by (6.37) [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the optimal mean values $\dot{H}(\theta,r)$ and $\dot{\bar{\bar{H}}}(\theta,r)$ for the repairable critical infrastructure with ignored and non-ignored renovation times existing in the formulae (6.48) and (6.49), respectively defined by (6.29) and (6.37) [Kołowrocki, Soszyńska-Budny, 2011], may be obtain by replacing the transient probabilities pq_{bl} by their optimal values $\dot{p}q_{bl}$ in the expressions for $H(\theta,r)$ and $\bar{\bar{H}}(\theta,r)$ defined by (3.58) and (3.92) in [Kołowrocki, Soszyńska-Budny, 2011], that are determined in Chapter 3 [Kołowrocki, Soszyńska-Budny, 2011], for typical multistate repaired critical infrastructure operating at the variable operation conditions.

The application of the formulae (35)-(37) and (38)-(40) allow us to compare the costs of the non-repairable and repairable critical infrastructure with ignored and non-ignored times of renovations operating at the variable operation conditions before and after the optimization of their operation processes.

6. Conclusions

The proposed optimization method presented in this paper can be used in critical infrastructure operation and safety optimization with considering climate-weather change impact. The optimization method application can be the basis for the elaboration of practical procedures of critical infrastructure safety improvement. The optimization model will be applied in Case Study 2 supported by suitable computer software that is placed at the GMU Safety Interactive Platform http://gmu.safety.am.gdynia.pl/. The GMU platform can be linked with CIRP and SimICI platforms of EU-CIRCLE project.

Acknowledgments

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/.

References

Barbu V., Limnios N., Empirical estimation for discrete-time semi-Markov processes with applications in reliability. Journal of Nonparametric Statistics, Vol. 18, No. 7-8, 483-498, 2006

Blokus-Roszkowska A., Kolowrocki K. Modeling safety of interconnected critical infrastructure network cascading. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminnars, Vol 8, No 3, 97-114, 2017

Bogalecka M., Kołowrocki K. Integrated model of critical infrastructure accident consequences, Journal of Polish Safety and Reliability Association, Special Issue on EU-CIRCLE Project Critical Infrastructure Impact Models for Operation Threats and Climate Hazards, Part 1, Critical Infrastructure Connections, 8(3), 43-54, 2017

Caldwell H., Quinn K.H., Meunier J., Suhrbier J., Grenzeback L., Potential impacts of climate change on freight
transport. In: U.S. DOT, 2002. The Potential Impacts of Climate Change on Transportation, Summary and Discussion Papers. U.S. Department of Transportation, 2002.

EU-CIRCLE Report D1.1, EU-CIRCLE Taxonomy, 2015

EU-CIRCLE Report D3.3, Integrated Model of Critical Infrastructure Safety (IMCIS) Related to its Operation Process, 2016

Ferreira F., Pacheco A., Comparison of level-crossing times for Markov and semi-Markov processes. Statistics and Probability Letters, Vol. 7, No 2, 151-157, 2007

Glynn P.W., Haas P.J., Laws of large numbers and functional central limit theorems for generalized semi Markov processes. Stochastic Models,Vol. 22, No 2, 201-231, 2006

Grabski F., (2002) Semi-Markov Models of Systems Safety and Operations Analysis. System Research Institute, Polish Academy of Science, 2002 (*in Polish*)

Guze S., Kołowrocki K.: Modelling Operation Process of Baltic Port, Shipping and Ship Traffic and Operation Information Critical Infrastructure Network. TransNav, the International Journal on

Marine Navigation and Safety of Sea Transportation, Vol. 10, No. 2, pp. 275-284, 2016

Guze S., Kołowrocki K., WP4-Task4.2-D4.4-Critical Infrastructure Climate Related Business Continuity Models-V1.0-GMU-2017

Guze S., Kołowrocki K., Safety Modeling of Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Join Network related to Its Operation Process, ESREL 2017, 2017 (submitted)

Klabjan D., Adelman D., Existence of optimal policies for semi-Markov decision processes using duality for infinite linear programming. Siam Journal on Control and Optimization, Vol. 44, No. 6, 2104-2122, 2006

Kołowrocki K., Reliability of Large and Complex Systems, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014

Kolowrocki K., Soszynska J., Reliability, risk and availability based optimization of complex technical systems operation processes. Part 1. Theoretical backgrounds. Electronic Journal Reliability & Risk Analysis: Theory & Applications, Vol. 2, No 4, 141-152, 2009

Kolowrocki K., Soszynska J., Reliability, risk and availability based optimization of complex technical systems operation processes. Part 2. Application in Port Transportation. Electronic Journal Reliability & Risk Analysis: Theory & Applications, Vol. 2, No 4, 153-167, 2009

Kolowrocki K., Soszynska J., Safety and risk optimization of a ferry technical system. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 1, 159-172, 2010

Kołowrocki K., Soszyńska-Budny J., Reliability and Safety of Complex Technical Systems and Processes: Modeling - Identification - Prediction - Optimization, London, Dordrecht, Heildeberg, New York, Springer, 2011

Kołowrocki K., Soszyńska-Budny J., Introduction to safety analysis of critical infrastructures. Proc. International Conference on Quality, Reliability, Risk, Maintenance and Safety Engineering - QR2MSE, Chendgu, China, 1-6, 2012a

Kołowrocki K., Soszyńska-Budny J., Introduction to safety analysis of critical infrastructures. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars 3, 73-88, 2012b

Kołowrocki K., Soszyńska-Budn, J., Prediction of Critical Infrastructures Safety. Proc. of The International Conference on Digital Technologies, Zilina, 141-149, 2014

Kołowrocki K., Soszyńska-Budny J. General approach to modelling operation threats and extreme weather hazards impact on critical infrastructure safety, Journal of Polish Safety and Reliability Association, Special Issue on EU-CIRCLE Project Critical Infrastructure Impact Models for Operation Threats and Climate Hazards, Part 1, Critical Infrastructure Connections, 8(3), 1-8, 2017a

Kołowrocki K. Soszyńska-Budny J. Torbicki M. Critical infrastructure operating area climate-weather change process including extreme weather hazards, Journal of Polish Safety and Reliability Association, Special Issue on EU-CIRCLE Project, Critical Infrastructure Operation and Climate-Whether Change Modelling, Prediction and Data Processing, 8(2), 15-24, 2017b

Kolowrocki K., Soszynska-Budny J., Modeling safety of multistate ageing systems. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminnars, Vol 8, No 3, 9-22, 2017c

Kuo W., Prasad V. R., An annotated overview of system-reliability optimization. IEEE Transactions on Reliability, 49(2), 176-187, 2000

Kuo W., Zuo M. J., Optimal Reliability Modeling: Principles and Applications. Hoboken: John Wiley & Sons, Inc., 2003

Levitin G., Lisnianski A., Optimisation of imperfect preventive maintenance for multistate systems. Reliability Engineering and System Safety 67, 193-203, 2000

Levitin G., Lisnianski A., Optimal replacement scheduling in multi-state series-parallel systems. Quality and Reliability Engineering International 16, 157-162, 2000

Tang H., Yin B. Q., Xi H. S., Error bounds of optimization algorithms for semi-Markov decision processes. International Journal of Systems Science, Vol. 38, No. 9,725-736, 2007

Vercellis S., Data mining and optimization for decision making. John Wiley & Sons Ltd, 2009

Xue J., On multi-state system analysis, IEEE Transaction on Reliability, 34, 329-337, 1985

Xue J., Yang K., Dynamic reliability analysis of coherent multi-stste systems, IEEE Transaction on Reliability, 4(44), 683-688, 1995