Free Vibrations of the Partially Tensioned Geometrically Non-Linear System Subjected to Euler's Load

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Abstract

In this study the fixed-fixed column subjected to axial Euler's load has been investigated. The load is placed between the fixed ends of the structure and its location can be changed along column's length. The boundary problem of free vibrations of the mentioned system has been formulated on the basis of Bernoulli – Euler theory and taking into account non-linear axial deformation relationship. Due to non-linear expressions the solution of the problem was done with small parameter method. In the paper the change of the first vibration frequency in relation to location and magnitude of the loading force was obtained. The relationship between natural vibration frequency and the amplitude is also discussed.

Keywords: column, Bernoulli-Euler's theory, free vibrations frequency, nonlinear system, characteristic curves, amplitude of vibrations, nonlinear component of free vibrations frequency

1. Introduction

In the literature the papers in which the vibrations of beams [1, 3, 4, 5, 11], columns [6, 12, 7, 8, 9, 15-21] and frame [10, 13, 14] are investigated can be found. In the boundary problem formulation process of these systems the theory of Bernoulli – Euler is mostly used. (see [2, 8-22]). This theory is sufficient when slender systems are taken into account (structures in which the total length is much greater than transverse dimensions) and when the system is not connected to mass elements with translational and rotational inertia. In the other cases (especially then higher order vibration frequencies are considered) the theory of beams proposed by Timoshenko should be used in which the shear energy and the rotational inertia energy of cross section are considered [1, 3-7]. The second problem which is present in the boundary problems are the linear and non-linear theories. When the non-linear one is taken into account the deformation of the elastic element at moderately large deflections is written in the form:

$$\varepsilon_i(x_i,t)=\frac{\partial U_i(x_i,t)}{\partial x_i}+\frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2 \tag{1}$$

where: $U_i(x_i,t)$, $W_i(x_i,t)$ longitudinal and transversal displacements respectively.

In non-linear systems in which the boundary problem is described by non-linear differential equations [2, 8, 15, 16, 19-22] the components of vibration frequency can be computed as dependent on amplitude of vibration (non-linear components of vibration frequency). The non-linear components may have great influence on vibration frequency and can't be omitted. In relation to the method of solution of the boundary problem the estimation of the non-linear component may be hard and time consuming. Nonlinear components of vibration frequencies of complex non-linear systems were investigated by Tomski and Przybylski [16], Przybylski [9] and Sokół [12] in relation to the conservative and non-conservative loads. The estimated components of vibration frequency were computed at rectilinear components of static equilibrium. The non-linear component of vibration frequency at rectilinear as well as at curvilinear form of static equilibrium of the column loaded by Euler's force were discussed in [21, 22]. At specific load studies on an influence of an amplitude on natural vibration frequency can be found in the following publications [19, 20]. The results were discussed at rectilinear and curvilinear form of static equilibrium. It has been shown that an influence of an amplitude on vibration frequency highly depends on the magnitude of external load. The use of specific load allows one to choose such load magnitude along with the parameters of the loading structure that an influence of an amplitude is negligible.

The main purpose of this paper is to present the results of the studies on the magnitude and location of the external force on natural vibration frequency (both linear and non-linear components) of the partially tensioned geometrically non-linear column.

2. Boundary problem

The considered column is presented in the figure 1. The column is fixed on both ends and loaded by a force P with constant line of action regardless to the deflection of the host element. The line of action of the force is compatible to the undeformed axis of the column. The point of location of the force is described by ζ parameter which is calculated as a relationship between length l_1 to total length l:

$$\zeta=\frac{l_1}{l} \tag{2}$$

The bending stiffness and compression stiffness and mass of the tensioned part (above the point of external force location) and compressed one are as follows: $((EA)_1 = (EA)_2 = (EA)$; $(EJ)_1 = (EJ)_2 = (EJ)$; $(\rho A)_1 = (\rho A)_2 = (\rho A))$.

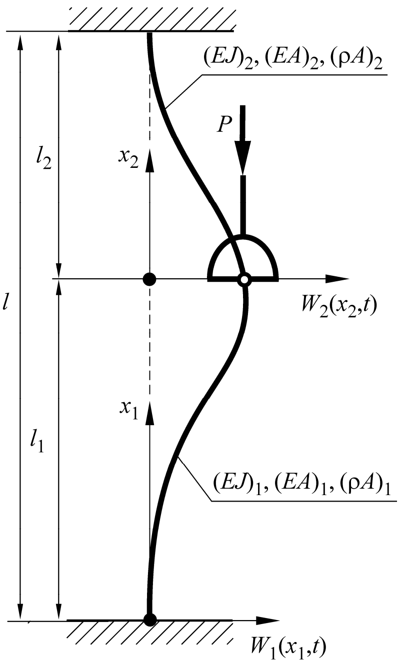

Figure 1. Considered column

The boundary problem is formulated on the basis of relation (1) and Bernoulli – Euler theory. The differential equations (in transversal and longitudinal direction) of vibration of the column are as follows:

$$\frac{\partial^4 W_i(x_i,t)}{\partial x_i^4}+S_i(t)\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}+(\rho A)_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2}=0 \tag{3}$$

$$U_i(x_i,t)-U_i(0,t)=-\frac{S_i(t)}{(EA)_i}x_i-\frac{1}{2}\int_0^{x_i}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2 dx_i \tag{4}$$

where: $S_i(t)$ – force in i – th element, $U_i(x_i,t)$, $W_i(x_i,t)$ – longitudinal and transversal displacements of the cross section of the i – th element described by coordinate x_i.

The boundary conditions of the considered system are presented below (5a-l):

$$U_1(0,t)=U_2(l_2.t)=W_1(0,t)=\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=0}=W_2(l_2,t)=\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|^{x_2=l_2}=0 \tag{5a-f}$$

$$U_1(l_1,t)=U_2(0,t),\ W_1(l_1,t)=W_2(0,t),\ S_1-S_2=P \tag{5g-i}$$

$$(EJ)_1\left.\frac{\partial^3 W_1(x_1,t)}{\partial x_1^3}\right|^{x_1=l_1}-(EJ)_2\left.\frac{\partial^3 W_2(x_2,t)}{\partial x_2^3}\right|_{x_2=0}+P\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=0}=0 \tag{5j}$$

$$\left.(EJ)_1 \frac{\partial^2 W_1(x_1,t)}{\partial x_1^2}\right|^{x_1=l_1} - \left.(EJ)_2 \frac{\partial^2 W_2(x_2,t)}{\partial x_2^2}\right|_{x_2=0} = 0 \tag{5k}$$

$$\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|^{x_1=l_1} = \left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=0} \tag{5l}$$

The further consideration are performed in non-dimensional form with the following relations:

$$\xi_i = \frac{x_i}{l_i},\ w_i(\xi_i,\tau) = \frac{W_i(x_i,\tau)}{l_i},\ u_i(\xi_i,\tau) = \frac{U_i(x_i,\tau)}{l_i},\ k_i^2(\tau) = \frac{S_i(\tau) l_i^2}{(EJ)_i}, \tag{6a-d}$$

$$\Omega_i^2 = \frac{(\rho A)_i \omega^2 l_i^4}{(EJ)_i},\ \tau = \omega t,\ \Theta_i = \frac{A_i l_i^2}{J_i},\ i = 1, 2. \tag{6e-g}$$

where ω is the natural vibration frequency.
The parameters presented in (6) are substituted into differential equations and boundary conditions what leads to their non-dimensional forms. The non-linear elements of the differential equations and boundary conditions are written into power series of the small parameter of an amplitude. In this study only the rectilinear form of static equilibrium is investigated at which the series are as follows:

$$w_i(\xi,\tau) = \sum_{j=1}^{N} \varepsilon^{2j-1} w_{i2j-1}(\xi,\tau) + O\left(\varepsilon^{2(N+1)}\right), \tag{7a}$$

$$u_i(\xi,\tau) = u_{i0}(\xi) + \sum_{j=1}^{N} \varepsilon^{2j} u_{i2j}(\xi,\tau) + O\left(\varepsilon^{2(N+1)}\right) \tag{7b}$$

$$k_i^2(\tau) = k_{i0}^2 + \sum_{j=1}^{N} \varepsilon^{2j} k_{i2j}^2(\tau) + O\left(\varepsilon^{2(N+1)}\right), \tag{7c}$$

$$\Omega_i^2 = \Omega_{i0}^2 + \sum_{j=1}^{N} \varepsilon^{2j} \Omega_{i2j}^2 + O\left(\varepsilon^{2(N+1)}\right) \tag{7d}$$

where:

$$w_{i1}(\xi,\tau) = \overset{(1)}{w}_{i1}(\xi)\cos\tau,\ w_{i3}(\xi,\tau) = \overset{(1)}{w}_{i3}(\xi)\cos\tau + \overset{(3)}{w}_{i3}(\xi)\cos 3\tau;\ \ldots \tag{8a,b}$$

$$u_{i2}(\xi,\tau) = \overset{(0)}{u}_{i2}(\xi) + \overset{(2)}{u}_{i2}(\xi)\cos 2\tau;\ \ldots \tag{8c}$$

$$k_{i2}^2(\tau) = \overset{(0)}{k}{}_{i2}^2 + \overset{(2)}{k}{}_{i2}^2 \cos 2\tau;\ \ldots \tag{8d}$$

On the basis of the obtained equations and boundary conditions the distribution of the external load on the elements of the structure can be found as well as magnitudes of the axial forces during vibrations and basic (ω_0) and nonlinear (ω_2) components of natural vibrations.

3. Results of numerical simulations

The results of numerical simulations are presented with the use of the following parameters:

$$\zeta_\Omega = \frac{\Omega - \Omega_0}{\Omega_0} 100\%, \ \lambda = \frac{Pl^2}{(EJ)}, \ \Omega = \frac{\omega^2 (\rho A) l^4}{(EJ)}, \tag{9a-c}$$

$$\Omega_i = \frac{\omega_i^2 (\rho A) l^4}{(EJ)}; \ i = 0, 2; \ \omega^2 = \omega_0^2 + \varepsilon^2 \omega_2^2 \tag{9d,e}$$

The parameters expressed by the formulas (9a-e) are the non-dimensional ones. Wherefore no information about material properties and cross-section area of the column can be found in this paper.

In the numerical calculations of an influence of a non-linear component ω_2 on natural vibration frequency ω the magnitude of the small parameter of an amplitude was defined as $\varepsilon = 0.008$.

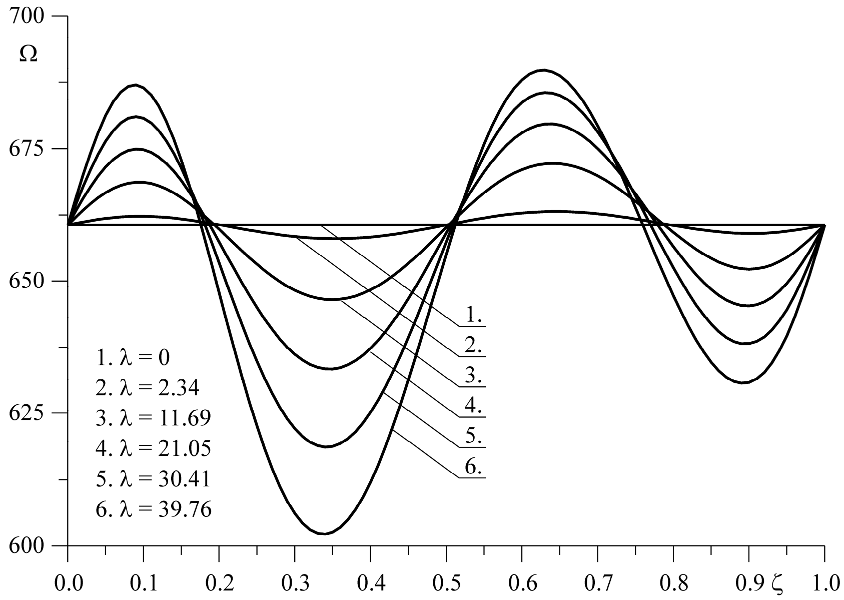

Figure 2. Magnitude of vibration frequency Ω parameter in relation to the point of location of external load ζ

In the figure 2 the change of vibration frequency parameter Ω (taking into account the non-linear component) in relation to the point of location of external load ζ has been presented. The calculations were performed at different magnitudes of external load parameter - λ. The vibration frequency highly depends on the magnitude and point of location of the external load. An increase of the magnitude of the external load causes an increase of the difference between the highest and the lowest magnitudes of vibrations in the investigated range of ζ. In this range the three points along the length of the column can be found in which the natural vibration frequency is not highly dependent on external load.

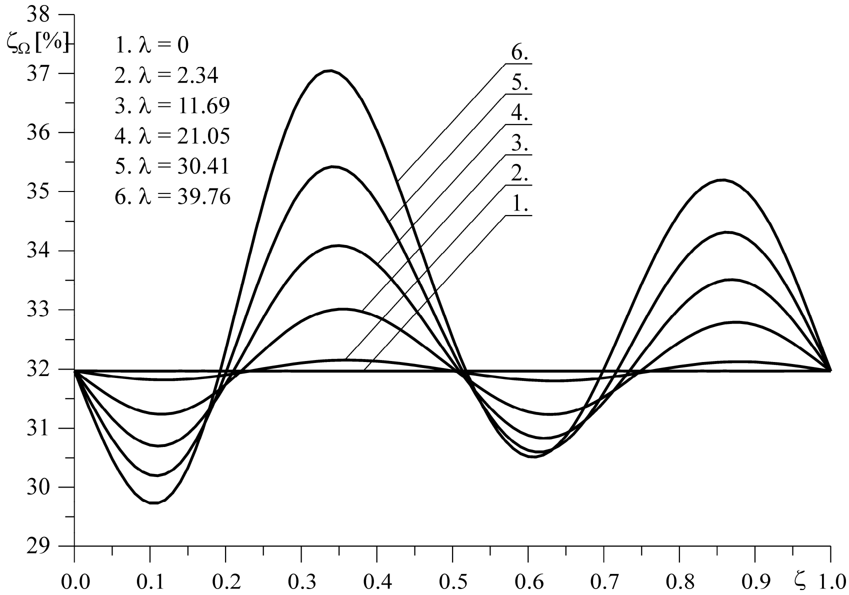

Figure 3. Magnitude of an influence on an amplitude on vibration frequency ζ_ω parameter in relation to the point of location of external load ζ

In the figure 3 the change of ζ_Ω parameter along length of the column at different magnitudes of external load has been plotted. It has been shown that an influence of an amplitude on natural vibrations depends on both external load magnitude and point of location of the external force. The highest magnitude of ζ_Ω has been found at $\zeta \approx 0.34$. In the unloaded system an influence of the second component of vibrations on vibration frequency is about 31.97 % at given amplitude corresponding to small parameter ε = 0.008.

4. Conclusions

In this paper the non-linear column fixed on both ends subjected to Euler's load (the load with constant line of action) has been investigated. The loading force was placed between the fixed ends of the structure. The boundary problem has been formulated on the basis of the Bernoulli – Euler theory and with taking into account the non-linear relationship of the axial deformation. In the final step of formulation of the boundary problem the small parameter method was used on the basis of which the computations of natural vibration frequency with consideration of linear and non-linear components (which depends on amplitude) were done. It has been shown that the natural vibration frequency of the investigated structure depends on both point of location and magnitude of the external force. The similar relationship can be observed at component which depends on amplitude of vibrations. It has been stated that the non-linear component of vibration can't be omitted especially at higher magnitudes of external load as well as at some point of location of external load. It's influence on final magnitude of vibration frequency can be significant but on the other hand it depends on amplitude.

In the future it is planned to develop of the studies started in this paper by addition of the elements which can have an influence on the behavior of the column during vibrations. The presented in this study results of numerical simulations may have engineering importance in investigation on the systems in which the point of location of the external load changes along their length (for example the screw along which the nut transferring loads changes position).

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