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Dynamic pricing policy in a growing market for a deteriorating product*

by

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Abstract: In this paper, we suggest an optimal pricing policy in a growing market for a deteriorating product. Here, the demand rate is considered as a function of selling price, time and stock level. When the product is introduced into the market, the demand for the product increases slowly according to the performance of the product and depending on market conditions. In maturity stage, the demand is gradually increasing and hence the need arises of developing a pricing policy in a growing market. This paper proves that there exists an optimal increasing-price policy, especially in a growing market for a product. The final recommendations are made based on a sensitivity analysis.

Keywords: inventory, dynamic pricing, quadratic demand, optimal price settings, deterioration

1. Introduction

In the real world, each business tries to achieve a possibly high profit with a possibly low production cost. Customer satisfaction is another big issue for any business, and for that purpose, the quality of the product should also be improved in order to secure the growing market. As in the present scenario, pricing strategies play an important role in customer satisfaction. A small investment can improve the quality of a product and thereby ensure a suitable price and enhance the brand image, which is beneficial for any industry. Existence of a pricing strategy is useful for every product and it is necessary to determine the best strategy depending on market conditions. In 1955, Whitin (1955) developed the idea of price theory for inventory models. This study established

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the importance of pricing policy for any business. Lau and Lau (1988) incorporated an explicit variable pricing strategy and compared it with a constant pricing strategy. While running along a similar path, Abad (1996) developed an inventory model, in which price and lot-size dependent demand were considered. Also, he considered shortage with partial back-ordering. Urban and Baker (1997) established the economic order quantity model with a new idea. In this model, the demand function was considered to be price, time and inventory level dependent. Arcelus and Srinivasan (1987) introduced the concept of discounts in price. Having used the concept of pricing policy an economic order quantity model was developed and presented by Datta and Paul (2001).

Actually, demand is less price-sensitive, if the price increases slowly, and then there is no effect on demand, but if price increases suddenly, then the demand decreases certainly. Datta and Paul (2001) analyzed a multi-period economic order quantity model that is useful in the retail business. They demonstrated how the selling price could be changed when considering a stock-dependent demand situation. Ray, Gerchak and Jewkes (2005) considered selling price as a decision variable and established an analogous model, where price is a mark-up over cost per unit. They suggested that good profits can be obtained when the manager is aggressive on price rather than when s/he is reducing the price too much. Wen and Chen (2005) developed an inventory model with a dynamic pricing strategy. They suggested that if the objective of a seller is to apply a dynamic pricing policy in the conduct of business, then they can maximize the total expected profit. In the context of the price changing strategy You and Hsieh (2006) estimated the sum of all relevant component costs and adjustment of the profit function.

Moreover, Roy and Chaudhuri (2008) designed an inventory model, in which they considered the stock level and the selling price dependent demand function and they also considered the selling price dependent production rates. They considered deterioration as a constant function and also extended the proposed demand function to the time-price or quadratic-price dependent demand or stochastically fluctuating demand pattern. Joglekar, Lee and Farahani (2008) designed an inventory model for e-tailers, in which they proved that the increasing price strategy is better than a constant price strategy. This model is applicable to such products that are more price sensitive. However, the deterioration factor is not considered in this model. Furthermore, Sajadieh, Akbari and Jokar (2009) proposed a model to find the relevant profit-maximizing decision variable values. This model is based on the joint total profit of both the vendor and the buyer. If the buyer and vendor cooperate with each other, and demand is more price-sensitive, then they can earn the maximum profit.

We observe that inventory has a seasonal track when the rate of demand depends upon both price and time. Banerjee and Sharma (2010) developed the model for the seasonal product, i.e. when the considered item has generally the seasonal demand rate, depending on both time and price. They considered the price as a decision variable and in the model, the profit function is a concave function of time and conditionally joint concave function of selling price and time. The demand rate, being a function of selling price and the time dependent holding cost were considered in an economic quantity model that was presented by Tripathi and Mishra (2010). This model is a deterministic inventory model for deteriorating items. In their model, the authors mentioned two cases, one with shortage and the second without shortage. They found that the optimum average profit without shortage is higher than that obtained with the assumption of shortage.

Many researchers have considered definite specific conditions in their investigations, such as time-dependent demand rate, price dependent demand, stock dependent demand, variable holding cost, preservation technology for preserving the deteriorating type items, etc. In this vein of approach, Sana (2010) designed an EOQ model in which the author considered the demand function as a function of selling price, and time proportional deterioration rate. Sana (2010) developed this model over an infinite time horizon for perishable products. Then, Sana (2011) proposed a pricing policy for deteriorating products under quadratic type of demand, which is a function of the selling price.

It is more than obvious that virtually all products are price sensitive. Some of them are highly price sensitive and others are less so. Correspondingly, a more general and representative inventory model is the one that considers the demand as a function of both stock level and selling price. Understandably, demand is a decreasing function of selling price and an increasing function of stock level. Shah, Patel and Lou (2011) extended the model of Sony and Shah (2008, 2009), who developed an inventory model by assuming the selling price to be a decision variable and ending the inventory at positive or zero. They also assumed limited floor space, profit maximization and kept deterioration rate as a constant; with these assumptions they developed an algorithm to find the optimal decision policy. Yang et al. (2013) designed a piecewise production inventory model for deteriorating products with price-sensitive demand. They proved that the multiple production cycles are better than a single production cycle. It certainly is a good opportunity to raise product prices if demand parameters increase. Khedlekar and Tiwari (2019) studied a dynamic pricing inventory model with constant deterioration rate. This study also revealed that the dynamic pricing policy provides better performance over the static pricing policy. By considering the stock dependent demand, this paper has been extended in the present study.

Deterioration is regarded as a natural phenomenon for inventories, as this is phenomenon observed very extensively for agricultural products, volatile liquids, pharmaceutical products, perfumes, radioactive substances, gasoline, food, medicine, semiconductor chips, and many others, which often have a high deterioration rate. In general, it is found that items always deteriorate continuously with respect to time, but deterioration not only varies with respect to products, but also can be controlled by applying some suitable preservation technology. Having used the preservation concept Khedlekar, Shukla and Namdeo (2016) developed an EOQ model, in which the demand for items is price sensitive and linearly decreases with respect to time. In this model, they proved that the profit function is a concave function of the optimal selling price. Mishra (2016) proposed a single-manufacturer single-retailer inventory model by incorporating the preservation technology cost for deteriorating items and determined the optimal retail price, replenishment cycle and the cost of preservation technology. Having assumed the concept of joint replenishment, an inventory model was developed by Mishra and Shanker (2017). In this model they studied the situation, in which, when an item is out of stock, the demand for it is met by an other item and any part of demand not met due to unavailability of the other item is lost.

Dynamic prices are take it or leave it prices, in the situation where the seller changes prices dynamically over time, based on factors such as the time of sale, demand information, and supply availability. As for our knowledge and the evidence from the literature survey, there is lack of contributions considering price-sensitive, time and stock dependent demand and there is lack of suggestions for a growing market. Price could be increased according to increasing demand in the growing market. So, motivation is derived to apply the increasing-price scenarios, especially in a growing market.

2. Assumptions & notations

We designed the proposed model by using the following assumptions and notations:

Assumptions:

1. The model is designed for the finite time horizon.
2. This model is designed for a single item.
3. In this model the deterioration rate of a defective item is considered.
4. Shortages are not allowed.
5. In this model in each distinct cycle the selling price is considered as a sequence of decision variables.
6. The demand function is quadratic with respect to selling price in each cycle and depends on time and stock level also.

Notations:

$D(p_j, t)$ – demand function, which varies quadratically with p_j and depends on time t (units/time unit),
$I_j(t)$ – on-hand inventory at time t in the j^{th} cycle (units),
p_j – selling price at the j^{th} period, which is the decision variable,
j – index of period, j refers to $[(j\text{-}1)T, jT]$,
n – number of cycles of different price,
T – length of each cycle, $T = \frac{L}{n}$ (time unit),
Q – preliminary lot-size (units),
L – time horizon,
q_j – inventory level at the start of cycle j,
C_h – inventory holding cost per unit per unit of time,
C_p – purchasing cost per unit of item,
θ – deterioration rate $(0 < \theta < 1)$ per unit of time,
K – price setting cost; this cost includes the resetting of the price label,
a – time scale parameter,
β & γ – price sensitivity parameters ($\gamma > 0$, $\beta > 0$),
ϕ – stock dependent consumption rate parameter $(0 \le \phi \le 1)$,
α – initial constant demand $(\alpha > 0)$,
H_j – the inventory holding cost at the j^{th} cycle per unit time,
H – the total inventory holding cost for n periods per unit time,
Π – the total profit per unit time.

3. The mathematical model

Consider an economic order quantity inventory system for price-sensitive seasonal items over a finite planning horizon of T periods. At the beginning of each sale period, the entrepreneur, purchases/manufactures the quantity Q of the seasonal product. The time horizon L is divided into n equal parts as $T = \frac{L}{n}$. So, the entrepreneur decides to increase the selling price in different sub-cycles according to market conditions and demand. The demand for seasonal and household products follows a quadratic function. It is assumed that α is the initial demand of the product, β and γ are positive price-sensitivity parameters. Then the demand will be

$$D(p_j, t) = \alpha e^{at} - \beta p_j - \gamma {p_j}^2 + \phi I_j(t),\ \alpha > 0,\ \beta > 0,\ \gamma > 0,\ j = 1, 2, 3..., n. \quad (3.1)$$

The inventory considered is supposed to change as the sum of the demand for products and deterioration, then the governing differential equation in the j^{th} cycle is

$$\frac{d}{dt} I_j(t) = -\theta I_j(t) - D(P_j, t) \quad (3.2)$$

with initial condition

$$I_j(0) = q_j,\ I_{j-1}(T) = I_j(0).$$

Here, q_j $(j = 1, 2..., n)$ are the stock levels at the beginning of the consecutive cycles j.

From Eq. (3.1), we have

$$\frac{d}{dt} I_j(t) = -(\alpha e^{at} - \beta p_j - \gamma {p_j}^2 + \theta I_j(t) + \phi I_j(t)) \tag{3.3}$$

and then the solution of this differential equation is provided by

$$I_j(t) = q_j e^{-(\theta+\phi)t} + \frac{\alpha}{a + (\theta+\phi)}(e^{-\theta t} - e^{at}) + \frac{1}{(\theta+\phi)}(\beta p_j + \gamma {p_j}^2)(1 - e^{-(\theta+\phi)t}),\quad j = 1, 2..., n \tag{3.4}$$

$$I_{j-1}(t) = q_{j-1} e^{-(\theta+\phi)t} + \frac{\alpha}{a + (\theta+\phi)}(e^{-(\theta+\phi)t} - e^{at}) + \frac{1}{(\theta+\phi)}(\beta p_{j-1} + \gamma {p_{j-1}}^2)(1 - e^{-(\theta+\phi)t}),\ j = 1, 2..., n. \tag{3.5}$$

Now, assuming that $I_{j-1}(T) = I_j(0)$

$$q_j = q_{j-1} e^{-(\theta+\phi)T} + \frac{\alpha}{a + (\theta+\phi)}(e^{-(\theta+\phi)T} - e^{aT}) + \frac{1}{(\theta+\phi)}(\beta p_{j-1} + \gamma {p_{j-1}}^2)(1 - e^{-(\theta+\phi)T}) \tag{3.6}$$

$$q_2 = q_1 e^{-(\theta+\phi)T} + \frac{\alpha}{a + (\theta+\phi)}(e^{-(\theta+\phi)T} - e^{aT}) + \frac{1}{(\theta+\phi)}(\beta p_1 + \gamma {p_1}^2)(1 - e^{-(\theta+\phi)T}) \tag{3.7}$$

if the lot-size $q_1 = Q$ and let $e^{(\theta+\phi)T} = x$, then we have from Eq.(3.7)

$$q_2 = Qx^{-1} + \frac{\alpha}{a + (\theta+\phi)}(x^{-1} - e^{aT}) + \frac{1}{(\theta+\phi)}(\beta p_1 + \gamma {p_1}^2)(1 - x^{-1})$$

$$q_3 = q_2 x^{-1} + \frac{\alpha}{a + (\theta+\phi)}(x^{-1} - e^{aT}) + \frac{1}{(\theta+\phi)}(\beta p_2 + \gamma {p_2}^2)(1 - x^{-1})$$

$$q_3 = Qx^{-2} + \frac{\alpha}{a+(\theta+\phi)}(x^{-1} - e^{aT})(1 + x^{-1}) + \frac{\beta}{(\theta+\phi)}((1 - x^{-1})(x^{-1}p_1 + p_2)$$
$$+ \frac{\gamma}{(\theta+\phi)}(1 - x^{-1})(x^{-1}{p_1}^2 + {p_2}^2),$$

and by mathematical induction

$$q_j = Qx^{j-1} + \frac{\alpha}{a+(\theta+\phi)}(x^{-1} - e^{aT})\sum_{i=2}^{j} x^{-(i-1)}$$
$$+\frac{\beta}{(\theta+\phi)}(1 - x^{-1})\sum_{i=2}^{j} x^{-(j-i)}p_{i-1} + \frac{\gamma}{(\theta+\phi)}(1 - x^{-1})\sum_{i=2}^{j} x^{-(j-i)}p_{i-1}^2, j \geq 2 \tag{3.8}$$

$$q_n x^{-1} + \frac{\alpha}{a+(\theta+\phi)}(x^{-1} - e^{aT}) + \frac{1}{(\theta+\phi)}(\beta p_j + \gamma {p_j}^2)(1 - x^{-1}) = 0$$

$$q_n = \frac{\alpha}{a+(\theta+\phi)}(xe^{aT} - 1) - \frac{1}{(\theta+\phi)}(\beta p_n + \gamma {p_n}^2)(x - 1). \tag{3.9}$$

Equations (3.8) and (3.9) lead to

$$Qx^{n-1} + \frac{\alpha}{a+(\theta+\phi)}(x^{-1} - e^{aT})\sum_{i=2}^{n} x^{-(i-1)} + \frac{\beta}{(\theta+\phi)}(1 - x^{-1})\sum_{i=2}^{n} x^{-(n-i)}p_{i-1}$$
$$+\frac{\gamma}{(\theta+\phi)}(1 - x^{-1})\sum_{i=2}^{n} x^{-(n-i)}p_{i-1}^2$$
$$= \frac{\alpha}{a+(\theta+\phi)}(xe^{aT} - 1) - \frac{1}{(\theta+\phi)}(\beta p_n + \gamma {p_n}^2)(x - 1)$$
$$Q = \frac{\alpha x^{n-1}}{a+(\theta+\phi)}[(e^{aT} - x^{-1})\sum_{i=1}^{n-1} x^{-(i-1)} + xe^{aT} - 1] - \frac{\beta}{(\theta+\phi)}(1 - x^{-1})\sum_{i=1}^{n} x^i p_i$$
$$-\frac{\gamma}{(\theta+\phi)}(1 - x^{-1})\sum_{i=1}^{n} x^i p_i^2. \tag{3.10}$$

The inventory carrying cost at the j^{th} cycle is

$$H_j = C_h \int_0^T I_{j(t)} dt.$$

$$H_j = C_h \int_0^T \{q_j e^{-(\theta+\phi)t} + \frac{\alpha}{a+(\theta+\phi)}(e^{-(\theta+\phi)t} - e^{at})$$
$$+ \frac{1}{(\theta+\phi)}(\beta p_j + \gamma {p_j}^2)(1 - e^{-(\theta+\phi)t})\}dt$$
$$H_j = C_h \Bigg[\frac{q_j}{(\theta+\phi)}(1-x^{-1}) + \frac{\alpha}{(\theta+\phi)(a+(\theta+\phi))}(1-x^{-1})$$
$$+ \frac{\alpha}{a(a+(\theta+\phi))}(1-e^{aT}) + \frac{1}{(\theta+\phi)^2}(\beta p_j + \gamma {p_j}^2)((\theta+\phi)T + x^{-1} - 1)\Bigg]. \tag{3.11}$$

Equations (3.8) and (3.11) lead to

$$H_j = C_h \Bigg[\left(\frac{(1-x^{-1})}{(\theta+\phi)} \right) \{Qx^{j-1} + \frac{\alpha}{a+(\theta+\phi)}(x^{-1} - e^{aT}) \sum_{i=2}^{j} x^{-(i-1)}$$
$$+ \frac{\beta}{(\theta+\phi)}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1} + \frac{\gamma}{(\theta+\phi)}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1}^2\}$$
$$+ \frac{\alpha}{(\theta+\phi)(a+(\theta+\phi))}(1-x^{-1}) + \frac{\alpha}{a(a+(\theta+\phi))}(1-e^{aT})$$
$$+ \frac{1}{(\theta+\phi)^2}(\beta p_j + \gamma {p_j}^2)((\theta+\phi)T + x^{-1} - 1)\Bigg]. \tag{3.12}$$

The total inventory carrying cost for n periods is

$$H = \sum_{j=1}^{n} C_h \Bigg[\left(\frac{(1-x^{-1})}{(\theta+\phi)} \right) \{Qx^{j-1} + \frac{\alpha}{a+(\theta+\phi)}(x^{-1} - e^{aT}) \sum_{i=2}^{j} x^{-(i-1)}$$
$$+ \frac{\beta}{(\theta+\phi)}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1} + \frac{\gamma}{(\theta+\phi)}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1}^2\}$$
$$+ \frac{\alpha}{(\theta+\phi)(a+(\theta+\phi))}(1-x^{-1}) + \frac{\alpha}{a(a+(\theta+\phi))}(1-e^{aT})$$
$$+ \frac{1}{(\theta+\phi)^2}(\beta p_j + \gamma {p_j}^2)((\theta+\phi)T + x^{-1} - 1)\Bigg].$$

Putting the value of Q into the above, we get

$$
\begin{aligned}
H = {} & \\
\frac{C_h}{(\theta+\phi)^2} & \Bigg[(\theta+\phi)(1-x^{-n})\left(\frac{\alpha x^{n-1}}{a+(\theta+\phi)}[(e^{aT}-x^{-1})\sum_{i=1}^{n-1} x^{-(i-1)} + xe^{aT}-1\right) \\
& -\beta(1-x^{-1})\sum_{i=1}^{n} x^i p_i - \gamma(1-x^{-1})\sum_{i=1}^{n} x^i p_i^2 \\
& +\frac{\alpha(\theta+\phi)}{a+(\theta+\phi)}(1-x^{-1})(x^{-1}-e^{aT})\sum_{j=1}^{n}\sum_{i=2}^{j} x^{-(i-2)} \\
& +\beta(1-x^{-1})^2\sum_{j=1}^{n}\sum_{i=2}^{j} x^{-(j-i)} p_{i-1} + \gamma(1-x^{-1})^2\sum_{j=1}^{n}\sum_{i=2}^{j} x^{-(j-i)} p_{i-1}^2 \\
& +\frac{n\alpha(\theta+\phi)^2}{a+(\theta+\phi)}\left(\frac{(1-x^{-1})}{(\theta+\phi)} + \frac{(1-e^{aT})}{a}\right) \\
& +((\theta+\phi)T + x^{-1} - 1)\sum_{j=1}^{n}(\beta p_j + \gamma {p_j}^2)\Bigg].
\end{aligned}
$$

After solving the above equation, we obtain

$$
\begin{aligned}
H = \frac{C_h}{(\theta+\phi)^2} & \Bigg[\frac{\alpha(\theta+\phi)}{a+(\theta+\phi)}(x^{n-1}-x^{-1})\left((e^{aT}-x^{-1})\sum_{j=1}^{n-1} x^{-(j-1)} + xe^{aT}-1\right) \\
& -\beta(1-x^{-1})\left\{\sum_{j=1}^{n}(x^j+1)p_j - 2\sum_{j=1}^{n} x^{-(n-j)} p_j\right\} \\
& -\gamma(1-x^{-1})\left\{\sum_{j=1}^{n}(x^j+1)p_j^2 - 2\sum_{j=1}^{n} x^{-(n-j)} {p_j}^2\right\} \\
& +\frac{\alpha(\theta+\phi)}{a+(\theta+\phi)}(x^{-1}-e^{aT})\sum_{j=1}^{n}(1-x^{-(j-1)}) \\
& +\frac{n\alpha(\theta+\phi)^2}{a+(\theta+\phi)}\left(\frac{(1-x^{-1})}{(\theta+\phi)} + \frac{(1-e^{aT})}{a}\right) \\
& +((\theta+\phi)T + x^{-1} - 1)\left\{\beta\sum_{j=1}^{n} p_j + \gamma\sum_{j=1}^{n} {p_j}^2\right\}\Bigg]. \qquad (3.13)
\end{aligned}
$$

The sales revenue from n cycles is

$$R = \sum_{j=1}^{n} \left(\int_{(j-1)T}^{jT} D(p_j)(t)dt \right) p_j$$
$$= \sum_{j=1}^{n} \left(\int_{(j-1)T}^{jT} (\alpha e^{at} - \beta p_j - \gamma {p_j}^2)dt \right) p_j$$
$$= \sum_{j=1}^{n} \left[\frac{\alpha}{a} \left(e^{ajT} - e^{a(j-1)T} \right) p_j - (\beta {p_j}^2 + \gamma {p_j}^3)T \right]$$
$$R = \frac{\alpha}{a} \sum_{j=1}^{n} p_j \left(e^{a_j T} - e^{a(j-1)T} \right) - \left\{ \beta \sum_{j=1}^{n} {p_j}^2 + \gamma \sum_{j=1}^{n} {p_j}^3 \right\} T. \tag{3.14}$$

Then, the total net profit is calculated as

$$\Pi(n,p) = R - H - C_p Q - nK$$

meaning that

$$\Pi(n,p) = R = \frac{\alpha}{a} \sum_{j=1}^{n} p_j \left(e^{ajT} - e^{a(j-1)T} \right) - \left\{ \beta \sum_{j=1}^{n} {p_j}^2 + \gamma \sum_{j=1}^{n} {p_j}^3 \right\} T$$
$$- \frac{C_h}{(\theta+\phi)^2} \left[\frac{\alpha(\theta+\phi)}{a+(\theta+\phi)} (x^{n-1} - x^{-1}) \left((e^{aT} - x^{-1}) \sum_{j=1}^{n-1} x^{-(j-1)} + x e^{aT} - 1 \right) \right.$$
$$- \beta(1 - x^{-1}) \left\{ \sum_{j=1}^{n} (x^j + 1) p_j - 2 \sum_{j=1}^{n} x^{-(n-j)} p_j \right\}$$
$$- \gamma(1 - x^{-1}) \left\{ \sum_{j=1}^{n} (x^j + 1) p_j^2 - 2 \sum_{j=1}^{n} x^{-(n-j)} {p_j}^2 \right\}$$
$$+ \frac{\alpha(\theta+\phi)}{a+(\theta+\phi)} (x^{-1} - e^{aT}) \sum_{j=1}^{n} (1 - x^{-(j-1)})$$
$$+ \frac{n\alpha(\theta+\phi)^2}{a+(\theta+\phi)} \left(\frac{(1-x^{-1})}{(\theta+\phi)} + \frac{(1-e^{aT})}{a} \right)$$

$$
\begin{aligned}
&+((\theta+\phi)T+x^{-1}-1)\left\{\beta\sum_{j=1}^{n}p_j+\gamma\sum_{j=1}^{n}{p_j}^2\right\}\Bigg]\\
&-C_p\left[\frac{\alpha x^{n-1}}{a+(\theta+\phi)}\left((e^{aT}-x^{-1})\sum_{i=1}^{n-1}x^{-(i-1)}+xe^{aT}-1\right)\right. \qquad (3.15)\\
&\left.-\frac{\beta}{(\theta+\phi)}(1-x^{-1})\sum_{i=1}^{n}x^ip_i-\frac{\gamma}{(\theta+\phi)}(1-x^{-1})\sum_{i=1}^{n}x^ip_i^2\right]-nK.
\end{aligned}
$$

THEOREM 3.1 *Assume a solution p^* of equation ${p^*}^2+\eta_1p^*+\eta_2=0$, in the interval (C_p,∞), satisfying*

$$
\begin{aligned}
&\{-2\beta-6\gamma p\}-\\
&\frac{C_h}{(\theta+\phi)^2}[-(1-x^{-1})\gamma 2(x^j+1)-4x^{-(n-j)}+2\gamma((\theta+\phi)T+x^{-1}-1)]\\
&+\frac{2C_p}{(\theta+\phi)}\gamma x^j(1-x^{-1})<0,
\end{aligned}
$$

then $\Pi(n,p^*)$ has maximum value at p^*, for a fixed value of n.

PROOF Differentiate partially with respect to p_j the profit function $\Pi(n,p)$; we have

$$
\begin{aligned}
\frac{\partial\Pi}{\partial p_j}&=\left[\frac{\alpha}{a}(e^{ajT}-e^{a(j-1)T})-(2\beta p_j+3\gamma{p_j}^2)T\right]-\frac{C_h}{(\theta+\phi)^2}\Big[-(1-x^{-1})\beta\{(x^j+1)\\
&-2x^{-(n-j)}\}-(1-x^{-1})\gamma\{2(x^j+1)p_j-4x^{-(n-j)}p_j\}+(\theta T+x^{-1}-1)\{\beta+2\gamma p_j\}\Big]\\
&+C_p\left[\frac{\beta}{(\theta+\phi)}(1-x^{-1})x^j+\frac{2\gamma}{(\theta+\phi)}(1-x^{-1})x^jp_j\right]\frac{\partial^2\Pi}{\partial p_i\partial p_j}=0,\ \text{for } i\neq j
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2\Pi}{\partial^2{p_j}^2}&=-2\beta-6\gamma p_j\\
&-\frac{C_h}{(\theta+\phi)^2}\left[-(1-x^{-1})\gamma 2(x^j+1)-4x^{-(n-j)}+2\gamma((\theta+\phi)T+x^{-1}-1)\right]\\
&+\frac{2C_p}{(\theta+\phi)}\gamma x^j(1-x^{-1})
\end{aligned}
$$

Now,

$$
\frac{\partial\Pi}{\partial p_j}=0,\ \text{imply } {p_j}^2+\eta_1p_j+\eta_2=0
$$

where

$$\eta_1 = \frac{1}{3\gamma}\left[2\beta T + \frac{C_h}{(\theta+\phi)^2}\Big\{-2(1-x^{-1})\gamma(x^j + 1 - 2x^{-(n-j)}) \right.$$
$$\left. + 2\gamma((\theta+\phi)T + x^{-1} - 1)\Big\} - 2\frac{c_p}{(\theta+\phi)}\gamma(1-x^{-1})x^j\right]$$
$$\eta_2 = \frac{1}{3\gamma}\left[-\frac{\alpha}{a}(e^{a_j T} - e^{a(j-1)T}) + \frac{C_h}{(\theta+\phi)^2}\Big\{-(1-x^{-1})\beta(x^j + 1 - 2x^{-(n-j)} \right.$$
$$\left. + \beta((\theta+\phi)T + x^{-1} - 1)\Big\} - \frac{C_p}{(\theta+\phi)}\beta(1-x^{-1})x^j\right].$$

Then it is clear that for fixed n, if equation ${p^*}^2 + \eta_1 p^* + \eta_2 = 0$ has a solution $p*$ in the interval (C_p, ∞), satisfying

$$\{-2\beta - 6\gamma p\} - \frac{C_h}{(\theta+\phi)^2}[-(1-x^{-1})\gamma 2(x^j + 1) - 4x^{-(n-j)}$$
$$+2\gamma((\theta+\phi)T + x^{-1} - 1)] + \frac{2C_p}{(\theta+\phi)}\gamma x^j(1-x^{-1}) < 0,$$

then profit is maximum at p^*. □

4. Numerical example & simulation

4.1. Numerical example for the proposed model

Example 4.1 *To illustrate the proposed model, we are considering an exemplary data set,* $a = 0.0001$, $\alpha = 100$, $\beta = 4$, $\gamma = 0.006$, $\theta = 0.006$, $\phi = 0.004$, $L = 90$, $C_h = 0.007$, $C_p = 4$, $K = 800$, *and the demand function* $D(p_j, t) = \alpha e^{at} - \beta p_j - \gamma {p_j}^2 + \phi I_j(t)$*; on applying the output of the proposed model and using the solution procedure suggested, we get*

Put n=1, in Eq. (3.15) *and* $j = 1$, *obtaining* $p_1 = 15.63$, *in interval* $[0, 90]$ *by Theorem (3.1)*
$Q_1 = 5332$, *by Eq. (3.10)*
$R_1 = 51295.55$ *by Eq. (3.14)*
$\Pi(1, p) = 27731.67$ *by Eq. (3.15)*

Put n=2 in Eq. (3.15) *and* $j = 1, 2$, *obtaining* $p_1 = 14.97$ *in interval* $[0, 45)$
$p_2 = 16.49$, *in interval* $[45, 90]$ *by Eq.(3.1)*
$Q_2 = 5126$, *by Eq.(3.10)*
$R_2 = 50820.91$, *by Eq.(3.14)*
$\Pi(2, p) = 28548.17$ *by Eq.(3.15)*

Put n=3 in Eq. (3.15) *and* $j = 1, 2, 3$, *obtaining* $p_1 = 14.80$ *in interval* $[0, 30)$, $p_2 = 15.67$, *in interval* $[30, 60)$, $p_3 = 16.83$, *in interval* $[60, 90]$

$Q_3 = 5069$ *by Eq. (3.10)*
$R_3 = 50669.91$ *by Eq. (3.14)*
$\Pi(3, p) = 24333.41$ *by Eq. (3.15)*
because the profit for $n = 2$, $(\Pi(n^*, p) = 28548.17)$ *is higher (see Table 1) than for* $n = 1$ (27731.67) *and for* $n = 3$ (24331.41), *therefore the optimal selling price in the first cycle [0, 60] is* $p_1{}^* = 13.98$, *and the optimal selling price in the second cycle (60, 120](see Fig.1) is* $p_2{}^* = 15.27$.

Table 1. Optimal solution of the numerical example

n	p_1	p_2	p_3	Q	R	$\Pi(n,p)$
1	15.63	-	-	5332	51295.55	27731.67
2*	**14.97**	**16.49**	**-**	**5126**	**50820.91**	**28548.17**
3	14.80	15.67	16.83	5069	50669.91	24333.41

4.2. Sensitivity analysis

To simulate the proposed model, we used the original data as in Example (4.1.) To examine the effect of various parameters on the output, we vary only one parameter at a time, treating other parameter values as constant.

Table 2. Effect of changes of parameters β & γ

Sensitivity analysis for parameter β & γ						
β	p_1	p_2	p_3	Q	R	$\Pi(n,p)$
4	14.97	16.49	-	5126	50820.90	28548.17
5	12.64	14.15	-	4567	38787.22	18953.39
6	11.05	12.56	-	4016	30260.84	12837.54
7	9.90	11.41	-	3468	23785.54	8759.96
γ	p_1	p_2	p_3	Q	R	$\Pi(n,p)$
.006	14.97	16.49	-	5126	50820.90	28548.17
0.06	12.76	14.19	-	5037	42953.30	21329.44
0.5	8.29	9.69	-	3142	18674.11	6220.290
0.6	7.92	9.33	-	2698	15635.36	5259.000

Table 2 shows the variation in model output with respect to the price-sensitivity parameter β. If the price-sensitivity parameter β increases, then the sales revenue decreases (see Fig. 2) which means that for the product, which is more price-sensitive than it permits to change the price-setting more frequently. In this connection, the optimal net profit decreases accordingly. The optimal price in the subsequent cycle in the decreasing order. So, management have to decide to change the sub-cycle conform to their needs and change the price accordingly.

Moreover, Table 2 shows the variation in model output with respect to the price-sensitivity parameter γ. On increasing the price-sensitivity parameter γ, the optimal profit decreases and follows the parameter β. It is, therefore, suggested to inventory managers to keep the β parameter high. For this, the managers have to advertise regarding product performance (see Fig. 3). We considered the dynamic pricing policy and the output, provided in Table 2, shows the sensitivity of the parameter values.

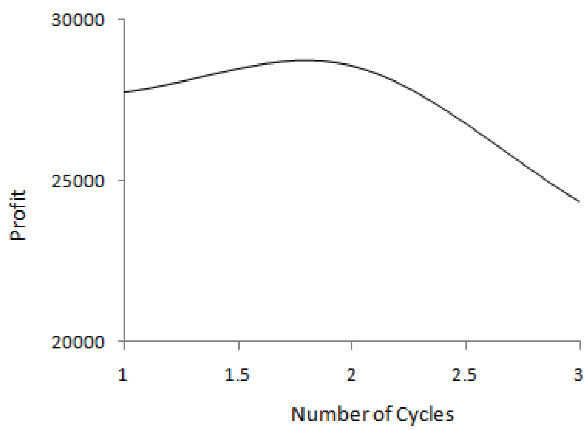

Figure 1. Total profit versus number of cycles n parameter

5. Conclusion

The benefits of dynamic pricing policies have long been known in many industries, such as airlines, hotels and electric utilities, railways, especially where the capacity is fixed in the short-term and is perishable. In this study, a dynamic pricing policy is developed for a product that follows an exponential increase of demand, which is also a quadratic function od the selling price. The sensitivity analysis reveals that highly price sensitive product permits a bigger optimal number of price settings. So, management have to decide on the number of price settings as per their need and find the respective model output accordingly. The pricing strategy in a growing market is entirely different than in the declining market. The realistic features are found for price sensitivity, especially for a growing market, i.e. the one, in which both demand and price are increasing. We incorporated the quadratic price-sensitive demand, which is more realistic

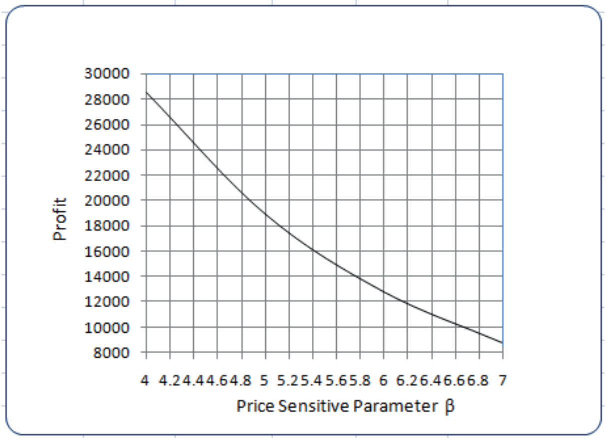

Figure 2. Total profit versus parameter β

for seasonal and also for some other products. An adequate numerical and computational study provides a better strategy for the vendor as well as for the retailers.

One can extend the model with unequal sub–cycle length and variable deterioration rates. One can also consider the demand as per requirement and simulate the model with different parameters.

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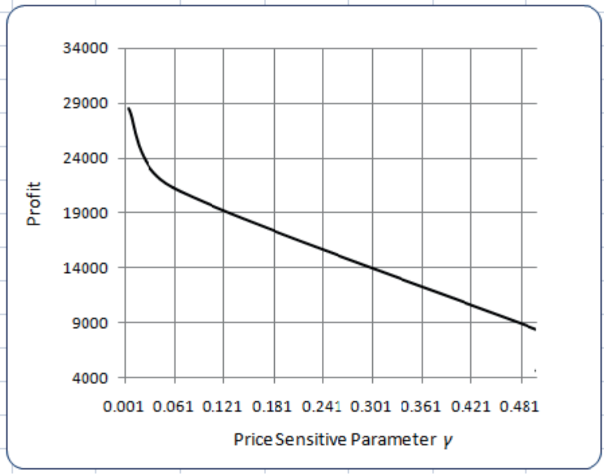

Figure 3. Total profit versus parameter γ

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