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## A METHOD OF COMPARATIVE EVALUATION OF CONTROL SYSTEMS BY THE SET OF PERFORMANCE MEASURES

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**Abstract.** It was proposed a method of comparative estimation of process control loops based on a complex of direct and indirect performance measurers taking into account stability margins. It was presented a comparison algorithm for complex performance parameter considering the influence of stability margins of a control loop on its properties. Features of the method of comparative estimation of process control systems performance have been considered as an example for the closed loop feedback control system with PI controller (4 variations) and the second order plus transport delay plant.

**Keywords:** feedback control systems, performance evaluation, stability, control

### METODA PORÓWNAWCZEJ OCENY UKŁADÓW AUTOMATYCZNEGO STEROWANIA WYKORZYSTUJĄCA ZBIÓR WSKAŹNIKÓW JAKOŚCI

**Streszczenie.** Zaproponowano metodę porównawczej oceny układów automatycznego sterowania procesami w oparciu o zbiór bezpośrednich i pośrednich wskaźników jakości, biorąc pod uwagę zapas stabilności. Został również przedstawiony algorytm porównania wg. złożonego parametru jakości, z uwzględnieniem wpływ zapasu stabilności układu regulacji na jego właściwości. Cechy charakterystyczne metody porównawczej oceny jakości układów regulacji zostały omówione na przykładzie zamkniętego układu sterowania z regulatorem PI (4 warianty) i obiektem regulacji drugiego rzędu z opóźnieniem transportowym.

**Słowa kluczowe:** systemy sterowania ze sprzężeniem zwrotnym, ocena jakości, stabilność, sterowanie

#### Introduction

Characteristics of an automatic control system which has a specific purpose depend on many factors. Main of them are: the structure of the system (closed loop, with local feedbacks, cascade, combined, etc.), a mathematical model of the plant, control algorithm, calculation method and criteria of performance control, technological requirements for process control response (oscillating or aperiodic). As a result, control of the same object in general can be achieved by different structural schemes and algorithms with different adjustment parameters.

This raises the problem of comparative performance assessment of control loops in order to choose the optimal one for a given plant. Most often it is carried out by transient response analysis for setpoint and disturbance channels. If a priority is one of the performance measures, such as the maximum dynamic error, then there is no problem, but in practice many different systems have different performance indexes [2]. The differences may be minor or very significant. So it's impossible to conclude confidently that one control loop is better than another, merely by comparing one or even several performance indexes. The task of comparative performance assessment becomes more difficult with increasing of the number of comparable indexes. Almost even with the same system structure and control algorithm using different methods of parametric synthesis it could be obtained a number of possible system parameters. They differ by controller parameters and performance measures. So the problem of choice appears. The problem of comparative assessment also occurs whenever it is necessary to make a comparison of control loops that have been synthesized by the same method, but with different control algorithms. However, it must be emphasized that the problem of comparative performance assessment in any case can be considered only for systems with the same purpose. Based on the fact that closed loop control system should compensate setpoint and disturbances changes, performance measures for both types of responses should be taken into account.

A number of control performance assessment techniques based on different performance measures were developed in recent decades [1, 2, 5, 6]. It is known that completely control loop performance can be characterized by a combination of direct and indirect performance indexes. Direct performance indexes can be  $y_{mg}$ ,  $y_{mf}$  – the maximum dynamic deviations of output variable;  $t_g$ ,  $t_f$  – settling time (the time of entering to the area 5% or 2% deviation of the settled value of controlled variable). In this

notation indices  $g$  and  $f$  are respectively related to setpoint and disturbance step responses.

At the control loop performance assessment, the Integral Squared Errors (ISE) of setpoint response ( $J_g$ ) and disturbance response ( $J_f$ ) could be used as indirect performance indexes. The ISEs can be calculated using the method described in [3]. It should be noted that the  $J_g$ ,  $J_f$  indexes have generalizing sense, but they are not sufficient, as they do not take into account forms of responses. Therefore, they should be considered together with the direct performance indexes. It is assumed that the static accuracy of the system is provided with appropriate selection of control algorithm.

In the analysis of control loops containing plants with time delay it is advisable to use the performance indexes in relative form:  $t_g / \tau$ ,  $t_f / \tau$ ,  $J_g / \tau$ ,  $J_f / \tau$ . However, in general, the absolute values of the indexes can be used.

Stability margin of control systems could be conveniently considered as the damping ratio  $M$  (the maximum of the amplitude absolute value divided by amplitude at the zero frequency point for closed loop control system). It is usually specified in definite limits, but it can assume different values in the limited range that affect the characteristics of the control system. The higher stability margin corresponds to decrease the value of  $M$ . However, this index loses its sense for aperiodic processes.

Comparative evaluation of control loops is not a trivial problem. The fact is that when controller parameters change, the performance indices of the control loop change simultaneously. And usually improving one or more of the indices causes deterioration of the other. For example, in a single-loop control systems, actions for improving of setpoint response lead to poor disturbances compensation and vice versa. Thus, it is necessary to compare control systems by a set of all direct and indirect performance indexes taking into account the grade of their differences [4].

#### 1. Description of the method

It has been proposed a method for comparative evaluation of control systems by a set of performance indexes considering the stability margin [4]. The method is based on the following main assumptions:

- 1) an important common feature of direct and indirect performance indexes is that the smaller are their values, the better is the control system; however, in relation to  $M$  index this statement is true only to a certain extent, because this

parameter can only be applied when implementing the control system with underdamped responses. If it is necessary to implement a feedback control system with damped response, then direct rates of the stability margins  $A_m$  (gain margin) and  $\varphi_m$  (phase margin) should be used;

- 2) all the performance indexes are considered as equally important, e.g. they have the same "weight". So it is assumed that the loss in one or more performance indexes can be compensated by gain in the others. If some indexes of one type differ for different control systems (for example, by one order and more), it is possible (though not necessarily) to exclude this index from the comparative analysis and consider it in the final evaluation of the control system;
- 3) only the relative values of the performance indexes should be compared as they are very different by their nature (e.g., some of them are dimensionless, while others, such as settling time, are dimensional quantities) and by absolute values.

## 2. Algorithm of the method implementation

Taking into account these assumptions comparative performance assessment of control systems can be implemented by the following algorithm:

- 1) For comparable control systems relative values of the same type indexes (performance indexes are defined in dimensionless form) are calculated by dividing the absolute value of this index by its maximum value:

$$\begin{aligned} \delta y_{gi} &= \frac{y_{mgi}}{(y_{mgi})_{\max}}, \quad \delta y_{fi} = \frac{y_{mfi}}{(y_{mfi})_{\max}}, \\ \delta t_{gi} &= \frac{t_{gi}/\tau}{(t_{gi}/\tau)_{\max}}, \quad \delta t_{fi} = \frac{t_{fi}/\tau}{(t_{fi}/\tau)_{\max}}, \\ \delta J_{gi} &= \frac{J_{gi}/\tau}{(J_{gi}/\tau)_{\max}}, \quad \delta J_{fi} = \frac{J_{fi}/\tau}{(J_{fi}/\tau)_{\max}}, \\ \delta M_i &= \frac{M_i}{(M_i)_{\max}}, \end{aligned}$$

where  $i = 1, 2, \dots$  – serial numbers of compared systems. And all the indexes of the same type for compared systems are rescaled to the same scale. This, in fact, is a basic requirement for correct comparison.

- 2) Calculation of the sums of relative values of all the performance indexes for each of the compared control systems

$$S_i = \delta y_{gi} + \delta y_{fi} + \delta t_{gi} + \delta t_{fi} + \delta J_{gi} + \delta J_{fi} + \delta M_i. \quad (1)$$

Decreasing of all the components in the expression (1) means improvement of the system and as a consequence the best control system by the set of performance indexes corresponds the minimum value of the sum  $S_i$ . Here should be taken into account not the absolute values of the sums  $S_i$ , but their relative values. However, it is more appropriate to rescale these values to a single range like it is done with the performance indexes. This means division the individual values by the maximum value among them. The result is an expression for optimality criterion of the control system by the set of performance indexes and stability margin which will be called a complex performance criterion

$$J_{com} = S_i / (S_i)_{\max} = \min \quad (2)$$

It is also necessary to pay attention to the stability margin influence on the properties of the control system. If the stability margin is too small, it threatens the disability of the control systems by large disturbances or changes of plant parameters. On the other hand, excessive stability margin leads to increasing of dynamic variations and settling time in the control loop. So there is the problem of choosing between dynamic accuracy and stability of the system. In practice, it is usually preferred the stability that is a factor of security. The above conflict can be

solved by finding the optimal balance between dynamic accuracy and stability margin of the control system. For this purpose, the method of comparative performance assessment (that is described above) by the  $J_{com}$  criterion can be used in a somewhat simplified version.

The equation (1) can be written in a form that takes into account only the parameters of dynamic accuracy and stability of the system, i.e.

$$S_i = \delta J_{gi} + \delta J_{fi} + \delta M_i. \quad (3)$$

Then the optimal ratio "dynamic accuracy / stability" will be reached in the control system for which

$$J_{JM} = S_i / (S_i)_{\max} = \min. \quad (4)$$

Calculations show that control systems which are optimal by the  $J_{com}$  criterion usually provide also the optimum or very close to it ratio of "dynamic accuracy / stability" which corresponds to the equation (4).

## 3. Example

Let's consider the features of the proposed method of comparative performance evaluation of control systems by the following example. For example, a simple closed loop feedback control system with PI controller is discussed below. The controlled process is described by the second order plus time delay model  $W_p(s) = K_p \cdot e^{-\tau s} / (Ts + 1)^2$ . The four versions of the control system have been analyzed which were named respectively System 1, 2, 3 and 4. Parameters of these systems and their performance measures are given in Table 1 and Table 2.

Table 1. Controller settings and stability margins of the compared systems

System #	$K_p K_c$	$T_i / \tau$	Stability margins		
			$A_m$	$\varphi_m$	$M$
System 1	1.153	7.574	6.111	56.82	1.051
<b>System 2</b>	<b>2.652</b>	<b>9.595</b>	<b>2.955</b>	<b>36.23</b>	<b>1.676</b>
System 3	1.446	7.970	5.002	51.70	1.152
System 4	3.945	11.779	2.125	25.83	2.402

Table 2. Performance measures of the compared control systems

System #	Setpoint response			Disturbance response		
	$y_{mg}$	$t_g / \tau$	$J_g / \tau$	$y_{mf}$	$t_f / \tau$	$J_f / \tau$
System 1	1.105	40.3	5.631	0.444	45.4	2.094
<b>System 2</b>	<b>1.345</b>	<b>44.67</b>	<b>4.342</b>	<b>0.33</b>	<b>36.2</b>	<b>0.752</b>
System 3	1.16	36.43	5.11	0.415	45.32	1.597
System 4	1.478	58.85	4.394	0.283	44.65	0.471

The relative values of performance measures and their sums are shown in Table 3.

Table 3. The components of the complex performance criterion

Indicator	System 1		System 2		System 3		System 4	
	abs.	rel.	abs.	rel.	abs.	rel.	abs.	rel.
$y_g, \delta y_g$	1.105	0.748	1.345	0.910	1.160	0.785	1.478	1.0
$y_f, \delta y_f$	0.444	1.0	0.330	0.743	0.415	0.935	0.283	0.638
$t_g, \delta t_g$	40.30	0.685	44.67	0.759	36.43	0.619	58.85	1.0
$t_f, \delta t_f$	45.40	1.0	36.20	0.797	45.32	0.998	44.65	0.983
$J_g, \delta J_g$	5.631	1.0	4.342	0.771	5.110	0.907	4.394	0.780
$J_f, \delta J_f$	2.094	1.0	0.752	0.358	1.597	0.763	0.471	0.225
$M, \delta M$	1.052	0.437	1.676	0.698	1.152	0.480	2.402	1.0
$S_i$	-	5.870	-	<b>5.037</b>	-	5.486	-	5.626
$\delta S$	-	1.0	-	<b>0.858</b>	-	0.935	-	0.958

The numerical data presented in Table 3, for greater clarity, are shown in Fig. 1.

As we can see, there is a clearly defined minimum for the System 2 (Fig. 2) that indexes its optimality by the criterion  $J_{com}$ . So the problem has been solved uniquely.

The data from Table 4 is shown in graphical form on Fig. 2.

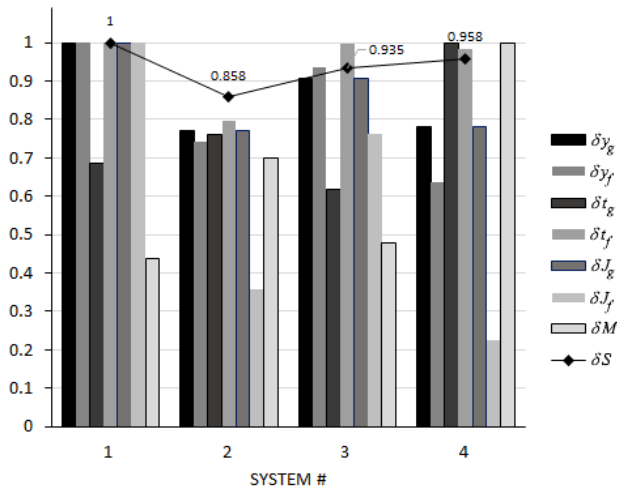


Fig. 1 a) the relative components of comparative performance assessment of control systems; b) resulting comparative assessment by set of performance indexes taking into account the stability margin

Table 4. The components of the complex performance criterion and the resulting

Indicator	System 1		System 2		System 3		System 4	
	abs.	rel.	abs.	rel.	abs.	rel.	abs.	rel.
$J_g, \delta J_g$	5.631	1.0	4.342	0.771	5.11	0.907	4.394	0.780
$J_f, \delta J_f$	2.094	1.0	0.752	0.358	1.597	0.763	0.471	0.225
$M, \delta M$	1.052	0.437	1.676	0.698	1.152	0.480	2.402	1.0
$S_i$	-	2.437	-	<b>1.828</b>	-	2.150	-	2.005
$\delta S$	-	1.0	-	<b>0.750</b>	-	0.882	-	0.822

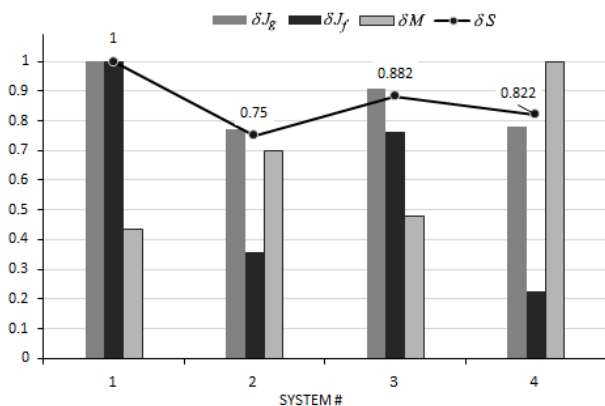


Fig. 2. Components of the complex performance criterion and the resulting assessment  $\delta S$  of the systems by the ratio "dynamic accuracy / stability"

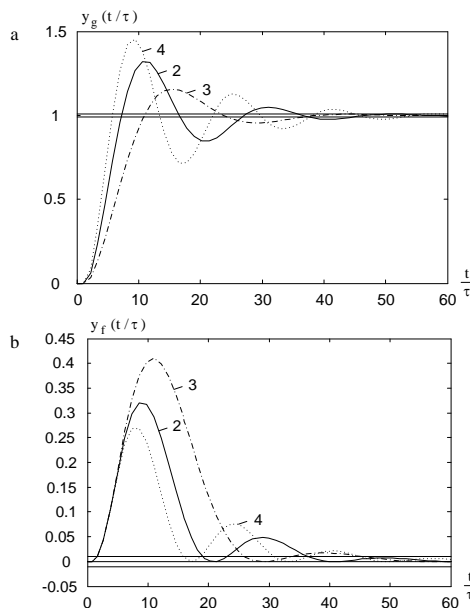


Fig. 3. Setpoint responses (a) and disturbance compensation responses (b) in compared systems; the digit labels corresponds to the system numbers

From Table 4 and Fig. 2 follows that the System 2 is also the best by the ratio "dynamic accuracy / stability".

Thus, as the results of the studies it was found that the worst by the  $J_{com}$  criterion is the System 1, so it could not be considered. The properties of the other three control systems are illustrated in Fig. 3, where the solid lines correspond to System 2 which is optimal by  $J_{com}$  criterion.

Fig. 3 clearly shows that improving the performance of setpoint response leads to the disturbance response deterioration. However, it could be seen that the system which is optimal by the  $J_{com}$  criterion provides a certain compromise between the performance of setpoint response and disturbance response.

### Conclusions

The described comparison algorithm can be easily programmed in different software packages for any number of comparable systems, but in practice it is sufficient to study three or four possible variants of control systems.

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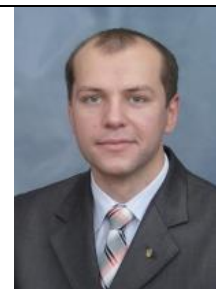
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