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Evolutionary computing approaches to optimum design of fuzzy logic controller for a flexible robot system

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This paper presents the design of a Fuzzy Logic Controller (FLC) whose parameters are optimized by using Genetic Algorithm (GA) and Bacteria Foraging Optimization (BFO) for tip position control of a single link flexible manipulator. The proposed FLC is designed by minimizing the fitness function, which is defined as a function of tip position error, through GA and BFO optimization algorithms achieving perfect tip position tracking of the single link flexible manipulator. Then the tip position responses obtained by using both the above controllers are compared to suggest the best controller for the tip position tracking.

Key words: flexible manipulator, fuzzy logic, genetic algorithm, bacteria foraging optimization, tip position tracking

1. Introduction

Flexible manipulators are suitable for some special applications such as in space, underwater, and high speed energy efficient manipulation due to their many advantages over their rigid-body counterparts in terms of an increased payload carrying capacity, less energy consumption, faster movements and longer reach. However link flexibility exhibits difficulties in control of tip position because settling of link vibration takes more time, therefore it prevents accurate tip positioning. Therefore, the control design of such manipulators is to be pursued such that the tip position will accurately tracks the desired position while suppressing the vibration in the link. The motivation lies in the optimum design of controller for such manipulators.

Fuzzy Logic Controllers (FLCs) have been successfully applied to a wide range of industrial processes as well as consumer products since its inception. FLC is one kind of nonlinear control scheme whose performances are quite robust for non-linear systems. So, it possesses the potential to achieve superior system performance over a linear control strategy. FLCs model the human decision making process with a collection of rules. With the help of the fuzzy sets, the linguistic terms used in these rules are converted into precise numeric values. Since both the rules and the fuzzy sets used in

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these rules play a crucial role in the outcome of the system, choosing the right rules and fuzzy sets becomes an important issue [1]. For a non-linear system like flexible manipulator, the design of FLC includes the finding of optimal fuzzy sets.

Evolutionary Computation (EC) provides an effective solution to optimization problems of such kind. Specific advantages of using EC over classic methods of optimization include the flexibility of the procedures, as well as the ability to self-adapt the search for optimum solutions on the fly. In this paper a new design method for FLC based on Genetic Algorithm (GA) and Bacteria Foraging Optimization (BFO) scheme is presented. Both GA and BFO are powerful optimization tools and the process of searching for best solution through minimization of a fitness function which in turn gives the best system performance makes both the optimization processes effective over other EC approaches. The controller is implemented on the flexible manipulator system in a loop and a certain fitness function is minimized over the loop thus ensuring better tip position control of the flexible manipulator. A common fitness function is chosen for both the processes and the simulation results are compared.

The paper is organized as follows: In Section 2 the dynamics of a single-link flexible manipulator system is described. The design of FLC for the manipulator tracking control problem is presented in Section 3. Section 4 presents the optimum FLC design techniques using GA optimization BFO is described in Section 5. Section 6 provides results and discussion using both GA and BFO based FLC controller. Conclusions are drawn in section 7.



Figure 1. A single-link flexible manipulator.

2. Dynamics of a single link flexible manipulator

Consider a single link flexible manipulator which is clamped at its base on the rotor of a motor and has a facing of attaching a pay load mass M_p , as shown in Fig. 1. Assuming the motion of the manipulator is in horizontal plane. The (X_0, Y_0) coordinate frame is considered as the base frame and the frame (X_1, Y_1) is considered as the local frame rotating with reference the hub. The rigid motion is described by the joint angle $\theta(t)$, while v(x,t) denotes the transversal deflection of the link measured with respect to the undeformed one. The length of the link is *l*, mass of the link is *m*, and moment of inertia of the hub is J_h .

The dynamic equations of motion of a single-link flexible manipulator can be obtained using the following Lagrangian approach. i.e by computing the kinetic energy E_k and the potential energy E_p of the manipulator and then forming the Lagrangian $L = E_k - E_p$. The dynamics of such single-link flexible manipulator using Lagrangian approach can be derived as [2].

The equations of motion for a planar single-link flexible arm can be obtained as

$$M(q)\ddot{q} + h(q,\dot{q}) + Kq = Qu \tag{1}$$

where $q = [\theta, \delta_1, \delta_2]^T$; θ is the link angle and δ_1 , δ_2 are the time-varying variables associated with the assumed spatial mode shapes $(\phi_1(x), \phi_2(x))$ which are used to define link deformation,

$$v(x,t) = \phi_1(x)\delta_1(t) + \phi_2(x)\delta_2(t).$$
⁽²⁾

Table 7. System parameters

l = 0.5 m	ho = 0.2 kg/m
m = 0.1 kg	$EI = 1 \text{ Nm}^2$
$J_h = 0.1 \text{ kgm}^2$	$M_p = 0.1 \text{ kg}$

M is the positive-definite symmetric inertia matrix which is found by using the system parameters given in Tab. 1 and is given by

$$M = \begin{bmatrix} 0.0446 + \delta_1^2 + \delta_2^2 & 0.0268 & 0.0226 \\ 0.0268 & 1 & 0 \\ 0.0226 & 0 & 1 \end{bmatrix}$$

and h is the vector of Coriolis and centrifugal forces and is given by

$$h = \begin{bmatrix} 2\dot{\theta}\dot{\delta}_1\delta_1 + 2\dot{\theta}\dot{\delta}_2\delta_2 & -\dot{\theta}^2\delta_1 & -\dot{\theta}^2\delta_2 \end{bmatrix}^T.$$

K is the stiffness matrix and is given by

$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.9096 & 0 \\ 0 & 0 & 12.7910 \end{bmatrix}$$

Q is the input weighting matrix due to the clamped link assumption and is given by

$$Q = \left[\begin{array}{rrr} 1 & 0 & 0 \end{array} \right]^T$$

and *u* is joint (actuator) torque.

3. Fuzzy logic controller design

Fig. 2 shows the structure of the FLC for tip position control of the single-link flexible manipulator. The error e(t) i.e. $e(t) = y(t) - y_{ref}(t)$ in the Universe of Discourse (UOD), and derivative of error \dot{e} in the UOD, CE are selected as the input variables to FLC where as torque (T) is considered as the output from the FLC that influences in damping out link vibration and steers actual position to the desired position.



Figure 2. Structure of fuzzy logic controller.

After having selected the input and output variables for FLC the membership functions are assigned as follows. Fuzzy membership functions for tip position error, error rate and controlled torque each consists of five membership functions. They are designated as Negative Large (NL), Negative Small (NS), Zero (ZE), Positive Small (PS) and Positive Large (PL) all with triangular fuzzy sets as shown in Fig. 3. The ranges of tip position error, error rate and torque considered for the tip position control of the single-link flexible arm are from -1.25 to 1.25, -2.50 to 2.50 and -2.50 to 2.50 respectively.

The fuzzy rule base of tip position control is adopted from operator's knowledge and experiences. Basically, the operator considers the target position, actual position and the end point speed during operation. Therefore, error and error rate are used in order to generate the rules. Tab. 2 lists the generated linguistic rules for tip position control [3].

The fuzzy inference for tip position control has adopted the Mamdani's Min-Max method which the fuzzy control output μ_T for the input μ_E and μ_{CE} is computed as

$$\mu_T = \vee [\mu_E \wedge \mu_{CE}] \tag{3}$$

where \vee and \wedge denote the maximum and minimum operators respectively while μ_E , μ_{CE} and μ_T denote degree of memberships of the error, error rate and control action respectively. Furthermore, in order to convert the fuzzy value to the crisp value of fuzzy

tip position control, the centre of area of defuzzification method is used

$$u_0 = \frac{\int \mu_u(u)udu}{\int \mu_u(u)du}$$
(4)

where u_0 is control input voltage obtained using Centre of Area (COA) defuzzification method.



Figure 3. Membership functions: (a) E, (b) CE, (c) T.



* $\Delta \rightarrow$ Link deflection angle

Figure 4. Evolutionary computation based FLC design.

CE E	NL	NS	ZE	PS	PL
NL	NL	NS	NS	ZE	ZE
NS	NL	NS	NS	ZE	ZE
ZE	NS	NS	ZE	PS	PS
PS	ZE	ZE	PS	PS	PL
PL	ZE	ZE	PS	PL	PL

Table 8. Fuzzy rule base of tip position control

4. GA based fuzzy logic controller design

Genetic algorithm (GA) is a search algorithm based on the observation that the reproduction and the principle of survival of the fittest, enables biological species to adapt to their environment and compete effectively for its resources. Control analysis for a single-link flexible manipulator includes tip position control and end effector path following control. In this paper tip position control analysis is considered for controller design. The objective of tip position control is to drive the tip of the flexible manipulator to a desired position as fast as possible with minimum vibration in the links.

Let P(x, y) is the desired position discussed above. Since the tip position P(x, y) is a vector with x and y coordinates being its elements, it is more convenient to consider the angle made by the tip mass at the joint as the tip position (in radian) for simulation purpose. Using inverse kinematics, the angles by which the link to is to be driven to reach P(x, y) is calculated as θ_d . A simple FLC controller is proposed for this proposes as shown in the Fig. 4, which takes the tip position error and error derivative as inputs and using the rule base presented in the Tab. 1 and membership functions shown in Fig 3 gives the controlled torque as output. The torque is then acts as an input to the single-link flexible manipulator system and controls the tip position guiding it to the desired point.

To improve the proposed FLC, Genetic Algorithm (GA) is used to find the optimal membership functions. The GA optimization technique consists of three basic processes; randomly initialization of the parameters to be optimized from a suitable search space, evaluation of performance fitness function and application of GA operators. First a suitable search space is chosen based on the convergence of tip position error and limitation of the model parameters. Then the first generation population is randomly generated within the range of the search space. The parameters set (population), which are called chromosomes, are of decimal form. For second process a proper fitness function should be chosen such that minimizing it will lead us to optimal parameter. In the case of flexible manipulator control, where the main objective being the tip position error minimization, the Integral Time-multiplied Absolute value of Errors (ITAE) is chosen as a suitable fitness. The fitness function that is used to optimize the membership functions which

influence the performance of FLC is given by

$$ff = ITAE = \int_{0}^{T} t |P(x,y) - p(x,y)| dt$$
(5)

where, P(x, y) is the desired tip position and p(x, y) is the actual tip position. For smaller values of the fitness functions, better tip performance can be achieved.

In the design of the proposed optimal FLC, two inputs, E, CE, and one output, T, are used. Each variable is described with five membership functions, as illustrated in Fig. 3.



Figure 5. Description of an individual [4].

The population consists of a set of individuals. Each individual is composed of three chromosomes (Fig. 5): chromosome I for E, chromosome II for CE and chromosome III for T [4]. The four intervals are the genes for each chromosome with each gene ranging from 0.01 to 0.99.

The shape of the fuzzy membership functions are determined by x_i , y_i , z_i . So to obtain those values, the relationship between (CX_i, CY_i, CZ_i) and (x_i, y_i, z_i) are established. As the maximum value of x_1 is -1.25 and the maximum value of extreme left in terms of intervals is $(CX_1 + CX_2)_{max} \approx 2$, there is a multiplication factor which is -0.625 (-1.25/2). Similarly the multiplication factor is calculated for other intervals and the relationships between (CX_i, CY_i, CZ_i) and (x_i, y_i, z_i) are given by [4]:

$$\begin{aligned} x_1 &= (CX_1 + CX_2)(-0.625) & x_2 = (CX_2)(-0.625) \\ y_1 &= (CY_1 + CY_2)(-1.25) & y_2 = (CY_2)(-1.25) \\ z_1 &= (CZ_1 + CZ_2)(-1.25) & z_2 = (CZ_2)(-1.25) \\ x_3 &= (CX_3)(0.625) & x_4 = (CX_3 + CX_4)(0.625) \\ y_3 &= (CY_3)(1.25) & y_4 = (CY_3 + CY_4)(1.25) \\ z_3 &= (CZ_3)(1.25) & z_4 = (CZ_3 + CZ_4)(1.25) \end{aligned}$$
(6)

The fitness function (5) is evaluated for each chromosome passing through the system model and then for that generation convergence condition is checked. If the optimization convergence limit is reached, then the optimization process is terminated as shown in Fig. 6. Otherwise the next generation is obtained through the genetic operators. GA operators play an important role in GA optimization process and are described in detail in the following.

A. Selection

This operator selects chromosomes in the population for reproduction. The fitter the chromosome, the more times it is likely to be selected to reproduce. So out of N chromosomes best N/2 chromosomes are selected based on the performance fitness function values determined using (5) (the chromosomes which give minimum fitness function values are chosen).

B. Cross-over

Randomly two parents are selected from the best N/2 chromosomes to exchange genetic information with each other and generate two new individuals. The cross-over operator is repeated until N/2 new individuals are generated so that population size remains constant for next generation. Mathematically the cross-over operator is described as follows.

If the randomly chosen parents are (CX_1, CY_1, CZ_1) and (CX_2, CY_3, CZ_4) , then [5]

$$CX_{1new} = rCX_1 + (1 - r)CX_2$$

$$CY_{1new} = rCY_1 + (1 - r)CY_3$$

$$CZ_{1new} = rCZ_1 + (1 - r)CZ_4$$
(7)

where $r \in (0,1)$ is a random number. Similarly for other pairs of parents, new genes (intervals) for FLC membership functions are formed as in (7) where cross-over operation is carried out taking *r* part from one parent and 1 - r part from the other.

C. Mutation

Mutation occurs with a certain probability knows as mutation rate. Mutation operator introduces new genetic information replacing that of the selected individual. Mathematically the mutation operator is described as [5],

$$CX_{inew} = |CX_{iold} + (r1 - 0.5)2CX_{imax}|$$

$$CY_{inew} = |CY_{iold} + (r1 - 0.5)2CY_{imax}|$$

$$CZ_{inew} = |CZ_{iold} + (r1 - 0.5)2CZ_{imax}|, \text{ for } i = 1, 2, 3, 4$$
(8)

where $r1 \in (0, 1)$ is another random number and $CX_{imax} = CY_{imax} = CZ_{imax} = 0.99$. The GA is repeated for certain generations or until the fitness function value reaches certain convergence value whichever is achieved first as shown in Fig. 6 and finally the optimal values of intervals of FLC membership functions are obtained for the flexible manipulator system for which the fitness function value is the least in the last generation. For flexible manipulator system the convergence value is considered as the minimum value of fitness function which gives the optimum values of intervals for the proposed FLC ensuring the best tip position tracking response and below which the tip position response doesn't vary so much. Here, the convergence value is taken as 0.01



Figure 6. Genetic algorithm optimization process.

5. BFO based fuzzy logic controller design

Bacteria Foraging Optimization (BFO) algorithm is a widely accepted global optimization technique inspired by the social foraging behavior of Escherichia coli [6]. Bacteria search for nutrients in a manner to maximize energy obtained per unit time. Individual bacterium also communicates with others by sending signals. A bacterium takes foraging decisions after considering two previous factors. The process, in which a bacterium moves by taking small steps while searching for nutrients, is called chemotaxis and key idea of BFO algorithm is mimicking chemotactic movement of virtual bacteria in the problem search space. The design of FLC for the single link flexible manipulator requires three quadruples of intervals, two for the two inputs E and CE, and one for output T, which are optimized using BFO technique. So the search space desired in the BFO algorithm is taken as twelve for the above twelve intervals

that influence the membership functions of the proposed FLC. In the bacterial foraging process, three motile behaviors are mimicked. They are described as follows.

A. Chemotaxis

In this process each bacteria cell moves through swimming and tumbling via flagella. It can swim for a period of time in the same direction or it may tumble and alternate between these two modes of operation for the entire lifetime. Suppose B(i, j, k, l) represents *i*-th bacterium at *j*-th chemotactic, *k*-th reproductive and *l*-th elimination-dispersal step and C(i) is the size of the step taken in the random direction specified by the tumble (run length unit). Then in computational chemotaxis the movement of the bacterium may be represented by [7,9]

$$B(i, j+1, k, l) = B(i, j, k, l) + \frac{C(i)}{\sqrt{\Delta^T(i)\Delta(i)}}$$
(9)

where, Δ indicates a vector in the random direction whose elements lie in [-1, 1]. As there are twelve intervals of the membership functions (two inputs *E* and *CE*, and one output *T* each having quadruple intervals), i = n number of bacterium represented by B(i, j, k, l) have twelve search spaces where they search for the optimal values of the intervals of FLC through minimization of an objective function [8], for which the system gives a better tip position response. The objective function, J(i, j, k, l), evaluated using the above n bacterium for twelve search spaces in each Chemotaxis process loop is defined as

$$J(i, j, k, l) = \int_{0}^{T} t \left| P(x, y) - p(x(i, j, k, l), y(i, j, k, l)) \right| dt$$
(10)

where, P(x,y) is the desired tip position and p(x(i, j, k, l), y(i, j, k, l)) is the actual tip position for the FLC membership function intervals in the Chemotaxis process.

B. Reproduction

The least healthy bacteria are eventually eliminated while each of the healthier bacteria (those yielding lower value of the objective function (10)) is asexually split into two bacteria, which are then placed in the same location. This keeps the swarm size constant. Mathematically this process can be represented as

$$B(i+Sr,k+1,l) = B(i,k+1,l)$$
(11)

where, B(i,k,l) represents the *i*-th bacterium at *k*-th reproductive and *l*-th eliminationdispersal step and *Sr* is the number of bacteria reproductions (splits) per generation.

C. Elimination and dispersal

Occurrence of any sudden change in the local environment, due to various reasons, where a bacterium population lives leads up Elimination and Dispersal process. In this

process all the bacteria in a region can be killed or a group can be dispersed into a new location. They have the effect of possibly destroying the chemotactic progress, but they also have the effect of assisting in chemotaxis, since dispersal process has the probability of placing the bacteria near lower objective function (10) region. To simulate this phenomenon in BFO algorithm some bacteria are liquidated at random with a very small probability while the new replacements are randomly initialized over the search space.

The above processes are repeated in loops for S number of bacteria for certain number of generations. Each generation has the Chemotaxis process loop of Nc with bacteria swim length taken as Ns and the Reproduction process loop of Nre and the elimination and dispersal loop of Ned as shown in Fig. 7. And finally the optimal intervals for membership functions of FLC in forms of bacteria B are found for which the objective function (10) value is the least.

6. Simulation results and discussion

In this section the simulation results for a single link flexible manipulator described in section II using optimized FLC is presented. The manipulator system is simulated for both the optimization algorithms and the optimized intervals, thus shape of the membership functions of FLC by using (6), are obtained. Using the optimized FLC found from both the algorithm the tip position is analyzed.

The desired tip position P(x, y) is mathematically represented as

$$P(x,y) = \begin{bmatrix} x_d \\ y_d \end{bmatrix}$$
(12)

where, $x_d = l \cdot \cos(\theta_d)$ and $y_d = l \cdot \sin(\theta_d)$.

In the simulation the desired link angle, θ_d , is taken as 20 degree or 0.3491 rad. p(x,y) vector, that denotes the real-time x and y coordinates of the tip of the manipulator, uses the link angle $\theta(t)$ and link deflection $y_e = v(x,t)|_{x=l}$, as shown in Fig. 1 and is defined by

$$p(x,y) = \begin{bmatrix} x \\ y \end{bmatrix}$$
(13)

where, $x = l \cdot \cos(\theta) - y_e \cdot \sin(\theta)$ and $y = l \cdot \sin(\theta) + y_e \cdot \cos(\theta)$.

The fitness function for GA optimization process (5) and objective function for BFO algorithm (10) both are considered as same function for comparison purpose and the function use the absolute value of the tip position error. So, the absolute value of error is defined as

$$|P(x,y) - p(x,y)| = \sqrt{(x_d - x)^2 + (y_d - y)^2}$$
(14)

For GA optimization process population size of 10 is considered and simulated in a loop for 50 no. of generations. The fitness function values over 50 generations are shown



Figure 7. BFO algorithm.



Figure 8. Fitness function versus number of generation.

in Fig. 8. From the figure the minimization of the fitness function can be realized. After 50 generations, the optimal values of membership functions for GA-FLC are obtained and are shown in Fig. 9.

In BFO process, 10 number of bacteria with 5 number of chemotactic steps, 4 number of reproduction steps, and 3 number of elimination-dispersal events are considered for simulation. In this process the twelve intervals that shape the membership functions of the FLC are chosen from different regions of search space and only those values are considered which are found in local minima of objective function. The BFO-FLC yields optimal values for membership functions for objective function of 0.0193 as compared to the final fitness function value of 0.0464 for GA-FLC as evident from Fig. 8. The optimal membership functions obtained for BFO-FLC are shown in Fig. 10.

The tip position responses for the single link flexible manipulator for both GA-FLC and BFO-FLC are shown in the Fig. 11 and Fig. 12 respectively. And the observed overall tip response data of the manipulator using both GA-FLC and BFO-FLC are provided in the Tab. 3.

Optimization process	Settling time (s)	Rise time (s)	Peak Overshoot (%)
GA	1.9	1.1	3.72
BFO	0.83	0.52	4.86

Table 9. Fuzzy rule base of tip position control

From the table it can be inferred that in case of BFO-FLC both rise time and settling time is less but percentage of peak overshoot is slightly more compared to the case of GA-FLC. The tip position responses of the manipulator is not affected much by the slight



Figure 9. GA optimized FLC membership functions: (a) E, (b) CE, (c) T.

difference in peak overshoot values. Therefore we concluded that optimization of FLC is best found in BFO technique. The cause of the above result can be deduced as this way;



Figure 10. BFO optimized FLC membership functions: (a) E, (b) CE, (c) T.

the BFO process involves in finding most of local minima of fitness (objective) function and global best among them, whereas GA optimization process sticks to local minima until another local minima is found by mutation operator. So, sometimes GA process



Figure 11. Tip position response for GA optimized FLC.



Figure 12. Tip position response for BFO optimized FLC.

could not find the global best of fitness function under consideration as it get stuck in local minima. Comparing the responses of tip positions shown in Fig. 11 and Fig. 12, it also observed that BFO optimized FLC yields faster and smooth tip performances.

7. Conclusion

The paper has presented optimization design of Fuzzy Controllers using two Evolutionary computing approaches namely GA and BFO for achieving tip position tracking of single link flexible manipulator. Both GA and BFO optimization process ensure the stability of the closed-loop system simply by a choice of a set of membership functions for FLC. And also both show good tip motion performance suppressing the unwanted vibration in the links. But, from the results presented, it is observed that the BFO approach to FLC design is better because it exhibits improved transient and steady state tip position control performances.

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