Journal of Polish Safety and Reliability Association
Summer Safety and Reliability Seminars, Volume 9, Number 1, 2018

Dziula Przemysław

Kołowrocki Krzysztof
Maritime University, Gdynia, Poland

Optimization of Operation and Safety of Global Baltic Network of Critical Infrastructure Networks (GBNCIN) with Considering Climate-Weather Change Process (C-WCP) influence – Maximizing GBNCIN Lifetime in the Set of Safety States not worse than a Critical Safety State

Keywords

critical infrastructure, critical infrastructure network, operation process, network of critical infrastructure networks

Abstract

The paper presents a method of the GBNCIN operation and safety, with considering the climate-weather change process, safety optimization. Basic characteristics of the critical infrastructure operation process related to climate-weather change process are shown. Then, optimal transient probabilities of the GBNCIN Operation Process at Operation States Related to Climate-Weather Change Process, and the GBNCIN optimal safety and resilience indicators, are introduced. By defining unconditional multistate safety function of the GBNCIN, and corresponding optimal risk function, the optimal coefficients of the operation process related to the climate-weather change impact on the GBNCIN intensities of degradation, have been determined. Finally, optimal sojourn times of the GBNCIN operation process at operation states, related to climate-weather change process and operation strategy, are presented.

1. Introduction

Critical infrastructure systems protection has become last years very important part of many public institutions and entrepreneurs activities. This is coming out of both: increasing menace of terrorist attacks usually concentrated on critical infrastructures, and increasing amount of different kinds of elemental disasters taking place in the near past, that also caused significant negative impact on critical infrastructure systems [Dziula, 2015]. It is also predicted, forecasted climate changes will significantly impact on critical infrastructure assets, too. Thus, intensive works on adapting infrastructures to possible climate fluctuations, have been processed for last couple of years [European Commission, 2013].

The Baltic Sea area is a region significantly fitted with various critical infrastructure systems. Additionally, geographical conditions of the area, cause potential failure of one of the systems, can lead to a massive negative impact on natural environment and societies located within and around. Also, predicted climate changes do have significant meaning for the Baltic Sea and critical infrastructures located within: one of the greatest (within Europe) increases in sea surface temperature; decreasing trend in the Baltic Sea's ice cover; falling level of the Baltic in the northern shores and rising to the south; increased beach erosion due to increased storminess in the eastern Baltic Sea; and increasing eutrophication problems in coastal waters [Dziula, 2015]. Main socio-economic implications coming out of forecasted climate changes are: high vulnerability of southern part of the Baltic coast to sea level rise flooding; impact on sea-life and therefore on fisheries and aquaculture of warmer, more acidic seawater; increasing risks of inundation and erosion of coastal road transport networks, causing disruptions in the transport of goods and in the mobility of local communities; energy production located in coastal areas, threatened by climate change induced storm surges, sea-level rise and flooding; impacts on agriculture, resulting in extreme cases, a reduction in suitable areas for cultivation in certain European regions; and erosion and flooding of sensitive coastal ecosystems such as brackish waters and tidal pools [European Commission, 2013].

2. The GBNCIN Operation Process Related to Climate-Weather Change Process

We consider the Global Baltic Network of Critical Infrastructure Networks (*GBNCIN*) impacted by the operation process related to the climate-weather change process $ZC_{GBNCIN}(t)$, $t \in <0,\infty)$, in a various way at this process states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$. We assume that the changes of the states of operation process related to the climate-weather change process $ZC_{GBNCIN}(t)$, $t \in <0,\infty)$, at the *GBNCIN* operating area have an influence on the *GBNCIN* safety structure and on the safety of particular *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,n$, as well [Kołowrocki, Soszyńska-Budny, 2011].

We assume, the *GBNCIN* during its operation process is taking $\iota, \iota \in N$, different operation states $z_1, z_2, \ldots, z_\iota$. We define the *GBNCIN* operation process $Z_{GBNCIN}(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_\iota\}$.

Moreover, we assume the climate-weather change process $C(t)$, $t \in <0,+\infty)$, at the *GBNCIN* operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. Climate-weather conditions can have also influence on *GBNCIN* safety.

Then, the joint process of *GBNCIN* operation process and climate-weather change process called the *GBNCIN* operation process related to climate-weather change is proposed and it is marked by $ZC_{GBNCIN}(t)$, $t \in <0,+\infty)$. Further, we assume that it can take $\iota w, \iota, w \in N$, different operation states related to the climate-weather change $zc_{11}, zc_{12}, \ldots, zc_{\iota w}$.

We assume that the *GBNCIN* operation process related to climate-weather change $ZC_{GBNCIN}(t)$, at the moment $t \in <0,+\infty)$, is at the state zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, if and only if at that moment, the operation process $Z_{GBNCIN}(t)$ is at the operation states z_b, $b = 1,2,...,\iota$, and the climate-weather change process $C(t)$ is at the climate-weather state c_l, $l = 1,2,...,w$, can be expressed as follows:

$$(ZC_{GBNCIN}(t) = zc_{bl}) \Leftrightarrow (Z_{GBNCIN}(t) = z_b \cap C(t) = c_l),$$
$$t \in <0,+\infty),\ b = 1,2,...,\iota,\ \ l = 1,2,...,w.$$

The transient probabilities of the *GBNCIN* operation process related to climate-weather change $ZC_{GBNCIN}(t)$ at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are defined as below:

$$pq_{bl}^{GBNCIN}(t) = P(ZC_{GBNCIN}(t) = zc_{bl}),\ t \in <0,+\infty),$$
$$b = 1,2,...,\iota,\ \ l = 1,2,...,w.$$

Further, the limit values of the transient probabilities of the *GBNCIN* operation process related to climate-weather change process $ZC_{GBNCIN}(t)$ at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are given by

$$pq_{bl}^{GBNCIN} = \lim_{t \to \infty} pq_{bl}^{GBNCIN}(t),\ b = 1,2,...,\iota,$$
$$l = 1,2,...,w, \tag{1}$$

and in case when the processes $Z_{GBNCIN}(t)$ and $C(t)$ are independent, they can be found from [EU-CIRCLE, 2017]

$$pq_{bl}^{GBNCIN} = p_b^{GBNCIN} q_l,\ b = 1,2,...,\iota,\ l = 1,2,...,w, \tag{2}$$

where p_b^{GBNCIN}, $b = 1,2,...,\iota$, are the limit transient probabilities of the operation process $Z_{GBNCIN}(t)$ at the particular operation startes z_b, $b = 1,2,...,\iota$, and q_l, $l = 1,2,...,w$, are the limit transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $l = 1,2,...,w$.

Other interesting characteristics of the *GBNCIN* operation process $ZC_{GBNCIN}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}^{GBNCIN}$ at the particular operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, during the fixed sufficiently large *GBNCIN* operation time θ. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta,\ b = 1,2,...,\iota,$$
$$l = 1,2,...,w, \tag{3}$$

where pq_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are defined by (1) and given by (2) in the case the processes $Z_{GBNCIN}(t)$ and $C(t)$ are independent.

3. Optimization of Operation and Safety of the Global Baltic Network of Critical Infrastructure Networks

3.1. Optimal Transient Probabilities of the Global Baltic Network of Critical Infrastructure Networks Operation Process at

Operation States Related to Climate-Weather Change Process

Considering that the coordinates of the unconditional safety function of the *GBNCIN* impacted by the operation process related to the climate-weather change process $ZC_{GBNCIN}(t)$, $t \in <0,\infty)$,

$$\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,\cdot) = [1, \boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,1), ..., \boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,z)],$$
$$t \in <0,\infty),$$

are given by

$$\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,u) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN} [\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)}$$
$$\text{for } t \geq 0,\ u = 1,2,...,z, \tag{4}$$

where

$$[\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)}, u = 1,2,...,z,\ b = 1,2,...,\iota,$$
$$l = 1,2,...,w,$$

are the coordinates of the *GBNCIN* impacted by the operation process related to the climate-weather change process $ZC_{GBNCIN}(t)$, $t \in <0,\infty)$, conditional safety functions [EU-CIRCLE, 2017]

$$[\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,\cdot)]^{(bl)} =$$
$$= [1, [\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,1)]^{(bl)}, ..., [\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,z)]^{(bl)}],$$
$$t \in <0,\infty),\ b = 1,2,...,\iota,\ l = 1,2,...,w,$$

and pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are the operation process related to the climate-weather change process $ZC(t), t \in <0,\infty)$, at the *GBNCIN* operating area limit transient probabilities at the states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, defined by (3), it is natural to assume that the *GBNCIN* operation process has a significant influence on the *GBNCIN* safety.

This influence is also clearly expressed in the equation for the mean lifetime of the *GBNCIN* in the safety state subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, given by

$$\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(u) = \int_0^\infty [\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,u)]dt$$
$$\cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN} [\mu^{4}_{GBNCIN}(u)]^{(bl)},\ u = 1,2,...,z, \tag{5}$$

where $[\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are the mean values of the *GBNCIN* conditional lifetimes $[T^{4}_{GBNCIN}(u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, in the safety state subset $\{u, u+1,...,z\}$ at the *GBNCIN* operating process related to the climate-weather change state zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, given by

$$[\mu^{4}_{GBNCIN}(u)]^{(bl)} = \int_0^\infty [\boldsymbol{S}^{4}_{GBNCIN}(t,u)]^{(bl)} dt,$$
$$u = 1,2,...,z,\ b = 1,2,...,\iota,\ l = 1,2,...,w \tag{6}$$

From the linear equation

$$\boldsymbol{\mu}_{\boldsymbol{GBNCIN}}(u) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN} [\mu^{4}_{GBNCIN}(u)]^{(bl)},$$
$$u = 1,2,...,z, \tag{7}$$

we can see that the mean value of the *GBNCIN* unconditional lifetime $\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(u)$, $u = 1,2,...,z$, is determined by the limit values of transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the *GBNCIN* operation process at the operation states given by (3) and the mean values $[\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are the mean values of the *GBNCIN* conditional lifetimes in the safety state subset $\{u, u+1,...,z\}$ at the *GBNCIN* operating process related to the climate-weather change process zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, given by (5).

Therefore, the *GBNCIN* lifetime optimization approach based on the linear programming [Klabjan, Adelman, 2006] and [Kołowrocki, Soszyńska-Budny, 2011] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the limit transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the *GBNCIN* operation process at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, to maximize the mean value $\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(u)$, $u = 1,2,...,z$, of the unconditional $b = 1,2,...,\iota$, lifetimes in the safety state subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z$, under the assumption that the mean values $[\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the $b = 1,2,...,\iota$, conditional lifetimes in the safety state subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z$, are fixed. As a special case of the above formulated *GBNCIN* lifetime optimization problem, if r_{GBNCIN}, $r_{GBNCIN} = 1,2,...,z$, is a *GBNCIN* critical safety state, we want to find the optimal values $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the *GBNCIN* operation process limit transient probabilities $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, to maximize the mean value $\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})$ of the unconditional *GBNCIN* lifetimes in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,...,z\}$, under the assumption that the mean values $[\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl)}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the *GBNCIN* conditional lifetimes in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,...,z\}$, are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN}) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN}[\mu^{4}_{GBNCIN}(r_{GBNCIN})]^{(bl)}, \quad (8)$$

for a fixed $r_{GBNCIN} \in \{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}q_{bl}^{GBNCIN} \le pq_{bl}^{GBNCIN} \le \hat{p}q_{bl}^{GBNCIN},\ b = 1,2,...,\iota,\ l = 1,2,...,w, \quad (9)$$

$$\sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN} = 1, \quad (10)$$

where

$$[\boldsymbol{\mu}^{4}_{GBNCIN}(r_{GBNCIN})]^{(bl)},\ [\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl)} \ge 0,\ b = 1,2,...,\iota,\ l = 1,2,...,w, \quad (11)$$

are fixed mean values of the *GBNCIN* conditional lifetimes in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,...,z\}$, and

$$\breve{p}q_{bl}^{GBNCIN},\ 0 \le \breve{p}q_{bl}^{GBNCIN} \le 1 \text{ and } \hat{p}q_{bl}^{GBNCIN},\ 0 \le \hat{p}q_{bl}^{GBNCIN} \le 1,\ \breve{p}q_{bl}^{GBNCIN} \le \hat{p}q_{bl}^{GBNCIN},\ b = 1,2,...,\iota,\ l = 1,2,...,w, \quad (12)$$

are lower and upper bounds of the transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, respectively.

Now, we can obtain the optimal solution of the formulated by (8)-(12) the linear programming problem, i.e. we can find the optimal values $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, that maximize the objective function given by (7).

First, we arrange the *GBNCIN* conditional lifetime mean values $[\boldsymbol{\mu}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl)}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, in non-increasing order

$$[\mu^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl_1)} \ge [\mu^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl_2)} \ge \ldots \ge [\mu^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN})]^{(bl_{\iota w})}$$

where $bl_i \in \{1,2,...,\iota w\}$ for $i = 1,2,...,\iota w$.

Next, we substitute

$$x_i = p_{bl}^{GBNCIN},\ \breve{x}_i = \breve{p}_{bl_i}^{GBNCIN},\ \hat{x}_i = \hat{p}_{bl_i}^{GBNCIN} \text{ for } i = 1,2,...,\iota w, \quad (13)$$

and we maximize with respect to x_i, $i = 1,2,...,\iota w$, the linear form (7) that after this transformation takes the form

$$\mu^{4}_{GBNCIN}(r_{GBNCIN}) = \sum_{i=1}^{\iota w} x_i[\mu^{4}_{GBNCIN}(r_{GBNCIN})]^{bl_i} \quad (14)$$

for a fixed $r_{GBNCIN} \in \{1,2,...,z\}$ with the following bound constraints

$$\breve{x}_i \le x_i \le \hat{x}_i,\ i = 1,2,...,\iota w, \quad (15)$$

$$\sum_{i=1}^{\iota w} x_i = 1, \quad (16)$$

where

$$\mu_{GBNCIN\,bl_i}(r_{GBNCIN}),\ \mu_{GBNCIN\,bl_i}(r_{GBNCIN}) \ge 0,\ i = 1,2,...,\iota w,$$

are fixed mean values of the *GBNCIN* conditional lifetimes in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,...,z\}$ arranged in non-increasing order and

$$\breve{x}_i,\ 0 \le \breve{x}_i \le 1 \text{ and } \hat{x}_i,\ 0 \le \hat{x}_i \le 1,\ \breve{x}_i \le \hat{x}_i,\ i = 1,2,...,\iota w, \quad (17)$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,...,\iota w$, respectively.

To find the optimal values of x_i, $i = 1,2,...,\iota w$, we define

$$\breve{x} = \sum_{i=1}^{\iota\varpi} \breve{x}_i,\ \hat{y} = 1 - \breve{x} \tag{18}$$

and

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \text{ for } I = 1,2,...,\iota w, \tag{19}$$

Next, we find the largest value $I \in \{0,1,...,\iota w\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \tag{20}$$

and we fix the optimal solution that maximize (13) in the following way:

i) if $I = 0,$ the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 1,2,...,\iota w; \tag{21}$$

ii) if $0 < I < \iota w,$ the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,..., I,\ \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1} \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,...,\iota w; \tag{22}$$

iii) if $I = \iota w,$ the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,...,\iota w. \tag{23}$$

Finally, after making the inverse to (12) substitution, we get the optimal limit transient probabilities

$$\dot{p}q_{bl_i}^{GBNCIN} = \dot{x}_i \text{ for } i = 1,2,...,\iota w, \tag{24}$$

that maximize the GBNCIN mean lifetime in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,...,z\}$, defined by the linear form (7), giving its maximum value in the following form

$$\dot{\boldsymbol{\mu}}^{4}_{\boldsymbol{GBNCIN}}(r_{GBNCIN}) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_{bl}^{GBNCIN}[\mu^4_{GBNCIN}(r_{GBNCIN})]^{(bl)} \tag{25}$$

for a fixed $r_{GBNCIN} \in \{1,2,...,z\}$.

3.2. The Global Baltic Network of Critical Infrastructure Networks Optimal Safety and Resilience Indicators

From the expression (25) for the maximum mean value $\dot{\mu}^4_{GBNCIN}(r_{GBNCIN})$ of the *GBNCIN* unconditional lifetime in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,...,z\}$, replacing in it the critical safety state r_{GBNCIN} by the safety state u, $u = 1,2,...,z$, we obtain the corresponding optimal solutions for the mean values of the *GBNCIN* unconditional lifetimes in the safety state subsets $\{u, u+1,...,z\}$ of the form

$$\dot{\mu}^4_{GBNCIN}(u) = \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_b[\mu^4_{GBNCIN}(u)]^{(bl)} \text{ for } u = 1,2,...,z. \tag{26}$$

Further, according to (4), the corresponding optimal unconditional multistate safety function of the *GBNCIN* is the vector

$$\dot{\boldsymbol{S}}^4_{GBNCIN}(t,\cdot) = [1,\ \dot{\boldsymbol{S}}^4_{GBNCIN}(t,1),...,\ \dot{\boldsymbol{S}}^4_{GBNCIN}(t,z)],\ t \ge 0, \tag{27}$$

with the coordinates given by

$$\dot{\boldsymbol{S}}^4_{GBNCIN}(t,u) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_{bl}[\boldsymbol{S}^4_{GBNCIN}(t,u)]^{(bl)} \text{ for } u = 1,2,...,z. \tag{28}$$

By applying (23) from [EU-CIRCLE, 2017], the corresponding optimal values of the variances of the *GBNCIN* unconditional lifetimes in the safety state subsets are

$$\dot{\sigma}^4_{GBNCIN}{}^2(u) = 2\int_0^\infty t\ \dot{\boldsymbol{S}}^4_{GBNCIN}(t,u)dt - [\dot{\mu}^4_{GBNCIN}(u)]^2,\ u = 1,2,...,z, \tag{29}$$

where $\dot{\mu}^4_{GBNCIN}(u)$ is given by (26) and $\dot{\boldsymbol{S}}^4_{GBNCIN}(t,u)$ is given by (28).

And, by (25) from [EU-CIRCLE, 2017], the optimal solutions for the mean values of the *GBNCIN* unconditional lifetimes in the particular safety states are

$$\dot{\bar{\mu}}^4_{GBNCIN}(u) = \dot{\mu}^4_{GBNCIN}(u) - \dot{\mu}^4_{GBNCIN}(u+1),\ u = ,1,..., z-1,\ \dot{\bar{\mu}}^4_{GBNCIN}(z) = \dot{\mu}^4_{GBNCIN}(z). \tag{30}$$

Moreover, considering (7) and (12) from [EU-CIRCLE, 2017], the corresponding optimal risk function of the *GBNCIN* and the optimal moment when the risk exceeds a permitted level δ_{GBNCIN}, respectively are given by

$$\dot{\boldsymbol{r}}^4_{GBNCIN}(t) = 1 - \dot{\boldsymbol{S}}^4_{GBNCIN}(t, r_{GBNCIN}), \ t \geq 0, \qquad (31)$$

and

$$\dot{\tau}^4_{GBNCIN} = \dot{\boldsymbol{r}}^4_{GBNCIN}{}^{-1}(\delta_{GBNCIN}), \qquad (32)$$

where $\dot{\boldsymbol{S}}^4_{GBNCIN}(t, r_{GBNCIN})$ is given by (28) for $u = r_{GBNCIN}$ and $\dot{\boldsymbol{r}}^4_{GBNCIN}{}^{-1}(t)$, if it exists, is the inverse function of the optimal risk function $\dot{\boldsymbol{r}}^4_{GBNCIN}(t)$.

The optimal intensities of degradation of the *GBNCIN* / the optimal intensities of *GBNCIN* departure from the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, impacted by the operation process related to the climate-weather change, i.e. the coordinates of the vector

$$\dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t, \cdot) = [0, \ \dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,1), \ ..., \ \dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,z)], \ t \in <0, +\infty), \qquad (33)$$

are given by

$$\dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,u) = \frac{-\frac{d\dot{\boldsymbol{S}}^4_{GBNCIN}(t,u)}{dt}}{\dot{\boldsymbol{S}}^4_{GBNCIN}(t,u)}, \ u = 1,2,...,z. \ (34)$$

The optimal coefficients of the operation process related to the climate-weather change impact on the *GBNCIN* intensities of degradation/ the coefficients of the operation process related to the climate-weather change impact on *GBNCIN* intensities of departure from the safety state subset $\{u, u+1, ..., z\}$, i.e. the coordinates of the vector are given by

$$\dot{\boldsymbol{\rho}}^4_{GBNCIN}(t, \cdot) = [0, \ \dot{\boldsymbol{\rho}}^4_{GBNCIN}(t,1), \ ..., \ \dot{\boldsymbol{\rho}}^4_{GBNCIN}(t,z)], \ t \in <0, +\infty), \qquad (35)$$

where

$$\dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,u) = \dot{\boldsymbol{\rho}}^4_{GBNCIN}(t,u) \cdot \boldsymbol{\lambda}^0_{GBNCIN}(t,u), \ u = 1,2,...,z, \qquad (36)$$

i.e.

$$\dot{\boldsymbol{\rho}}^4_{GBNCIN}(t,u) = \frac{\dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,u)}{\boldsymbol{\lambda}^0_{GBNCIN}(t,u)}, \ u = 1,2,...,z, \qquad (37)$$

and $\boldsymbol{\lambda}^0_{GBNCIN}(t,u)$, $t \in <0, +\infty)$, $u = 1,2,...,z$, are the intensities of degradation of the *GBNCIN* without the operation process related to the climate-weather change impact, i.e. the coordinate of the vector [EU-CIRCLE, 2017]

$$\boldsymbol{\lambda}^0_{GBNCIN}(t, \cdot) = [0, \ \boldsymbol{\lambda}^0_{GBNCIN}(t,1), \ ..., \ \boldsymbol{\lambda}^0_{GBNCIN}(t,z)], \ t \in <0, +\infty), \qquad (38)$$

And $\dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,u)$, $t \in <0, +\infty)$, $u = 1,2,...,z$, are the optimal intensities of degradation of the *GBNCIN* impacted by the operation process related to the climate-weather change, i.e. the coordinate of the vector

$$\dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t, \cdot) = [0, \ \dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,1), \ ..., \ \dot{\boldsymbol{\lambda}}^4_{GBNCIN}(t,z)], \ t \in <0, +\infty). \qquad (39)$$

The optimal indicator of the *GBNCIN* resilience to operation process related to climate-weather change impact is given by

$$\dot{\boldsymbol{RI}}^4_{GBNCIN}(t, r_{GBNCIN}) = \frac{1}{\dot{\boldsymbol{\rho}}^4_{GBNCIN}(t, r_{GBNCIN})}, \ t \in <0, +\infty), \qquad (40)$$

where $\dot{\boldsymbol{\rho}}^4_{GBNCIN}(t, r_{GBNCIN})$, $t \in <0, +\infty)$, is the optimal coefficients of operation process related to climate-weather change impact on the *GBNCIN* intensities of degradation given by (37) for $u = r_{GBNCIN}$.

3.3. Optimal Sojourn Times of Global Baltic Network of Critical Infrastructure Networks Operation Process at Operation States Related to Climate-Weather Change Process and Operation Strategy

Assuming that the *GBNCIN* operation process and the climate-weather change process are independent, i.e.

$$\dot{p}q^{GBNCIN}_{bl} = \dot{p}^{GBNCIN}_b \cdot q_l, \ b = 1,2,...,\iota, \ l = 1,2,...,w,$$

and replacing in (22) from [Kołowrocki, Soszyńska-Budny, 2011], the limit transient probabilities p^{GBNCIN}_b of the *GBNCIN* operation process at the operation states by

$$\dot{p}^{GBNCIN}_b = \frac{\dot{p}q^{GBNCIN}_{bl}}{q_l} = \frac{\dot{p}^{GBNCIN}_b \cdot q_l}{q_l} = \dot{p}^{GBNCIN}_b, \ b = 1,2,...,\iota, \ l = 1,2,...,w, \qquad (41)$$

where

$$\dot{p}q_{bl}^{GBNCIN} = \dot{p}_b^{GBNCIN} \cdot q_l, \; b = 1,2,...,\iota, \; l = 1,2,...,w,$$

are the optimal values of transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, found by the equations (21) and the mean values M_b^{GBNCIN} of the unconditional sojourn times at the operation states z_b, $b = 1,2,...,\iota$, defined by (21) in [Kołowrocki, Soszyńska-Budny, 2011] by their corresponding unknown optimal values $\dot{M}_b^{GBNCIN}$ maximizing the mean value of the $GBNCIN$ lifetime in the safety states subset $\{r_{GBNCIN}, r_{GBNCIN}+1,..., z\}$ defined by (7), we get the system of equations

$$\dot{p}_b^{GBNCIN} = \frac{\pi_b \dot{M}_b^{GBNCIN}}{\sum_{l=1}^{v} \pi_l \dot{M}_l^{GBNCIN}}, \; b = 1,2,...,\iota. \tag{42}$$

After simple transformations the above system takes the form

$$\begin{aligned}
&(\dot{p}_1^{GBNCIN} - 1)\pi_1 \dot{M}_1^{GBNCIN} + \dot{p}_1^{GBNCIN} \pi_2 \dot{M}_2^{GBNCIN} + ... \\
&... + \dot{p}_1^{GBNCIN} \pi_\iota \dot{M}_\iota^{GBNCIN} = 0 \\
&\dot{p}_2^{GBNCIN} \pi_1 \dot{M}_1^{GBNCIN} + (\dot{p}_2^{GBNCIN} - 1)\pi_2 \dot{M}_2^{GBNCIN} + ... \\
&... + \dot{p}_2^{GBNCIN} \pi_\iota \dot{M}_\iota^{GBNCIN} = 0 \\
&\dot{p}_\iota^{GBNCIN} \pi_1 \dot{M}_1^{GBNCIN} + \dot{p}_\iota^{GBNCIN} \pi_2 \dot{M}_2^{GBNCIN} + ... \\
&... + (\dot{p}_\iota^{GBNCIN} - 1)\pi_\iota \dot{M}_\iota^{GBNCIN} = 0,
\end{aligned} \tag{43}$$

where $\dot{M}_b^{GBNCIN}$ are unknown variables we want to find, $\dot{p}_b^{GBNCIN}$ are optimal transient probabilities determined by (41) and π_b are steady probabilities determined by (23) in [Kołowrocki, Soszyńska-Budny, 2011].

Since the system of equations (43) is homogeneous and it can be proved that the determinant of its main matrix is equal to zero, then it has nonzero solutions and moreover, these solutions are ambiguous. Thus, if we fix some of the optimal values $\dot{M}_b^{GBNCIN}$ of the mean values M_b^{GBNCIN} of the unconditional sojourn times at the operation states, for instance by arbitrary fixing one or a few of them, we may find the values of the remaining once and this way to get the solution of this equation.

Having this solution, it is also possible to look for the optimal values $\dot{M}_{bl}^{GBNCIN}$ of the mean values M_{bl}^{GBNCIN} of the conditional sojourn times at the operation states using the following system of equations

$$\sum_{l=1}^{\iota} p_{bl}^{GBNCIN} \dot{M}_{bl}^{GBNCIN} = \dot{M}_b^{GBNCIN}, \; b = 1,2,...,\iota, \tag{44}$$

obtained from (21) given in [Kołowrocki, Soszyńska-Budny, 2011] by replacing M_b^{GBNCIN} by $\dot{M}_b^{GBNCIN}$ and M_{bl}^{GBNCIN} by $\dot{M}_{bl}^{GBNCIN}$, were p_{bl}^{GBNCIN} are known probabilities of the $GBNCIN$ operation process transitions between the operation states z_b and z_l, $b, l = 1,2,...,\iota$, $b \neq l$, defined by (2) in [Kołowrocki, Soszyńska-Budny, 2011].

The knowledge of the optimal values $\dot{M}_b^{GBNCIN}$ of the mean values M_b^{GBNCIN} of the unconditional sojourn times at the operation states and the conditional the optimal values $\dot{M}_{bl}^{GBNCIN}$ of the mean values M_{bl}^{GBNCIN} of the conditional sojourn times at the operation states maximizing the mean value of the $GBNCIN$ lifetime in the safety states subset not worse than the critical safety state can help in the planning better strategy in the critical infrastructure networks operation processes related to the climate-weather change resulting in higher safety and safety of these critical infrastructure networks operation.

Another very useful and much easier to be applied in practice tool that can help in planning more reliable and safe operation process of the critical infrastructure networks are the optimal mean values of the total $GBNCIN$ operation process sojourn times $\hat{\theta}_b^{GBNCIN}$ at the particular operation states z_b, $b = 1,2,...,\iota$, during the fixed $GBNCIN$ operation time θ^{GBNCIN}, that can be obtain by the replacing in the formula (24) from [Kołowrocki, Soszyńska-Budny, 2011], the transient probabilities p_b^{GBNCIN} at the operation states z_b by their optimal values $\dot{p}_b^{GBNCIN}$ and resulting in the following expression

$$\dot{\hat{M}}_b^{GBNCIN} = \dot{E}[\hat{\theta}_b^{GBNCIN}] = \dot{p}_b^{GBNCIN} \theta^{GBNCIN},$$
$$b = 1,2,...,\iota. \tag{45}$$

Similarly, replacing in the formula (3) the transient probabilities pq_{bl}^{GBNCIN} at the operation states z_{bl} by their optimal values $\dot{p}q_{bl}^{GBNCIN}$, we get the optimal mean values of the total $GBNCIN$ operation process sojourn times $\hat{\theta}\hat{C}_{bl}^{GBNCIN}$ at the particular operation states zc_{bl}, $b=1,2,...,\iota,$ $l=1,2,...,w,$ during the fixed *GBNCIN* operation time θ^{GBNCIN}, given by

$$\dot{\hat{M}}\hat{N}_{bl}^{GBNCIN} = \dot{E}[\hat{\theta}\hat{C}_{bl}^{GBNCIN}] = \dot{p}q_{bl}^{GBNCIN}\theta^{GBNCIN},$$
$$b=1,2,...,\iota, \quad l=1,2,...,w. \tag{46}$$

The knowledge of the optimal values $\dot{M}_{b}^{GBNCIN}$ of the mean values of the unconditional sojourn times and the optimal values $\dot{M}_{bl}^{GBNCIN}$ of the mean values of the conditional sojourn times at the operation states and the optimal mean values $\dot{\hat{M}}_{b}^{GBNCIN}$ of the total sojourn times at the particular operation states z_b, $b=1,2,...,\iota,$ and the optimal mean values $\dot{\hat{M}}\hat{N}_{bl}^{GBNCIN}$ of the total sojourn times at the particular operation states related to climate-weather change zc_{bl}, $b=1,2,...,\iota,$ $l=1,2,...,w,$ during the fixed $GBNCIN$ operation time may by the basis for changing the critical infrastructure networks operation processes in order to ensure these critical infrastructure networks operation more safe. Their knowledge may also be useful in these critical infrastructure networks operation cost analysis.

4. Conclusion

The tools presented in the article are useful for the optimization of operation and safety of the GBNCIN, operating at the varying conditions that have an influence on changing its safety structures and its components safety characteristics. Presented: optimal transient probabilities of the GBNCIN operation process at operation states related to climate-weather change process, the GBNCIN optimal safety and resilience indicators, and optimal sojourn times of the GBNCIN operation process at operation states related to climate-weather change process and operation strategy, can be applied for different critical infrastructure networks, including maritime transport industry.

Acknowledgments

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/.

References

Dziula P. (2015). Selected aspects of acts of law concerning critical infrastructure protection within the Baltic Sea area, *Scientific Journals of the Maritime University of Szczecin*, Vol. 44, No 116, 173-181.

EU-CIRCLE Report D1.4-GMU2. (2016). *Selected critical infrastructures at the Southern Baltic Sea area and its surroundings identification and their parameters analysis - port, maritime transport and information critical infrastructures network defining*.

EU-CIRCLE Report D2.1-GMU2. (2016). *Modelling outside dependences influence on Critical Infrastructure Safety (CIS) – Modelling Critical Infrastructure Operation Process (CIOP) including Operating Environment Threats (OET)*.

EU-CIRCLE Report D2.1-GMU3. (2016). *Modelling outside dependences influence on Critical Infrastructure Safety (CIS) – Modelling Climate-Weather Change Process (C-WCP) including Extreme Weather Events (EWE)*.

EU-CIRCLE Report D2.1-GMU4. (2016). *Modelling outside dependences influence on Critical Infrastructure Safety (CIS) - Designing Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Events (EWE) by linking CIOP and C-WCP models*.

EU-CIRCLE Report D2.2-GMU4. (2016). *Modeling the operation process of the Baltic Sea critical infrastructures general network of interconnected and interdependent critical infrastructures located within the Baltic Sea and ashore around that function collaboratively using the Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Events (EWE) in its operating environment ("network of networks" approach)*.

EU-CIRCLE Report D2.3-GMU1. (2016). *Identification methods and procedures of Critical Infrastructure Operation Process (CIOP) including Operating Environment Threats (OET)*.

EU-CIRCLE Report D2.3-GMU2. (2016). *Identification methods and procedures of Climate-Weather Change Process (C-WCP) including*

Extreme Weather Events (EWE).

EU-CIRCLE Report D2.3-GMU3. (2016). *Identification methods and procedures of unknown parameters of Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Events*.

EU-CIRCLE Report D2.3-GMU7. (2016). *Evaluation of unknown parameters of the Baltic Sea critical infrastructures global/general network ("network of networks") of interconnected and interdependent critical infrastructures located within the Baltic Sea and ashore around that function collaboratively using the Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Wheather Events (EWE) in its operating environment*.

EU-CIRCLE Report D3.3. (2017). *Critical Infrastructure Safety and Resilience Indicators*.

European Union, European Commission. (2013). Commission Staff working document: *Climate change adaptation, coastal and marine issues*, SWD(2013) 133 final. Brussels.

European Union, European Commission. (2013). Commission Staff working document: *Adapting infrastructure to climate change*, SWD(2013) 137 final. Brussels.

Grabski F. (2015). *Semi-Markov Processes: Applications in System Reliability and Maintenance*. Elsevier.

Klabjan D., Adelman D. (2006). Existence of optimal policies for semi-Markov decision processes using duality for infinite linear programming. *Siam Journal on Control and Optimization*, Vol. 44, No. 6, 2104-2122.

Kołowrocki K., Soszyńska-Budny J. (2011). *Reliability and Safety of Complex Technical Systems and Processes: Modeling - Identification - Prediction - Optimization*. Springer, London, Dordrecht, Heildeberg, New York.

Limnios N., Oprisan G. (2005). *Semi-Markov Processes and Reliability*. Birkhauser, Boston.

Mercier S. (2008). Numerical bounds for semi-Markovian quantities and application to reliability. *Methodology and Computing in Applied Probability*, 10(2), 179-198.

Soszyńska J. (2007). *Systems reliability analysis in variable operation conditions*. PhD Thesis. Gdynia Maritime University – System Research Institute Warsaw, (in Polish).