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M. SULIGA*, L. BOROWIK**, K. CHWASTEK***

ESTIMATION OF THE LEVEL OF RESIDUAL STRESS IN WIRES WITH A MAGNETIC METHOD

OCENA POZIOMU NAPRĘŻEŃ WŁASNYCH W DRUTACH METODĄ MAGNETYCZNĄ

Residual stress present in wires after drawing process affects their magnetic properties. The paper presents a concept to estimate the level of residual stress on the basis of measurements of hysteresis loops. In order to describe the effect qualitatively the Jiles-Atherton-Sablik description is adapted. On the basis of variations in hysteresis loop shapes the average values of residual stress in wires for different single draft values are determined. It was found that the estimated average values by magnetic stresses are comparable with the results of numerical modeling and experimental studies.

Keywords: residual stress, wire drawing, hysteresis loop, effective field

Naprężenia własne istniejące w drutach po procesie ciągnienia mają wpływ na ich właściwości magnetyczne. W pracy przedstawiono koncepcję oszacowania poziomu naprężeń własnych drutów na podstawie pomiarów pętli histerezy. W celu jakościowego opisu zjawiska zaadaptowano model Jilesa-Athertona-Sablika. Na podstawie zmian kształtu pętli histerezy oszacowano średnie wartości naprężeń własnych dla drutów ciągnionych z różnymi wartościami gniotu pojedynczego. Stwierdzono, że oszacowane metodą magnetyczną średnie wartości naprężeń własnych są porównywalne z wynikami z modelowania numerycznego i badań eksperymentalnych.

1. Introduction

Determination of residual stress level in drawn wires is an important problem in contemporary metallurgy. Increasing demands of the customers of metallurgical products stimulate the efforts of the producers to supply high-quality reliable and safe products. Residual stresses, which occur after the drawing process, have a negative impact on the quality and properties of drawn wires and can disqualify them for applications as ropes, tyres or springs. These stresses are generated as a consequence of the inhomogeneous deformation and heat generation associated with the drawing process [1, 2]. They affect geometrical precision of the drawn work-pieces and may have substantial impact on the mechanical properties and durability of ready-made wires [3, 4]. Therefore it is crucial to develop methods which make it possible to predict the residual stress level.

Conventional approaches to solve the problem are based on mechanical methods i.e. wire polishing and longitudinal wire cutting [5, 6]. In recent years, due to a progress in computer science, simulations have become important tools for the metallurgists.

The present paper focuses on a non-destructive method based on magnetic measurements and modelling. Hysteresis loop subject to stress becomes deformed; this property may be used in practice to estimate the averaged value of stress present in a ready-made sample, provided the measurement data for annealed (stress-free) sample is also available [7].

There are many mathematical models of hysteresis loop available [8]. They differ in their mathematical backgrounds, accuracies of representation and application scope. The Preisach, Jiles-Atherton and Stoner-Wohlfarth descriptions have gained a lot of attention of the scientific and engineering community. In the present paper we have focused on the phenomenological model proposed in the eighties of the last century by D. C. Jiles and D. L. Atherton [9], as it has a number of advantages from the engineering perspective:

- it has a relatively simple mathematical structure, which consists of a set of coupled nonlinear and first-order ordinary differential equations;
- the effects of stress, temperature, demagnetization, eddy currents etc. may be easily introduced into model equations using an appropriate extension of the “effective field”, which plays a crucial role in the description;
- a physical interpretation may be attributed to model parameters;
- the authors have developed the description having in mind the magneto-elastic effects [10-14]; this concept has been further explored in numerous publications, just to mention Refs. [15-18].

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* CZĘSTOCHOWA UNIVERSITY OF TECHNOLOGY, INSTITUTE OF METAL FORMING AND SAFETY ENGINEERING, 19 ARMII KRAJOWEJ AV., 42-200 CZĘSTOCHOWA, POLAND
** CZĘSTOCHOWA UNIVERSITY OF TECHNOLOGY, INSTITUTE OF TELECOMMUNICATION AND ELECTROMAGNETIC COMPATIBILITY, 19 ARMII KRAJOWEJ AV., 42-200 CZĘSTOCHOWA, POLAND
*** CZĘSTOCHOWA UNIVERSITY OF TECHNOLOGY, INSTITUTE OF ELECTRIC POWER ENGINEERING, 19 ARMII KRAJOWEJ AV., 42-200 CZĘSTOCHOWA, POLAND

2. Jiles-Atherton-Sablik (JAS) model

Jiles and Atherton have envisaged the hysteresis loop as related to the movement of domain walls. Within ferromagnetic material there exist defects, irregularities of crystalline lattice, impurities etc., which hamper the movement. The obstacles to domain wall displacement are termed pinning sites. Domain walls either bow on these obstacles or, if the magnetic field is strong enough, translate through them and some energy is lost.

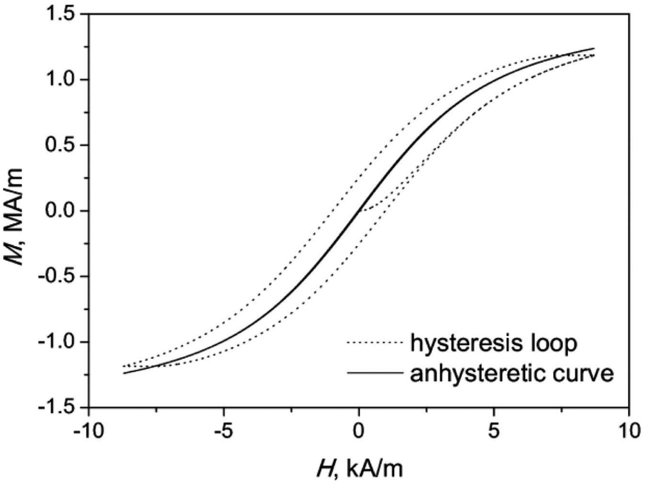

Fig. 1. An exemplary hysteresis loop and the anhysteretic curve

The anhysteretic (hysteresis-free) curve plays the role of the „spine" for the hysteresis loop, cf. Fig. 1. It is given with the modified Langevin function in the original description

$$M_{\mathrm{an}} = M_{\mathrm{s}}\left[\coth\frac{H_{\mathrm{eff}}}{a} - \frac{a}{H_{\mathrm{eff}}}\right] \tag{1}$$

where the argument of the function is the so-called effective field, which is the true field within the magnetic material. The following definition of the effective field is used in this paper [14]

$$\begin{aligned} H_{\mathrm{eff}} &= H + \alpha M + H_{\sigma} = \\ &= H + \alpha M + \tfrac{3}{2}\tfrac{\sigma}{\mu_0}\left(\tfrac{\mathrm{d}\lambda}{\mathrm{d}M}\right) \end{aligned} \tag{2}$$

The term αM describes the cooperative action between magnetic moments within the material, whereas the third term takes into account the effect of stress σ. The third term has been introduced into the model by Sablik. μ_0 is the free space permeability, $\mu_0 = 4\pi 10^{-7}$H/m, and λ is magnetostriction. The dependence $\lambda(M)$ may be given in the first approximation as parabolic, with $\lambda_0 = 2 \cdot 10^{-5}$ [13]:

$$\lambda(M) \cong \lambda_0\left(\frac{M}{M_{\mathrm{s}}}\right)^2 \tag{3}$$

The fundamental JA model equation may be written as [19]

$$\frac{\mathrm{d}\,M}{\mathrm{d}\,H_{\mathrm{eff}}} = \frac{\delta_{\mathrm{M}}\,(M_{\mathrm{an}} - M)}{k\delta} \tag{4}$$

where $\delta = \pm 1$ is introduced in order to distinguish the ascending and the descending loop branch, whereas $\delta_{\mathrm{M}} = 0.5\,[1 + \mathrm{sign}\,((M_{\mathrm{an}} - M)\cdot \mathrm{d}H/\mathrm{d}t)]$

Application of chain rule for differentiation and regrouping leads to a closed form of expression for differential susceptibility [19]

$$\frac{\mathrm{d}M}{\mathrm{d}H} = \frac{\delta_{\mathrm{M}}\,(M_{\mathrm{an}} - M)}{k\delta - \alpha^{*}\delta_{\mathrm{M}}\,(M_{\mathrm{an}} - M)} \tag{5}$$

where the modified value of parameter α^{*} for the stressed sample is $\alpha^{*} = \alpha + \frac{3\sigma\lambda_0}{\mu_0 M_s^2}$. Therefore it is possible to determine the average level of stress in the wire on the basis of magnetic measurements for the ready-made and the annealed wires; the latter one may be considered as devoid of residual stress [20]. The slope of for hysteresis loop for a wire with a certain level of tensile stress shall be steeper than for the annealed wire, as proven by Naus [21].

3. Measurements

Measurements of major (saturating) hysteresis loops for wires drawn at different values of single draft were carried out using a Vibrating Sample Magnetometer VSM 7301 from Lakeshore. The details concerning the applied drawing technologies are described in detail in the paper [6]. Three alternative designs were considered: mode A with average single draft D_{av} = 26.5%, mode B with average single draft D_{av} = 6.5% and mode C with average single draft D_{av} = 10.4%. The fragments of relevant $M(H)$ dependencies (descending branches of hysteresis loops) are depicted in Fig. 2. It is easy to notice that the value of chosen single draft affects the shape of the $M(H)$dependence. The steepest slope is obtained for D_{av} = 26.5 %, what implies the highest level of residual stress.

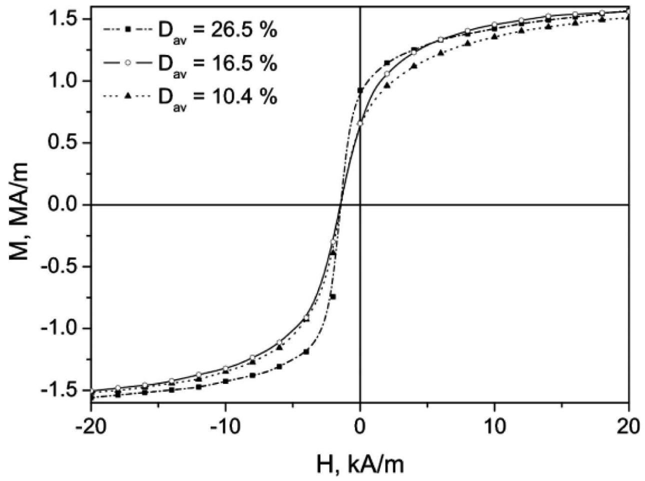

Fig. 2. Measured descending branches of hysteresis loops for different values of single draft

Additionally, the measurements were carried out for the annealed wire, cf. Fig. 3. The values of model parameters for that case have been determined with the „branch-and-bound" algorithm [22]. The obtained set of model parameters was: $\alpha = 2.9 \cdot 10^{-3}$, a = 5537 A/m, k = 1102.1 A/m, $M_s = 1.647 \cdot 10^{6}$A/m.

In order to determine the level of residual stress in the considered wires, the value of parameter α was varied in order to obtain the best match of the modelled dependencies to the measured ones (a nonlinear least square fit problem). Other parameters were kept fixed. The values of mean field parameter α^* (increased due to the existence of residual stress) as well as the values of the average stress, as determined from the relationship $(\alpha^* - \alpha)\mu_0 M_s^2/(3\lambda_0)$ are given in Table 1.

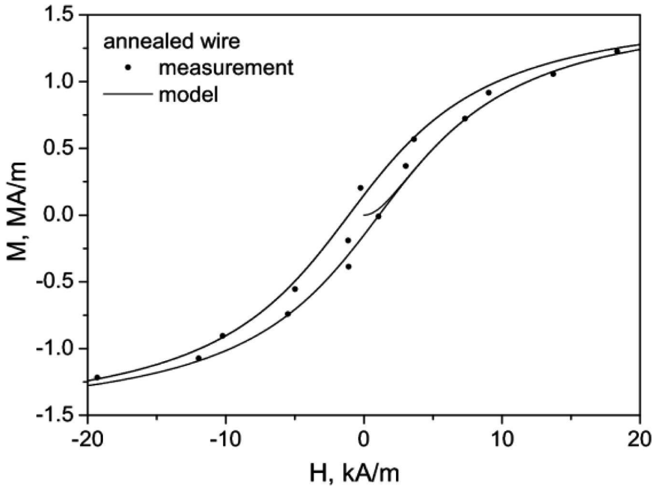

Fig. 3. Hysteresis loops for the annealed wire

TABLE 1

The modelling results

Single draft	α^*, –	$\langle\sigma\rangle$, MPa
D_{av} =26.5%	0.0100	403.4
D_{av} = 16.5%	0.0080	289.8
D_{av} = 10.4%	0.0078	278.4

$\langle\sigma\rangle$ denotes the value of residual stress averaged on the wire cross-section.

The results are in a qualitative agreement with those obtained during previous analyses based on the mechanical (Sachs-Linicus) method and FEM simulations [6], what is depicted visually in Figure 4. It should however be borne in mind, that the values for the latter two methods are given for the wire surface.

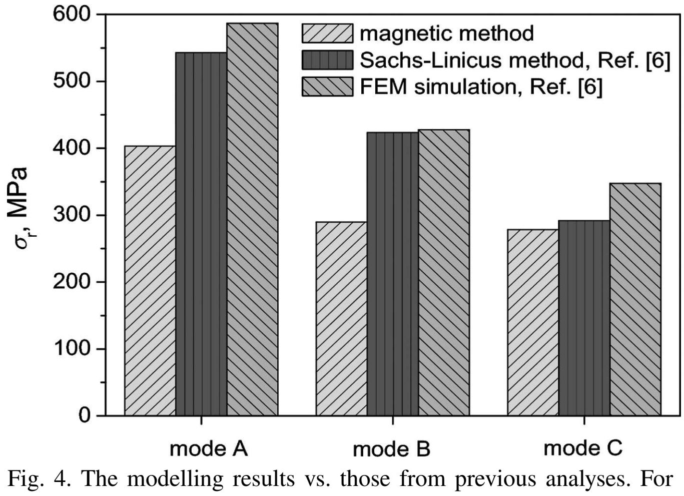

Fig. 4. The modelling results vs. those from previous analyses. For the magnetic method (yellow bars) the average value of residual stress is given, whereas for the Sachs-Linicus and FEM methods the bars denote maximum residual stress values at wire surface

4. Discussion

In recent years the interest of scientific community in the study of magneto-mechanical effects for diagnostics purposes is clearly noticeable [7, 18, 23-27]. Historically, most probably the first attempt to avail of magnetic methods in order to determine residual stress in cylindrical metal bars was undertaken in 1971 by Abuku and Cullity, who have carried out a number of measurements of the reversible effective permeability at different bias field strengths for nickel and steel rods and concluded that this quantity increases almost linearly with tensile stress [28].

In the present paper the saturating (major) hysteresis loop is used as the indicator of residual stress level for different values of single draft. For modelling purposes we have used the Jiles-Atherton description with Sablik's extension to the effective field. In our opinion this modelling framework is simpler to understand and implement than the phenomenological Preisach approach [8, 29]. On the other hand it should be remarked that the concept of effective field itself and Sablik's proposal have been accepted by the scientific community as valuable add-ons also for the Preisach model [30-32]. The approach to determine the average level of residual stress presented in this paper may be considered as based on the inverse magnetostrictive effect. The changes of residual stress level due to varying processing conditions are reflected in the variation of a single model parameter.

It would be desirable to extend the model to determine the effects of local stress on magnetic $M(H)$ dependencies, assuming a simple parabolic profile of residual stress in the wire cross-section [6,33]. However this task seems to be very difficult due to the following reasons:

– the Jiles-Atherton model should be considered as an averaged description of magnetic properties of the medium – it follows from the fact that some model parameters (α, k) are interpreted in terms of averaged quantities; α describes the strength of interactions between magnetic moments within the material (a magnetic counterpart of a many-body system), whereas k is interpreted as the product of pinning site density and their average energy [34]. Theoretically it might possible to apply the Jiles' concepts at micro-level, treating the relationships (1)-(5) as valid locally [35-37], but in that case new problems arise. It is well known that Jiles-Atherton model offers just a qualitative agreement for hysteresis loops that do not reach saturation, whereas it seems crucial to have an accurate representation of arbitrary magnetization curves at hand in order to develop the detailed model.

– In the presented approach anisotropy is neglected both for the $M(H)$ [38, 39] and $\lambda(M)$dependencies. It is possible to introduce in the first approximation a vectorized version of the JA model, similar to the one considered by Szymański and Waszak [40]. Anisotropy of magnetic properties may be considered in the first approximation by an appropriate modification of the value of model parameter a, which controls the loop shape. Such a model extension might be useful if residual stresses in the radial and perimeter directions were to be taken into account. However, the situation is even more complicated with the $\lambda(M)$ dependence. The value $\lambda_0 = 2\cdot 10^{-5}$ used in our simulations corresponds to λ_{100} for iron, i.e. the value for the rolling direction. For the polycrystalline material the saturation magnetostriction is determined as a simple weighted sum $\lambda_s = 0.4\lambda_{100} + 0.6\lambda_{111}$[16, 34], but the values $\lambda_{111}$and $\lambda_s$are negative for iron, what implies a convex shape of the parabolic $\lambda(M)$ dependence for certain directions ! Moreover, the parabolic profile $\lambda(M)$ for the rolling direction is approximately valid only in a limited magnetization range. For the saturation region the $\lambda(M)$ dependence takes a more complicated form, cf. e.g. [13]. In the present paper the same $\lambda(M)$ dependence is used regardless of the level (and sign) of stress and magnetization. Moreover, the hysteresis of the $\lambda(M)$ dependence [30] as well as the asymmetry of $\lambda(M)$dependencies for tensile and compressive stresses [13, 24, 41] are neglected in order to simplify the analysis.

– in the presented approach the effect of varying temperature in the wire cross-section is neglected. Most of the existing JA model extensions aimed at taking into account the influence of temperature on hysteresis [42-46] are focused on ferrites, thus they cannot be applied directly to carbon steels. In our opinion, however, the approach advanced by Perevertov [47], might be relevant for drawn wires.

– assumption of too large value for the α parameter may lead to numerical instability of the JA model. This effect has been noticed already by model developers in their most-cited paper [48], where for a large α value the anhysteretic curve given with Eq. 1 exhibits hysteresis itself. Thus the curve, which should describe purely reversible states from the thermodynamic point of view, becomes irreversible. The problems related to the reversibility issue and the Jiles-Atherton model and their implications are well described elsewhere [49-52]. Attempts to modify the definition of the effective field, leading to an implicit relationship between $M_{\mathrm{an}}$and H do not solve the problem, as instead of hysteresis loop, one obtains an S-shaped curve crossing the second and the fourth quadrant of the coordinate system. It should be noticed that in another hysteresis model considered by Harrison [53], the curve, whose argument is the effective field, is responsible for the irreversible magnetization processes.

Taking into account the above-given argumentation, in the future work aimed at a more precise description of the phenomena affecting the shape of hysteresis loop of drawn wires, we shall depart from the JA model in favour of other descriptions e.g. the Harrison [53] or the Takács [54] models, where the concept of effective field is already present or may be readily implemented [7, 51] and extended with the Sablik's term H_σ. The Jiles-Atherton model should be treated as a simple tool providing approximate results for quick reference.

5. Conclusions

Despite its apparent simplicity due to uncomplicated geometry, a sample of drawn wire is an interesting subject for studying the effects of coupled phenomena (residual stress and temperature) on magnetic properties. In the paper we have suggested that there existed a possibility to estimate the average stress level in drawn wires on the basis of measurements of hysteresis loops. The Jiles-Atherton-Sablik description has been applied in order to describe the phenomenon qualitatively. The wires drawn under varying processing conditions (with different values of single draft) exhibited different shapes of hysteresis loops. This effect has been described as related to the existence of additional term of the effective field in the material.

Future work shall be aimed at development of a more accurate model, which is able to take into account the complicate intricate relationships between its parameters, magnetostriction and temperature. The Jiles-Atherton model is found unsuitable for this purpose (the reasons are given in the paper), but Sablik's extension of the effective field seems to be useful for its coupling with other hysteresis models.

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