Design of the Novel Double-ring Dynamical Gravimeter

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Abstract:
In this paper, a description of the design and operation principle of the double-ring dynamically tuned gravimeter is given. This design allows for the gravimeter signal separation from the noise, which makes it possible to increase the measurement accuracy of Earth's gravitational field anomalies using gravimetric aviation system to 1 mGal.

Keywords: *gravimeter, gyroscope, aircraft gravimetric system*

1. Introduction

The gravimeter is an instrument used for measurements of local variations in Earth's gravitational field. The constant downward gravitational acceleration varies by about 0.5% on the surface of the Earth, due to the planet internal structure, land masses, metallic ores deposits, surface shape etc. The gravimeters operation principle is in essence the same as for other accelerometers, but there is need for exceptional measurement accuracy, due to the extremely small relative changes of the measured value [1]. Gravimeters are used in mineral prospecting, seismology, geodesy, geophysical surveys and research. They display the measured gravitational acceleration value in units of gals (cm^2/s), named after the famous Galileo Galilei [2].

Table. 1. Acceleration units conversion

Base unit	Gal (cm^2/s)	m^2/s	Standard gravity g_0
Gal (cm^2/s)	1	0.01	0.00101972
m^2/s	100	1	0.101972
Standard gravity g_0	980.665	9.80665	1

Existing gravimeter systems are either relative or absolute. Absolute gravimeter system measures the absolute value of the gravity acceleration, ex. by directly measuring the acceleration of a mass during free fall in a vacuum, when the accelerometer is rigidly attached to the ground, and the falling mass forms one of the arms of the Michelson interferometer. Relative gravimeters measure the ratio of the gravity between different measurement points. They can be constructed for example as a mass on a spring. Transportable systems would use extremely stable inertial platform to overcome distortions caused by the device's movement and vibrations [3, 4].

Most of the precise gravimeter systems are heavy, complicated and rigidly attached to the ground. To study the characteristics of the Earth's gravitational field in inaccessible parts of the globe, the aircraft gravimetric system (AGS) is necessary [5, 6]. In the available literature there are descriptions of known gravimeters working principles, and some accelerometers characteristics that are used as gravimeters in AGS's. However, there are no descriptions of the design, functional diagram and the working principle of the double-ring dynamically tuned gravimeter [7–10].

Accuracy of the Earth gravity anomalies measurement using gravimetric aviation system is largely dependent on the choice of system's sensing element. Today, the gyroscopic gravimeters are considered to be one of the most effective among the available solutions. Additionally, in [7, 11] it was proved that the double-ring dynamically tuned gravimeter (DG) has much greater accuracy and performance than the standard one-ring one.

Therefore, further research and development of the double-ring dynamically tuned gravimeter design is advisable, as the most promising of the known aviation gravimeters. The aim of this paper is to provide a description of the design and lay out the working principle of the double-ring dynamically tuned gravimeter.

2. Design

Double-ring dynamically tuned gravimeter development belongs to measurement techniques science, and can be used for moving platform gravimetric measurements in geodesy, geology, and in inertial navigation systems. A dynamically tuned gyroscope (DTG) is a rotor suspended by a universal joint with flexure pivots. The flexure spring stiffness is independent of the rotor spin rate. However, the dynamic inertia, from the gyroscopic reaction effect, from the gimbal suspension provides negative spring stiffness proportional to the square of the spin speed. Therefore, at a particular tuning speed, the two moments cancel each other, freeing the rotor from torque, which is a necessary condition for an ideal gyroscope. The design of the double-ring DTG [12], is shown in Fig. 1. The rotor 1, drive shaft 2 and internal suspension elements are driven by engine 3, which provides a constant rotational speed. Internal Kardanov suspension contains gimbal ring 4, two internal flexure pivots 5, linking gimbal ring to the rotor and two external flexure pivots 6 connecting gimbal ring with a drive shaft. Axes of the internal and external flexure pivots are perpendicular to each other and perpendicular to the axis of the drive shaft.

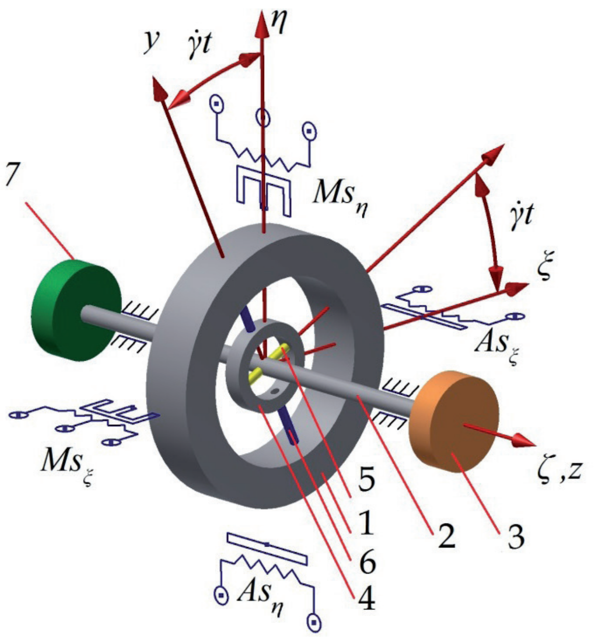

Fig. 1. Schematic diagram of the gravimeter construction

The reference voltage generator 7 is coupled with the drive shaft. The generator is used to generate control signals that drive the Ms_ξ and Ms_η moment sensors, affecting the rotor. The two electromagnetic torque sensors are attached to the body of the device. When supplying power to the sensors they create a torque around η or ξ axis, which does not rotate with the rotor Voltage, is applied between the middle and two extreme points of the windings. The angle sensors As_ξ and As_η are also attached to the body of the device. They convert the rotor plane deviation from the neutral position into electrical signals. In particular, the shown angle sensors respond to the change of the air gap between the windings center and the rotor surface, in device body points that do not revolve around the ξ and η axis. The basis of the design of double-ring dynamically tuned gravimeter was the known two-stage gyroscope based gravimeter [11, 12]. The common essential feature of the double-ring dynamically tuned gravimeter and the known one is that they contain two-stage gyroscope and the torque sensors. However, unlike the double-ring dynamically tuned gravimeter, in the known gravimeter construction the center of mass of the two-stage gyroscope rotor is located on the axis of rotation, and it contains the sensor of the angular velocity of the gyroscope rotor rotation relative to the base. As a result, the output value of the gravimeter is the angular velocity of rotation of the gyroscope rotor relative to the device's base. Thus, the known gyroscopic gravimeter require additional signal conversion to determine the gravity acceleration value, which contributes additional errors in measurement result.

Furthermore, there is no measurement errors calculation and compensation in the standard device. Therefore, measurement results obtained using this gravimeter contain*s* significant measurement errors. These errors are caused by non-linear distortions of the trajectory of the sensing element; precession oscillations damping through viscous type torques action on the sensor element; non-synchronization of precession oscillations; the discrepancy between the value of the angular precession vibrations frequency used in the estimation algorithms, and the value of the angular precession oscillation frequency of the sensing element; interferences that distort the sensing element mode of motion. Thus, significant drawback of the known solution is a low accuracy.

Thus, the presented work is focused on redesigning the gravimeter to improve the accuracy of measurements of the acceleration of gravity. The problem is solved by the fact that the center of rotor 2 mass *of* the dynamically tuned gyroscope 1 (Fig. 2) is shifted relative to its axis of rotation. Additionally, rotation angle sensor 3 and the calculation and measurement errors compensation unit 5 are introduced, where the angle of rotation sensor 3 input is connected to the dynamically tuned gyroscope 1 output and rotation angle sensor 3 output is connected to the input of measurement errors calculation and compensation device 6.

3. Operational Principle

Double-ring dynamically tuned gravimeter working principle is as follows. The center of mass C of the rotor 2 is shifted relative to the axis of rotation by the value of l (Fig. 2). With the drive motor (not shown in the Fig. 2, 3 in Fig. 1) rotor 2 rotates at a constant angular velocity γ. In the absence of external influences rotor 2 rotates in the horizontal plane. In the presence of gravitational acceleration along the axis of gyroscope rotation, the rotor 2 starts to deviate. The result is a signal at the output of the angle of rotation sensor 3 that goes to the input of the device 6, through the low-pass filter 5. The device 6 performs calculations and compensation of measurement errors. Torque sensor 4 is used for measuring compensation method based on dynamically tuned gyroscope. The output value of the double-ring dynamically tuned gravimeter is the output signal of the measurement errors calculation and compensation device 6. If 0_z axis is directed vertically, along the acceleration of gravity g, with the other accelerations excluded, there is a moment of the force of gravity M_g that will make the rotor 2 deviate relative to the axis 0_x. Moment of the force of gravity M_g is determined by the expression:

$$M_g = mgl \cos \alpha \tag{1}$$

where: m – rotor mass, l – displacement of the center of rotor mass relative to its axis of rotation, α – angle of rotor deviation.

Moment $M_{\ddot{h}}$ of the vertical acceleration $\ddot{h}$ on the axis of sensitivity of the device (Fig. 1) is:

$$M_{\ddot{h}} = m\ddot{h}l \cos \alpha \tag{2}$$

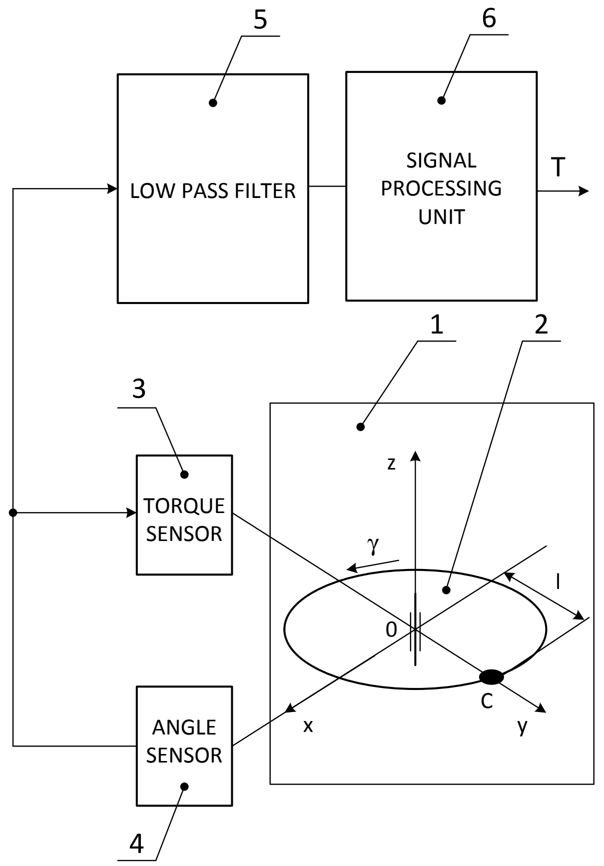

Fig. 2. Gravimeter operating schematic diagram

Elastic torque M_T of the rotor's flexure pivots is:

$$M_T = C_X \alpha \tag{3}$$

where C_x – Flexure pivots stiffness.

The centrifugal moment M_u is:

$$M_u = I\dot{\gamma}^2 \sin\alpha \tag{4}$$

where I – moment of inertia of the rotor, γ – angle velocity of the rotor.

If we assume that $\alpha \ll 1$ rad, then we can write :

$$C_X\alpha + I\dot{\gamma}^2\alpha = -mlg + ml\ddot{h} \tag{5}$$

where, denoting $k = C + I\dot{\gamma}^2$, we get:

$$\alpha = -\frac{ml}{C + I\dot{\gamma}^2}(g + \ddot{h}) = -\frac{ml}{k}(g + \ddot{h}) \tag{6}$$

Denoting $S = \frac{ml}{k}$, we get the low-pass filter input signal:

$$T = \frac{1}{S}\alpha = -g + \ddot{h} \tag{7}$$

where:

$$\alpha = (-g + \ddot{h})S \tag{8}$$

Thus, the angle of rotor deviation is proportional to the acceleration of gravity g and vertical acceleration $\ddot{h}$.

The output signal of the angle of rotation sensor passes through the amplifier (not shown in the drawing) with a gain of $1/S$ and enters the lowpass filter input. The output signal of the angle of rotation sensor also present errors caused by translational and angular vibrations of the gravimeter carrying aircraft. Therefore, given these circumstances, lowpass filter input is given by:

$$T = \frac{1}{S}\alpha = -g + \ddot{h} - (R_x\alpha - R_y)\beta - \frac{B}{ml}(\dot{\omega}_x + \dot{\omega}_y\alpha) + \frac{M_{i2}}{ml} \tag{9}$$

Where: R_x, R_y – translational acceleration of the aircraft projections on the axes 0_x, 0_y, β – proportionality factor, B – moment of inertia of the rotor, $\dot{\omega}_x, \dot{\omega}_y$ – angular acceleration of the aircraft projections on the axes 0_x, 0_y, M_{i2} – torque related to the instrumental errors of the dynamically tuned gyroscope.

Since the DG performs direct measurements of the acceleration of gravity, it is possible to filter the measurement signal. The frequency spectrum of the signal, which corresponds to the acceleration of gravity, and the signal corresponding to the vertical acceleration, angular and translational vibrations of the aircraft are different (Fig. 3, curve 2 and curve 1, respectively). The bulk of utility component of the measurement signal is focused at frequencies lower than 0.1 rad/s (curve 2, Fig. 3). The majority of the errors and noise in the measurement signal is concentrated

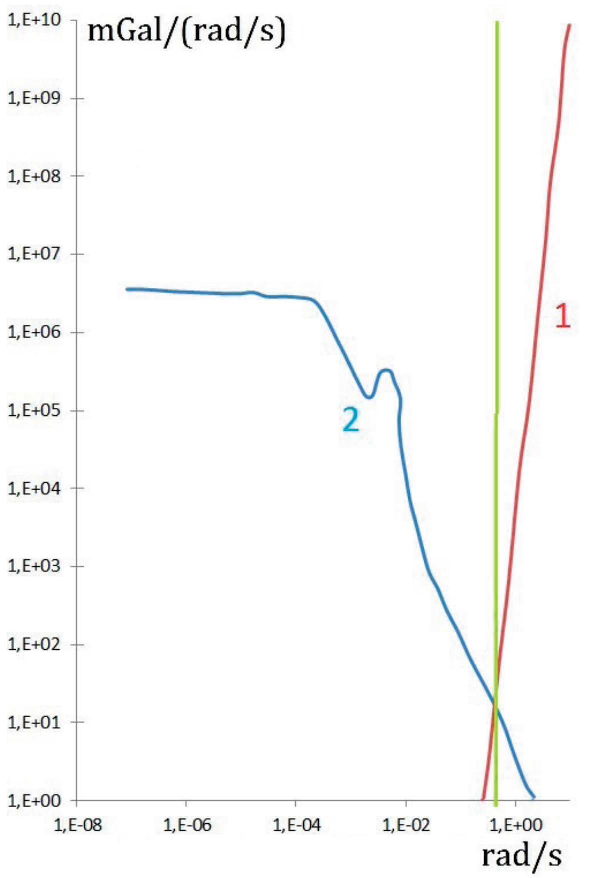

Fig. 3. The spectral density of the vertical acceleration $G_{\ddot{h}}(\omega)$ (1) and the acceleration of gravity anomalies $G_{\Delta g}(\omega)$ (2)

at frequencies greater than 0.1 rad/s (curve 1, Fig. 3). Because of this, the low-pass filter was incorporated in the gravimeter system. This filter performs filtering of the measurement signal to eliminate errors caused by vertical accelerations, angular and translational vibrations of the DG carrying aircraft. The signal processing unit converts the filtered signal to the DG output signal.

Most of the first frequency signal is 0.00175 rad/s, the majority of the second signal frequency is 0,269 rad/sec. Low-pass filter has a cutoff frequency of 0.1 rad/s and performs signal filtering to eliminate errors caused by vertical accelerations, angular and translational vibrations. For a low-pass filter, a relationship can be written [11]:

$$T'(\tau) = \int_{-\infty}^{\infty} w(t-\tau)T(\tau)d\tau \tag{10}$$

where $w(.)$ – lowpass filter weight function. It can be written as [15]:

$$w(t) = \int_{-\infty}^{\infty} W(j\omega)e^{j\omega\tau}d\omega = 2\omega_0\left[\frac{sin\omega_0 t}{\omega_0 t}\right] \tag{11}$$

where $W(j\omega)$ – transfer function of low-pass filter.

The low-pass filter output signal is fed to the input of signal processing unit, which calculates the DG output signal according to the formula:

$$T'(\tau) = \int_{-\infty}^{\infty} 2\omega_0\left(\frac{sin\omega_0(t-\tau)}{\omega_0(t-\tau)}\right)T(\tau)d\tau \tag{12}$$

In this case $\tau = 2\pi n$ – the time interval of the signal processing unit, $n = 1,2,...$ – the number of complete revolutions of the DNG outer frame.

The result is an DG output signal, which contains useful information about the gravity acceleration g. The errors in the signal are for the great part filtered out. These errors are caused by: translational acceleration with the dominant frequency of 3140 rad/s; angular acceleration with the dominant frequency of 20 rad/s, which is equal to the frequency of natural DG oscillations (the most dangerous case of resonance); angular acceleration with the dominant frequencies of 40 rad/s and 60 rad/s; angular acceleration, the frequency of which is equal to 6.7 and 10 rad/s (subharmonic oscillations).

In addition, the computation time $\tau = 2\pi n$ allow for almost entirely DG instrumental error elimination, because for a full rotation of its external frame these errors in turn take equal amplitude positive and negative values, that is, on average, they are zero.

Improving the accuracy of measurements in the double-ring dynamically tuned gravimeter is ensured by the fact that the center of mass of the DG rotor is shifted from its axis of rotation, and therefore this gravimeter takes the direct measurement of the acceleration of gravity. These measurements are performed using the rotation angle sensor, which was incorporated into the device.

During the preliminary tests of the proposed invention, the following hardware was used:

- Dynamically tuned gyroscope serially produced at "NVK GI Petrovsky Kyiv Automation Factory", Kyiv, Ukraine);
- Signal processing unit – onboard computer AN-124.

In the factory made gyroscope the said center of mass of the rotor is located on the axis of rotation. Therefore, the modification was made by fixing extra weight to the rotor, the location of which shifted the rotor center of mass relatively to its axis of rotation.

As a result of the tests it was determined that DG allows to separate the gravity acceleration signal from the disturbances caused by vertical accelerations, angular and translational vibrations of the aircraft carrying the gravimeter. Thus, the DG output signal is compensated for the impact of a number of measurement errors. However, the results of measuring the gravity acceleration contain systematic errors ΔR_i due to the following factors influencing the gravimeter:

- ΔR_1: nonlinear distortion of the gravimeter sensing element trajectory,
- ΔR_2: precession oscillations damping through viscous type torques action on the sensor element,
- ΔR_3: non-synchronization of the precession oscillations,
- ΔR_4: the discrepancy between the value of the angular precession vibrations frequency used in the estimation algorithms, and the value of the angular precession oscillation frequency of the sensing element,
- ΔR_5: interferences that distort the sensing element mode of motion.

Compensation of the systematic measurement errors of gravitational acceleration is performed by their subtraction according to the known formula [16]:

$$g = R - \sum_i \Delta R_i \tag{13}$$

where: g – gravimeter output signal after error compensation, R – the output signal of the rotation angle sensor proportional to the acceleration of the force of gravity.

Calculation of systematic measurement errors in the device is performed based on the approach outlined in [11]. Without the error compensation for the proposed method, the errors may be equal in magnitude to the signal value, i.e. be unacceptably large. Compensation of errors allows to significantly increasing of the measurement accuracy of the gravity acceleration.

Furthermore, from the literature [7, 8] it is clear that the dynamically tuned gravimeter has the major advantages over other gyroscopic gravimeters:

- The absence of friction in the rotor suspension node,
- in case when the stiffness of gravimeter's suspension is close to zero, the sensitivity and the accuracy of the device increases considerably,

– it has small dimensions (54×46 mm diameter) and low weight (0.35 kg) – less than any other gyroscopic gravimeter,
– it has an automated processing of the output signal, thus its performance is much higher than in other types of aircraft gravimeters.

4. Conclusions

As a result of the tests it was determined that the double-ring dynamically tuned gravimeter enhances gravitational acceleration measurement accuracy to 10 times compared with other known gyroscopic gravimeters. Improving the accuracy of measurements in the device is ensured by the fact that the center of mass of the DG rotor is shifted relative to the axis of its rotation, and therefore this gravimeter takes the direct measurements of the gravity acceleration, and it became possible to perform measurement signal filtering. The achieved gravitation measurement accuracy is 1 mGal, that is 0.00001 m/s^2 or $\sim 1*10^{-6}$ g .

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