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Influence of Axial Heat Conduction in the Wall on Convective Heat Transfer in the Microchannel

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1. Introduction

In literature there are reported many experimental results revealed significantly lower value of heat transfer coefficients for fully developed laminar flow. Some of the authors are trying to explain this phenomenon due to scale, aspect ratio of micro channels in micro heat exchanger, roughness of the wall or axial heat conduction. The importance of axial heat conduction increases with increasing ratio of channel thickness to channel length, and wall to fluid thermal conductivity ratio. The fluid temperature along the channel length in uniformly heated channel did not increase linearly, according to many researchers. This evidence is due to axial conduction. Davis and Gill [2] numerically investigated the effect of axial heat conduction. According to survey of literature clearly exhibit the effect of axial conduction on heat transfer. Herwig and Hausner [3] compared their numerical simulation with experimental data of Tso and Mahulikar [6] for water flow in an aluminium plate with 25 circular microchannels. Experimental evidence as well as extensive numerical studies confirm this effect.

Many authors present their results both on theoretical and experimental investigation concerning usage of mini- and microchannel heat exchangers used in environmentally friendly distributed energy installations based on ORC ([1]). However, proper design of microchannel heat exchangers have to take into account many additional effects, for example axial heat conduction in microchannel heat exchanger walls.

In the present work a new analytical model is developed based on the axial conduction effects on the local fluid temperature and on the local wall temperature. This model is compared with available similar theoretical models and experimental data for single-phase liquid flow.

2. Theoretical Model

Consider laminar developed flow in micro channel subjected to constant heat flux. In this work, the analysis is oriented at evaluating the local fluid and wall temperatures at any section of the channel in the fully developed flow region. The model is intended to determine heat transfer coefficient in the fully developed region taking into account conduction in the wall. The sketch of the problem is shown in Fig. 1. Fluid is flowing in the channel and the wall is subjected to constant heat flux on the outside surface. Additionally, in the model the following assumptions were made: steady state, fluid properties are constant, fluid flow is thermally and hydro dynamically fully developed.

Fluid in the channel is warmed or cooled by convective heat flux q_k which has lower value then heat flux q on the outside surface of the wall. This is the reason that that convective heat transfer coefficient on the inner surface of the wall is lower than heat transfer coefficient calculated on the basis of heat flux on outer surface of the wall.

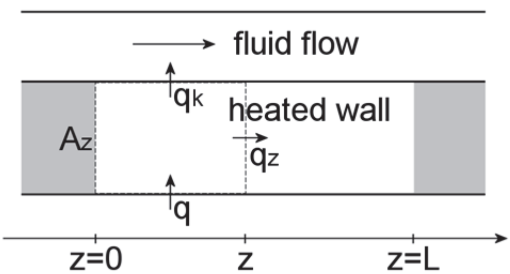

Fig. 1. The sketch of the problem of axial conduction in the wall
Rys. 1. Przedstawienie zagadnienia wzdłużnego przewodzenia ciepła w ściance

Considering the energy balance for:
1. the wall:

$$qU = q_k U + q_z A_z \tag{1}$$

Rewriting in details (1) we have;

$$qU = h_e U(T_w - T_f) - \lambda A_z \frac{d^2 T_w}{dz^2} \tag{2}$$

2. the fluid:

$$\dot{m} c_p dT_f = h_e U(T_w - T_f) dz$$

or

$$\dot{m} c_p \frac{dT_f}{dz} = h_e U(T_w - T_f) \tag{3}$$

Assuming that for constant heat fluxes:

$$\frac{dT_f}{dz} \cong \frac{dT_w}{dz} \tag{4}$$

we can obtain differential equation for the wall by introducing (3) and (4) to (2):

$$qU = \dot{m} c_p \frac{dT_w}{dz} - \lambda_z A_z \frac{d^2 T_w}{dz^2} \tag{5}$$

Solving (5) with boundary conditions $\frac{dT_w}{dz} = 0$ for z=L we get:

$$\frac{dT_w}{dz} = \frac{qU}{\dot{m} c_p}(1 - e^{-M(l-x)}) \tag{6}$$

where: $$M = \frac{\dot{m} c_p}{\lambda_z A_z} \tag{7}$$

Adiabatic boundary condition assumed for z=L is not exactly satisfied in the reality, but it simplifies mathematical calculation.

Introducing Reynolds Number $\mathrm{Re} = \frac{d_h G}{\mu}$ and Prandtl Number $\mathrm{Pr} = \frac{c_p \rho \gamma}{\lambda}$ we obtain dimensionless form of number M^+:

$$M^+ = \frac{\mathrm{Re}}{\mathrm{Pr}} \frac{\lambda_f}{\lambda_w} \frac{Ul}{A} \tag{8}$$

In the case when we neglect axial conduction in (2) we have heat transfer from wall to the fluid only through convection:

$$qU = h\,U(T_{wz} - T_{fz}) = \dot{m}c_p \frac{dT_{fz}}{dz} \tag{9}$$

where: h – nominal heat transfer coefficient for laminar developed heat transfer.

Assuming for constant heat flux that:

$$\frac{dT_{fz}}{dz} \cong \frac{dT_{wz}}{dz} \tag{10}$$

Introducing (10) to (9), we obtain constant gradient of temperature distribution in the wall.

$$\frac{dT_{wz}}{dz} = \frac{qU}{\dot{m}c_p} \tag{11}$$

Comparing (6) with (11) we can see that axial gradient temperature in the wall in case of taking into account axial conduction is less than in the case when we neglect axial conduction except of beginning conduction in the wall. Convective term in (2) is:

$$q_{con} = h_e(T_w - T_f) = \frac{\dot{m}c_p}{U}\frac{dT_f}{dz} \cong \frac{\dot{m}c_p}{U}\frac{dT_w}{dz} \tag{12}$$

Introducing (6) into (12) we get:

$$\frac{q_{con}}{q} = 1 - \exp(-M^+(1 - x^+)) \tag{13}$$

In the same relation are heat transfer coefficients. The ratio of the two heat transfer coefficients may be written as:

$$\frac{h_e}{h} = 1 - \exp(-M^+(1 - x^+)) \tag{14}$$

or

$$\frac{Nu_e}{Nu} = 1 - \exp(-M^+(1 - x^+)) \tag{15}$$

Relation (14) for M^+=1 is shown in Fig. 2.

Average value of equivalent heat transfer coefficient is:

$$\frac{h_e}{h} = \frac{1}{l}\int_0^1 \frac{h_e(x^+)}{h}dx^+ = 1 - \frac{1}{M^+}(1 - \exp(-M^+)) \tag{16}$$

or

$$\frac{Nu_e}{Nu} = 1 - \frac{1}{M^+}(1 - \exp(-M^+) \tag{17}$$

Dependence of $\frac{h_e}{h}$ against M^+ is presented in Fig. 3

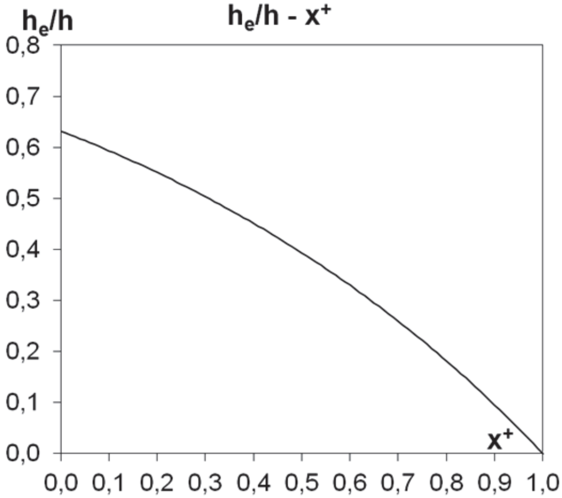

Fig. 2. Local ratio $\frac{h_e}{h}$ against reduced heated part's length x^+

Rys. 2. Lokalna iloraz $\frac{h_e}{h}$ w zależności od zredukowanej długości x^+

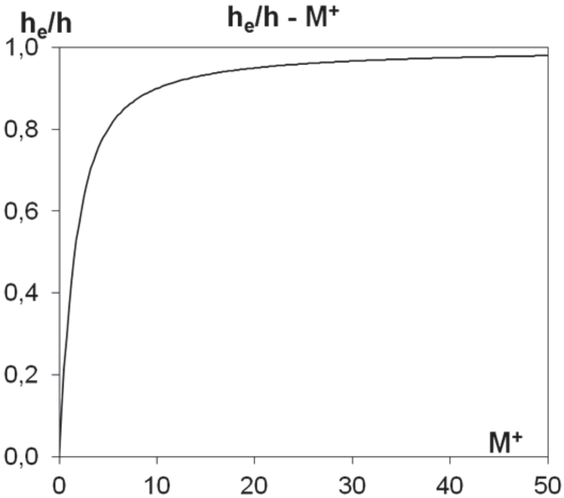

Fig. 3. Dependence of $\frac{h_e}{h}$ against M^+

Rys. 3. Zależność $\frac{h_e}{h}$ od M^+

3. Validation of the Model

Lin and Kandlikar [4] compared their model with numerical simulation of Maranzana et al [5] for water flow in a 100 μm channel formed by two 10 mm long and 500 μm thick silicon blocks. The channel was heated by 30 kW/m^2 uniform heat flux on one wall and the other wall was applied an adiabatic boundary condition. Fig. 4 shows the numerical simulation results by Maranzana et al [5] and Lin and Kandlikar [5]. The present model, Eq. (17) is also plotted in Fig.4 for comparison with simulation results.

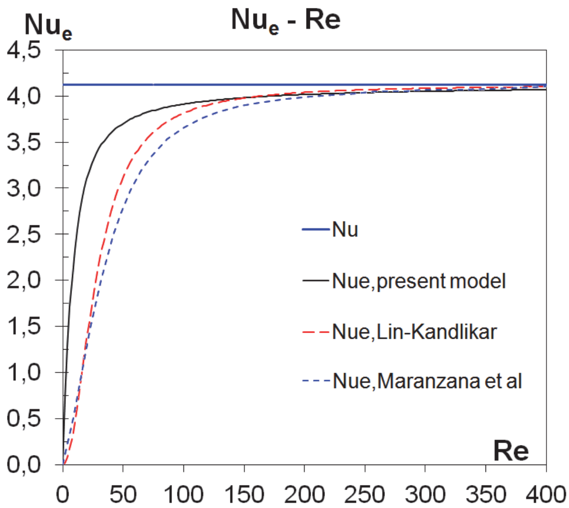

Fig. 4. Results of numerical simulations by Lin and Kandlikar [4] and Maranzana et al [5] with comparison to the present analytical model
Rys. 4. Wyniki porównania numerycznych symulacji wg Lin i Kandlikar [4] oraz Maranzana et al [5] z prezentowanym modelem analitycznym

4. Conclusions

The effect of axial conduction is important, while reducing the experimental data. A new model is developed to account for this effect. The effect of axial heat conduction in the fluid is shown to be negligible for air and water flow in microchannels for conditions generally encountered in cooling electronic equipment. The paper presents that the new model results are in agreement with numerical simulations and experimental evidences. The model shows that the axial conduction effect in the wall are important for gas flow in any wall material and is negligible for water flow in metal tubes.

Nomenclature

A	– channel wall cross-sectional area for axial conduction, m^2
c_p	– heat capacity, J/kgK
d_h	– hydraulic diameter of the channel, m
h	– heat transfer coefficient, W/m^2K
L	– length of the heated part, m
M	– nondimensional parameter
m	– mass flow rate, kg/s
Nu	– Nusselt number
Pr	– Prandtl number
q	– heat flux, W/m^2
Re	– Reynolds number
U	– perimeter, m
T	– temperature, K
x	– distance from the entrance of the heated part, m
z	– axial coordinate, m

Greek letters

λ	– thermal conductivity, W/mK

Subscripts

conv	– convection
e	– with conduction
f	– fluid
w	– wall
z	– conduction
+	– dimensionless (reduced)

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Wpływ podłużnego przewodzenia ciepła w ściance na konwekcyjną wymianę ciepła w mikrokanale

Streszczenie

Opracowany został nowy model umożliwiający obliczenie wpływu podłużnego przewodzenia ciepła w ściance kanalika na wymianę ciepła pomiędzy ścianką a przepływającym czynnikiem w mikrokanale. Zbadany został przypadek ustalonej wymiany ciepła. Znaleziono bezwymiarowe kryterium umożliwiające ocenę ważności wzdłużnego przewodzenia ciepła w ściance. W publikacji przedstawiono porównanie wyników modelu z wynikami przy zaniedbaniu podłużnego przewodzenia ciepła oraz z wynikami innego modelu opracowanego przez Kandlikara. Dodatkowo wykonano porównanie nowego modelu z wynikami eksperymentalnymi innych autorów. Osiągnięto zadawalające rezultaty.

Słowa kluczowe:
podłużne przewodzenie w ściance, konwekcyjna wymiana ciepła, mikrokanał

Keywords:
axial conduction in the wall, convective heat transfer, microchannel