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A NEW APPROACH FOR DETERMINING THE TWO-MASS MODEL PARAMETERS OF A RAILWAY CURRENT COLLECTOR

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Abstract
The paper presents two mathematical models of railway current collectors both with two degrees of freedom. The first one, hereinafter Pantograph Articulated Model (PAM), has one degree of freedom in rotational motion and the second degree of freedom in translational motion. The second model, called henceforth as Pantograph Reference Model (PRM), has both degrees of freedom in translational motion. Differential equations of the PAM contain very complex coefficients dependent on rotation angles of individual arms. These coefficients can be determined analytically, based on the dimensional and material data of the collector. The mathematical formulation of the PRM is relatively simple, but the coefficients in differential equations of this model are equivalent. Defining them by way of analysis makes it necessary to adopt numerous simplifying assumptions. Application of the PRM is justifiable in many cases, particularly while analysing the interaction between the collector and the contact line. In order to ensure that the results of the analysis are reliable, it is necessary to define, with appropriate accuracy, the equivalent values of the PRM coefficients. This is usually done through experiments. The paper shows the way in which the PRM coefficient values are defined based on the PAM simulation. The advantage of the presented method is that it does not require a complex experimental setup.

Introduction
System commonly used for supplying electric railway vehicles consists of the overhead catenary along the track and current collectors placed on the roof of vehicles [6]. With the ever increasing speeds and power of the vehicles, the problem of the quality of dynamic cooperation of these elements is gaining in importance [10]. It is necessary to maintain contact force within a specified range to ensure continuity of contact and eliminate the phenomenon of arc, and on the other hand to reduce the risk of mechanical damage and to minimize wear of the contact strips and wires. On an open market of railway transport in the European Union are introduced Technical Specifications for Interoperability (TSI) and the detailed technical standards [2-5], as a condition for approval vehicles as well as new or overhauled catenaries into operation. In terms of verification a correct dynamic cooperation between pantograph and catenary, among others, it is required to carry out simulations based on so-called reference models. In TSI for current collector of any type and of various design was adopted a unified traditional lumped mass model. It consists of two equivalent masses, connected by springs and damping elements, which remain only in a translational motion in one axis and are subjected to external forces - see Fig. 1 on the right. This model of current collector will be called hereafter as PRM – Pantograph Reference Model. Besides normative requirements, the simplicity of the PRM due to the absence of complex nonlinear relationships and of masses in rotational movement qualifies it for such applications where a short simulation time is important, e.g. for so-called active pantograph systems.

Although the structure of this reference model is relatively simple, it does not precisely reflect the real construction solutions of the current collector. The use of PRM for a specific type of pantograph therefore requires a determination of the relevant parameter values – the equivalent masses, stiffness and damping coefficients.

Values of PRM parameters are usually defined by analysing of displacements of the current collector elements in static and dynamic conditions, caused by the applied force with a known function in time domain. The aim of this analysis is to define the parameters of the PRM so, that it will respond similarly to the real current collector. The idea of such measurement is presented schematically in Fig. 1.

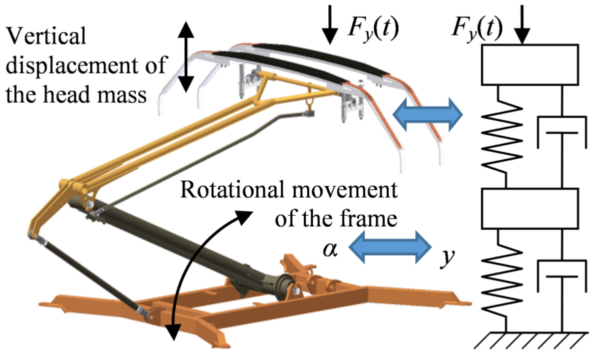

Fig.1. Equivalent pantograph model formulated in translational motion developed from experimental results on articulated type pantograph

The above-mentioned analysis can be conducted based on experiments but it requires a suitably equipped measurement stand and is rather expensive [11]. It is also possible to carry out such analysis based on simulations with the use of the current collector model which takes into account the actual articulated construction and real physical parameters of the pantograph [8]. Such model will be called hereafter as PAM – Pantograph Articulated Model. This paper shows the way of determining the parameters of the PRM based on the PAM simulations. On the other hand, the use of simulation method makes it necessary to elaborate with sufficient accuracy the PAM.

The joint construction of current collectors makes it possible to raise the contact strips by rotating its arms in relation to joint axes. As a result the mathematical formulation of the PAM takes into account degrees of freedom in rotational motion [1]. The coefficients in differential equations of PAM are relatively complicated functions of the rotation angle of pantograph arms [9]. The complexity of these functions is caused by the fact that the gravity centers of the collector arms are moving along the trajectory with varying radius. These functions can be defined by way of analysis, based on the Lagrange formalism and on the known construction details of the arms. These arms are subjected to dynamic forces in the direction which is tangential or perpendicular to the trajectory of movement. Both components of these forces significantly influence the movement dynamics of the entire collector.

The PAM and PRM models are formulated in this paper. Also the issues of analytical determining the parameters of the PAM are discussed. Afterwards, the method for defining the parameters of the PRM, based on the PAM simulation is presented. Selected experiment and simulation results are presented and compared.

Pantograph articulated model PAM

The mathematical model of current collector of PAM type was elaborated using the Lagrange's energy method. The collector was divided into lumped conservative elements (inertial and elastic) and dissipative elements (dampers), taking into consideration both the translational and rotational movement. The elaborated model is representative for current collectors whose construction is based on two articulated quadrilaterals – the collector type 160 EC or DSA150, … 380 are examples of such construction. The collector model, in its part connected with articulated arms, was elaborated with a high degree of accuracy. The simplifying assumptions for the model, which have been adopted in this paper are as follows:

- only one degree of freedom, i.e. the translational motion of the entire panhead unit comprising the panhead's springs set and the panhead itself, is taken into consideration,
- the upper arm of the pantograph together with panhead guide rod are represented by one equivalent beam.

The computer 3D collector model of the PAM type is presented in Fig. 2 and Fig. 3, where the individual lumped elements resulting from pantograph's structure and from physics of the phenomena are additionally labelled. The symbols marked in Fig. 2 and Fig. 3 denotes the following:

- M_i, J_i – mass and its moment of inertia with respect to the axis i of respectively: i=1 – the lower arm, i=2 – the lower guide rod, i=3 – the upper arm lever,
- M_4, J_4 – mass and its moment of inertia to axis 3 of the equivalent beam for upper arm and panhead guide rod,
- M_5, M_6 – masses of the supports and the panhead unit,
- D_1, D_2, D_3, D_4, and D_5 – equivalent damping coefficients in rotational movement which represent viscous and dry friction in the articulations 1, 2, 3, 4, and 5,
- D_7 – equivalent damper in translational movement representing dissipation of energy in the head supports,
- K_7 – compliance of the equivalent spring in translational movement.

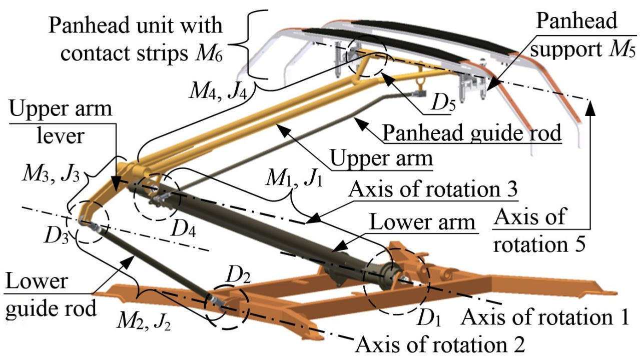

Fig.2. A set of rigid bodies that compose the mechanical system of tested pantograph; parameters used in model PAM are marked

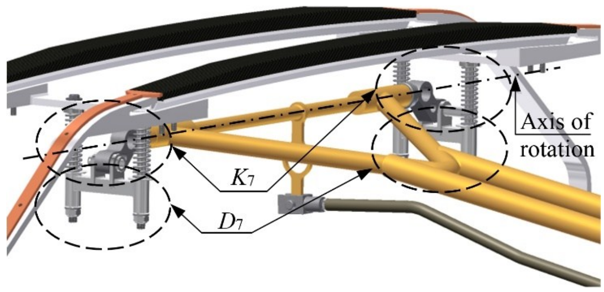

Fig.3. Elastic panhead supports and pantograph contact strips

The diagram of the kinematic system of the PAM is shown in Fig. 4.

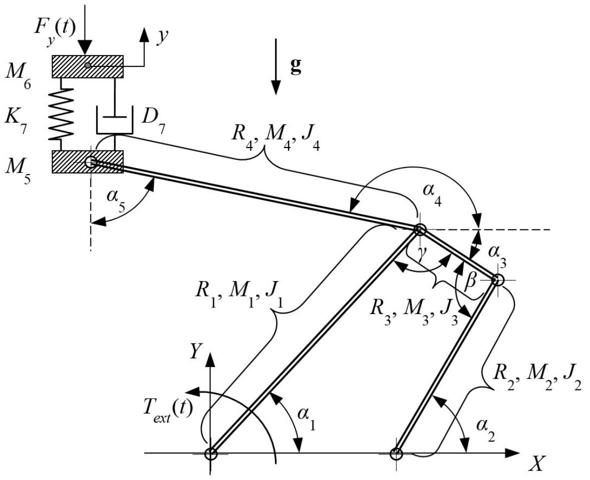

Fig.4. Kinematic diagram of articulated type pantograph PAM with two degrees of freedom – rotational and translational motion

Additional symbols in this diagram denote the following:

- R_i, α_i – length and angle to the x axis of respectively: i=1 – the lower arm, i=2 – the lower guide rod, i=3 – the upper arm lever, i=4 – the upper arm equivalent beam,
- α_5 – angle to the y axis of the upper arm,
- $T_{ext}(t)$ – external torque acting on the lower arm,
- $F_y(t)$ – force representing interaction between pantograph and a catenary system,
- $y(t)$ – vertical displacement,
- **g** – vector of gravity.

The Lagrangian defined in the coordinates assigned to individual lumped elements has the form

$$L(\dot{\alpha}_1,...,\dot{\alpha}_4,\dot{x}_5,...,\dot{y}_6,y_7)=\frac{1}{2}J_1\dot{\alpha}_1{}^2+\frac{1}{2}J_2\dot{\alpha}_2{}^2+\frac{1}{2}J_3\dot{\alpha}_3{}^2$$
$$+\frac{1}{2}J_4\dot{\alpha}_4{}^2+\frac{1}{2}M_5(\dot{x}_5^2+\dot{y}_5^2)+\frac{1}{2}M_6(\dot{x}_6^2+\dot{y}_6^2)-\frac{1}{2}\frac{y_7^2}{K_7} \tag{1}$$

where:

- $\dot{\alpha}_1,\dot{\alpha}_2,\dot{\alpha}_3,\dot{\alpha}_4$ are, respectively, angular velocities of corresponding parts of pantograph to related axes,
- $\dot{x}_5,\dot{y}_5,\dot{x}_6,\dot{y}_6$ are velocity components in translational motion, respectively for elements with the mass M_5, M_6,
- y_7 denotes relative terminals displacement of equivalent spring with K_7 compliance.

The particular expressions of equation (1) represent in turn: kinetic co-energies in rotational motion of the lower arm, the lower guide rod, the upper arm lever and the upper arm equivalent beam. The subsequent expressions are respective kinetic co-energies in translational motion of the panhead support springs set and the panhead unit. The last expression is the potential energy of the springs set.

The Rayleigh dissipation function, defined in the coordinates assigned to individual equivalent dampers, where only viscous friction is taken into account, is

$$P_m(\dot{\alpha}_1,\dot{\alpha}_2,\dot{\beta},\dot{\gamma},\dot{\alpha}_5,\dot{y}_7)=$$
$$\frac{1}{2}D_1\dot{\alpha}_1^2+\frac{1}{2}D_2\dot{\alpha}_2^2+\frac{1}{2}D_3\dot{\beta}^2+\frac{1}{2}D_4\dot{\gamma}^2+\frac{1}{2}D_5\dot{\alpha}_5^2+\frac{1}{2}D_7\dot{y}_7^2 \tag{2}$$

where:

$\dot{y}_7$ is relative velocity of equivalent spring terminals,

$\dot{\beta}$, $\dot{\gamma}$ are angular velocities of equivalent dampers, D_1, D_2, D_3, D_4 and D_5 are coefficients of viscous friction of dampers in rotational motion, D_7 is the coefficient of viscous friction of the damper in translational motion.

Furthermore dry friction was taken into consideration in the final form of the Euler-Lagrange equation of the collector.

In accordance with the adopted simplifying assumptions, the current collector model has two degrees of freedom. They are the y displacement of the gravity center of mass M_6 and the rotation angle of the pantograph arms – it has been assumed that it is the α_1 angle. With such assumption, the equation of constraints may be expressed in a general form, as

$$\alpha_2=\alpha_2(\alpha_1),\quad \alpha_3=\alpha_3(\alpha_1),\quad \alpha_4=\alpha_4(\alpha_1),\quad \alpha_5=\alpha_5(\alpha_1),$$
$$x_5=x_5(\alpha_1),\quad y_5=y_5(\alpha_1),\quad x_6=x_6(\alpha_1),\quad y_6=y_6(y),$$
$$y_7=y_7(\alpha_1,y),\quad \beta=\beta(\alpha_1),\quad \gamma=\gamma(\alpha_1) \tag{3}$$

The detailed form of these functions is very complex and will not be presented in this paper.

The general form of the Euler-Lagrange equation of the PAM is expressed in the formula

$$\frac{d}{dt}\left(\frac{\partial L(\dot{\alpha}_1,\alpha_1,\dot{y},y)}{\partial\dot{\alpha}_1}\right)-\frac{\partial L(\dot{\alpha}_1,\alpha_1,\dot{y},y)}{\partial\alpha_1}+\frac{\partial P_m(\dot{\alpha}_1,\dot{y})}{\partial\dot{\alpha}_1}=Q_\alpha(t),$$
$$\frac{d}{dt}\left(\frac{\partial L(\dot{\alpha}_1,\alpha_1,\dot{y},y)}{\partial\dot{y}}\right)-\frac{\partial L(\dot{\alpha}_1,\alpha_1,\dot{y},y)}{\partial y}+\frac{\partial P_m(\dot{\alpha}_1,\dot{y})}{\partial\dot{y}}=Q_y(t), \tag{4}$$

where: Q_α and Q_y are so-called generalized forces resulting from external forces and torques appropriately transformed with the use of coefficient derivatives of individual elements in relation to generalized coordinates.

Generalized force $Q_\alpha(t,\alpha_1)$ can be represented as a superposition of three torques acting on the collector

$$Q_\alpha(t,\alpha_1)=T_{ext}(t,\alpha_1)+T_g(\alpha_1)+T_{fy}(t,\alpha_1) \tag{5}$$

where: $T_{ext}(t,\alpha_1)$ is the torque raising the pantograph, $T_g(\alpha_1)$ is the torque resulting from gravity, $T_{fy}(t,\alpha_1)$ is the torque derived from the contact force $F_y(t)$ as a result of catenary interaction. All this torque components are functions of the rotation angle, even when the forces have a fixed value.

Taking into account the constraints (3), after calculating derivatives of the Lagrange function and the Rayleigh dissipation function in equation (4), we obtain

$$\begin{aligned} &J_\alpha(\alpha_1)\ddot{\alpha}_1 + h_\alpha(\alpha_1)\dot{\alpha}_1^{\,2} + k_y(\alpha_1)\ddot{y} + D_\alpha(\alpha_1)\dot{\alpha}_1 = Q_\alpha(t) \\ &k_y(\alpha_1)\ddot{\alpha}_1 + h_y(\alpha_1)\dot{\alpha}_1^{\,2} + M_6\ddot{y} + \frac{y}{K_7} + D_7\dot{y} = Q_y(t) \end{aligned} \tag{6}$$

The coefficients $J_\alpha(\alpha_1)$, $h_\alpha(\alpha_1)$, $k_y(\alpha_1)$, $h_y(\alpha_1)$ are very complex functions of the α_1 rotation angle, but they can be determined by an analytical approach. The equivalent coefficient values of viscous damping $D_\alpha(\alpha_1)$ and dry damping is usually determined from experiment. The matrix form of equation (6) is as follows

$$\begin{bmatrix} J_\alpha(\alpha_1) & k_y(\alpha_1) \\ k_y(\alpha_1) & M_6 \end{bmatrix}\begin{bmatrix} \ddot{\alpha}_1 \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} Q_\alpha(t) - h_\alpha(\alpha_1)\dot{\alpha}_1^{\,2} - D_\alpha(\alpha_1)\dot{\alpha}_1 \\ Q_y(t) - h_y(\alpha_1)\dot{\alpha}_1^{\,2} - D_7\dot{y} - \dfrac{y}{K_7} \end{bmatrix} \tag{7}$$

Pantograph two-mass reference model PRM

The structure of current collector model of PRM type, with two degrees of freedom, has been presented in Fig. 5. It contains the following lumped elements:

- m_b – mass of the panhead unit,
- K_d – compliance of the panhead support equivalent spring,
- D_d – equivalent damper of the panhead support system,
- m_a – equivalent mass of the frame-base system,
- K_c – equivalent compliance of the frame-base system,
- D_c – equivalent damper of the frame-base system.

The current collector is subjected to the external force $F_{ext}(t)$, which rises it, and to the $F_y(t)$ force, as the result of interaction with the contact line. The Lagrangian defined in the coordinates assigned to individual lumped elements is

$$L(\dot{y}_a, \dot{y}_b, y_c, y_d) = \frac{1}{2}m_a\dot{y}_a^2 + \frac{1}{2}m_b\dot{y}_b^2 - \frac{1}{2}\frac{y_c^2}{K_c} - \frac{1}{2}\frac{y_d^2}{K_d} \tag{8}$$

where:

$\dot{y}_a, \dot{y}_b$ re the velocities of lumped masses m_a and m_b; y_c, y_d are the relative terminals displacements of equivalent springs K_c and K_d.

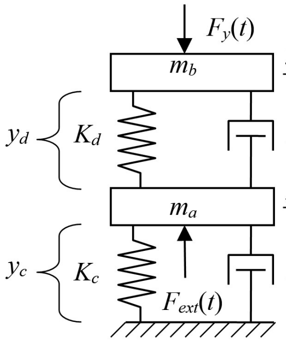

Fig.5. Two-mass model of a pantograph in translational motion

The Rayleigh dissipation function defined in the coordinates assigned to individual equivalent dampers, with only viscous friction taken into consideration, is

$$P_m(\dot{y}_c, \dot{y}_d) = \frac{1}{2}D_c\dot{y}_c^2 + \frac{1}{2}D_d\dot{y}_d^2 \tag{9}$$

In accordance with the adopted assumptions, the model of this current collector has two degrees of freedom, i.e. the y_1 displacement of the gravity center of mass m_a and the y_2 displacement of the gravity center of mass m_b. With such assumption, the constraint equation in its matrix form is

$$\begin{bmatrix} \dot{y}_a \\ \dot{y}_b \\ \dot{y}_c \\ \dot{y}_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} \tag{10}$$

The general form of the Euler-Lagrange equation of the PRM is expressed by the formula:

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L(\dot{y}_1, \dot{y}_2, y_1, y_2)}{\partial \dot{y}_1}\right) - \frac{\partial L(\dot{y}_1, \dot{y}_2, y_1, y_2)}{\partial y_1} + \frac{\partial P_m(\dot{y}_1, \dot{y}_2)}{\partial \dot{y}_1} = Q_1(t), \\ &\frac{d}{dt}\left(\frac{\partial L(\dot{y}_1, \dot{y}_2, y_1, y_2)}{\partial \dot{y}_2}\right) - \frac{\partial L(\dot{y}_1, \dot{y}_2, y_1, y_2)}{\partial y_2} + \frac{\partial P_m(\dot{y}_1, \dot{y}_2)}{\partial \dot{y}_2} = Q_2(t), \end{aligned} \tag{11}$$

where: Q_1 i Q_2 are so-called generalised forces which, for this case, are: $Q_1(t)=F_{ext}(t)$, $Q_2(t)=-F_y(t)$.

Having taken into account the constraint equations (10) and after calculating the derivatives of the Lagrange function and the Rayleigh dissipation function in the equation (11) we obtain

$$\begin{aligned} &m_a\ddot{y}_1 + \left(\frac{1}{K_c} + \frac{1}{K_d}\right)y_1 - \frac{1}{K_d}y_2 + (D_c + D_d)\dot{y}_1 - D_d\dot{y}_2 = F_{ext}(t) - m_a g \\ &m_b\ddot{y}_2 - \frac{1}{K_d}y_1 + \frac{1}{K_d}y_2 - D_d\dot{y}_1 + D_d\dot{y}_2 = -F_y(t) - m_b g \end{aligned} \tag{12}$$

Determination of parameters of a pantograph articulated model

In order to calculate the functions $J_\alpha(\alpha_1)$, $h_\alpha(\alpha_1)$, $k_y(\alpha_1)$, $h_y(\alpha_1)$ it is necessary to have the following data: the masses of individual elements of the current collector, gravity centres of these masses, the moments of inertia of the arms with respect to the defined rotation axes and the lengths of the arms. These data can be defined by an analytical approach. In this paper the authors used the Autodesk Inventor program, which facilitates the calculations of the inertia parameters. The values of the most important parameters have been presented in Table 1.

Table.1. Inertial parameters of rigid bodies from Fig.2

Part No i	Rigid body	Mass M_i [kg]	Moment of inertia J_i [kg·m^2]
1	Lower arm	38.5	12.8 (around axis 1)
2	Lower guide rod	2.9	0.65 (around axis 2)
3	Upper arm lever	3.8	0.079 (around axis 3)
4	Upper arm	14.3	10.0 (around axis 3)
5	Panhead support	3.4	—
6	Panhead unit	12.2	—

The equivalent compliance $K_7 = 6.25\times10^{-5}$ m/N of panhead support system (4 springs connected in parallel) has been evaluate experimentally. The parameters of equivalent dampers have also been established from experiments. The essence of those experiments was to define, separately, the damping coefficients in individual joints of the current collector. Generally, the research procedure was consisted in creating a separate physical pendulum for individual arms of the current collector in order to define the parameters D_1, D_2, D_3, D_4, D_5 and torques caused by dry friction. Fig. 6 shows the way in which the upper arm is mounted in order to specify the D_4 viscous damping coefficient and its dry friction torque.

a) b)

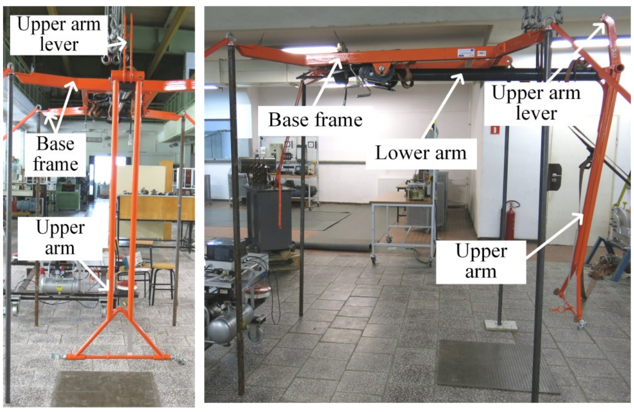

Fig.6. Determination of damping properties of articulated upper arm in movement as physical pendulum: a) front view, b) side view

The parameter D_7 and corresponding force produced by dry friction were determined separately from oscillation waveform in translational motion. Fig. 7 shows the oscillation waveforms of the: lower arm, upper arm and lower guide rod by movement in the character of a physical pendulum.

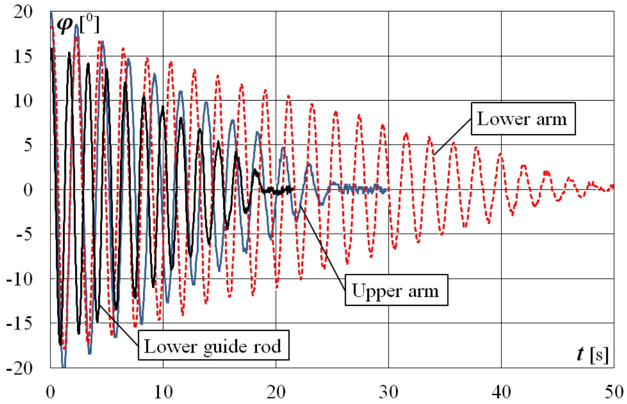

Fig.7. Waveforms of the angular position of the: lower arm, lower guide rod and upper arm in rotational movement as a pendulum

Fig. 8 shows the characteristics of viscous friction $T_v(\omega)$ and dry friction $T_d(\omega)$ for the equivalent damper of lower arm in rotational motion, each of them expressed as a torque dependent on angular velocity.

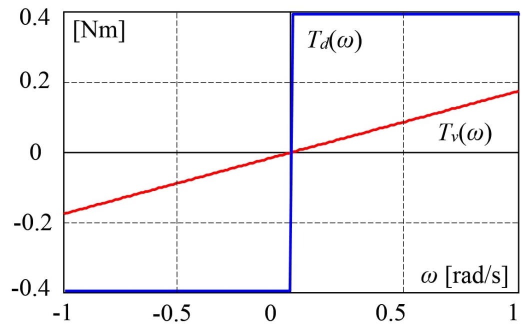

Fig.8. Characteristics of the viscous $T_v(\omega)$ and dry $T_d(\omega)$ friction as functions of angular velocities for D_1 equivalent damper

Determination of parameters of two-mass pantograph from articulated type pantograph

Proposed method for estimation of PRM parameter values involves obtaining the steady state of PAM in the simulation and next requires analysis of the transient state caused by force step of a specific value applied to the panhead. In steady state of PAM the following equation is performed

$$0 = T_{ext}(\alpha_1) + T_g(\alpha_1) + T_{fy}(\alpha_1) \tag{13}$$

Components of torque from equation (13) are shown in Fig. 9. In the simulation it was assumed that the torque $T_{ext}(\alpha_1)$ is a linear and decreasing function of the rotation angle of the current collector. The torque $T_g(\alpha_1)$ is a nonlinear function of the rotation angle, even though the gravity force are fixed. The torque $T_{fy}(t,\alpha_1)$ resulting from the force $F_y(\alpha_1)$ occurs only when the elevation angle of the simulated pantograph has a value greater than α_1 = 32 deg. This assumption is arbitrary and means that, at this angle a contact force occurs as the result of interaction between panhead and catenary. The force $F_y(\alpha_1)$ increases linearly with increasing angle α_1, and this torque is in opposite direction to the torque $T_{ext}(\alpha_1)$. The PAM steady state occurs when α_1 = 36 deg. The lumped mass M_5 is raised in this state at the height Y_5 = 1.18 m.

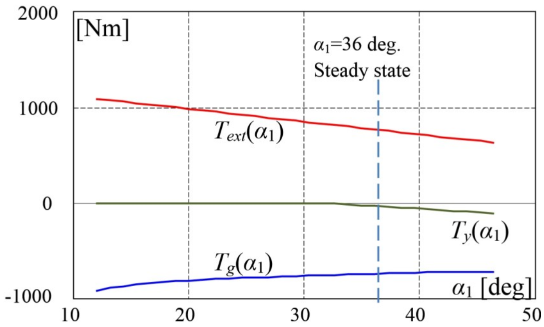

Fig.9. Torque curves versus angular position

After the steady state has been obtained, the force step ΔF_y= –20 N is applied to the current collector, to the node where the lumped mass M_6 is located. The value of this force should be specified in such a way that the minimum value of the angle is not smaller than α_1=32 deg. It is assumed that, during the simulation, reactive force from the contact line always occurs. The oscillation waveform of the relative displacement $y_5(t)$ of the M_5 mass gravity center is shown in Fig. 10. These torque components, which represent dry friction (Fig. 8) in particular joints of the current collector, significantly affect the damping characteristics.

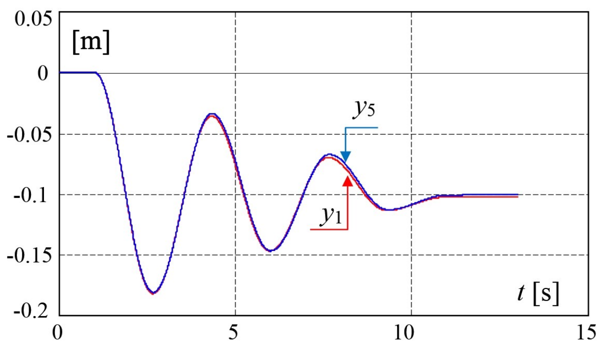

Fig.10. Waveforms of displacement: y_1 – displacement of M_1 mass in PRM, y_5 – displacement of M_5 mass in PAM

The $y_5(t)$ waveform is treated as a reference function for the PRM model. The next stage of the procedure is to estimate the parameters of the PRM model in such a way that its answer to the same excitation force is compatible with the answer of the PAM model.

It has been assumed that some parameters of the lumped elements in translational motion of the PRM and PAM models are equal, i.e. m_b = M_6 = 12.2 kg, $K_d = K_7 = 6.25 \cdot 10^{-5}$ m/N and $D_d = D_7$ = 0.9 Ns/m. Therefore, the parameters which still have to be determined are m_a, K_c, D_c and the dry friction force function $F_{dry}(v)$, where v is the relative velocity of damper terminals. If it is assumed that $F_{dry}(v)$=$F_c \cdot$sgn(v), four parameters, i.e. m_a, K_c, D_c and F_c with fixed values remain to be determined. In order to do this, the residuum function ε (m_a, K_c, D_c, F_c), which implements the least squares method has been formulated

$$\varepsilon(m_a, K_c, D_c, F_c) = \sum_{k=1}^{N} \left(y_{5,k}{}^{(PAM)} - y_{1,k}{}^{(PRM)}(t_k, m_a, K_c, D_c, F_c) \right)^2 \quad (14)$$

where: $y_5^{(PAM)}$ is the set of discreet values of the $y_5(t)$ function, obtained from the simulation on the PAM model, $y_1^{(PAM)}$ is the set of discreet values of the $y_1(t)$ function, obtained from the simulation on the PRM model, while N is the number of samples in the set.

In order to determine the optimal approximation, the Levenberg-Marquardt numerical method [7], implemented in the Mathcad software has been applied. The values of the parameters have been presented in Table 2. The waveform of the $y_1(t)$ function, which approximates the $y_5(t)$ function when using these parameter values has been shown in Fig. 10.

Table 2. Parameter values of PRM model obtained by PAM simulation

m_a [kg]	K_c [m/N]	D_c [Ns/m]	F_c [N]
15.5	0.01	0.62	0.81

To perform the simulation of the PRM model, it is necessary to define the value of the exciting force $F_{ext}(t)$. It has been assumed, for the purposes of the simulation, that the value of $F_{ext}(t)$ is fixed and fulfils the equation

$$-\frac{1}{K_d} y_2 = F_{ext} - m_a g$$
$$\frac{1}{K_d} y_2 = -F_y - m_b g \quad (15)$$

This equation results directly from the equation (12) for the steady state, with the assumption that the deformation of the equivalent spring K_c is y_1=0.

The authors of this paper suggest that the process of determining the minimum residuum (14) is much faster if divided into two stages. During the first stage lack of damping in the PAM and PRM simulations is assumed. This allows for relatively simple determination of the m_a^* and K_c^* parameters, which are close to the optimal values. During the second stage damping is taken into consideration and the assumed original values at the beginning of the Levenberg-Marquardt optimization procedure are $m_a = m_a^*$, $K_c = K_c^*$.

Conclusions

The model of a current collector which includes degrees of freedom in rotational motion takes into account the length of arms, the moments of inertia, the mass of arms and coordinates of their centers of gravity. Therefore it is a relatively accurate model, although, unfortunately, it is also rather complex and non-linear. All the coefficients in the equation of the current collector rotation dynamics are very complex functions of the rotation angle, which include trigonometric functions. The parameters of such a model, except from the friction coefficients, are defined in an unambiguous and relatively accurate way.

The model of a current collector which includes degrees of freedom only in the translational motion contains a number of parameters which are equivalent in their character. The values of these parameters are calculated based on approximate rules and are accepted within quite a wide scope without appropriate explanations. Determination of parameter values by way of experiments makes it necessary to prepare a suitably equipped and expensive measurement stand. Every modification of the construction of a current collector means, in this case, that an expensive prototype has to be built in order to determine the parameters again.

Any changes in the construction or material of a current collector can be unambiguously and easily transferred onto the parameters of the PAM model, which constitutes its significant advantage. It is possible, based on the PAM model, to determine unambiguously the equivalent parameters of the PRM model. In this paper the authors applied the Levenberg-Marquardt optimization method to define the minimum residuum, formulated on the basis of the least squares method. The parameters of this residuum were the desired values of equivalent parameters of the PRM model. The optimization procedure allowed for obtaining the parameters which ensure a high level of compatibility between the simulation results of the PAM and PRM models.

References

[1] J. Benet, N. Cuartero, F. Cuartero, T. Rojo, P. Tendero, E.Arias, *An advanced 3D-model for the study and simulation of the pantograph catenary system*, Transportation Research Part C, 36 (2013), 138-156

[2] Commission Decision of 23 July 2012 amending *Decisions 2006/679/EC and 2006/860/EC concerning technical specifications for interoperability* (notified under document C(2012) 4984) 2012/464/EU

[3] EN 50318:2002 *Railway applications. Current collection systems. Validation of simulation of the dynamic interaction between pantograph and overhead contact line*

[4] EN 50119:2009 *Railway applications. Fixed installations electric traction overhead contact lines*

[5] EN 50367:2012 *Railway applications. Current collection systems. Technical criteria for the interaction between pantograph and overhead line (to achieve free access)*

[6] F. Kiessling, R. Puschmann, A. Schmieder, E. Schneider, *Contact Lines for Electrical Railways: Planning – Design – Implementation – Maintenance*, Wiley VCH, 2009

[7] J. J. More, *The Levenberg-Marquardt algorithm: Implementation and theory*, Lecture Notes in Mathematics, Numerical Analysis, Vol. 630, 1978

[8] F. G. Rauter, J. Pombo, J. Ambrósio, J. Chalansonnet, A. Bobillot, M. S. Pereira, *Contact model for the pantograph-catenary interaction*, Journal of Design and Dynamics, vol.1, No.3 (2007), 447-453

[9] Z. Shufeng, Y. Jian, S. Ruigang, Y. Tianchen, *Optimum Design and Simulation of Structure Parameters of Pantograph Based on Equivalent Mass.* Science Research. Vol. 3, No. 1 (2015), 25-29