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Water Supply Networks – performance modelling and assessment

Keywords

water, supply network, supply, performance, probabilistic, dynamic, modelling, assessment

Abstract

Performance modelling and assessment of Water Supply System (WSS) is a critical activity in system management process. It contributes into producing indicators necessary for the optimisation of the system operation, maintenance, safety, and resources use. The Water Supply Network (WSN) is a major component of any WSS. Assessing the performance of the WSN requires the development of dynamic-probabilistic models and the use of performance notions that are beyond the local availability and reliability of a cluster of pipes (mains, connections, and distributions) or nodes. The proposed performance notions are fully described in terms of performance-levels. The proposed modelling scheme is applied on a real WSN that has slightly been modified to preserve the didactic quality of the chapter and render the modelling scheme accessible at its first uses. Once the use of the scheme is mastered, its exploitation for real and complex WSN is straight forward.

1. Introduction

Performance modelling and assessment of Water Supply System (WSS) is a critical activity in system management process. It requires developing performance indicators necessary for the management and the optimisation of the system operation, maintenance, safety, and resources use (Eid, 2010; Eid et al., 2014, 2015; EN 15975-1:2011, EN 15975-2:2013; Żywiec et al., 2023).

The Water Supply Network (WSN) is a major component of any WSS and the assessment of its performance requires the development of purpose-oriented performance models.

If the purpose is supporting system design activities, nominal operational optimisation, or system safety during nominal operation, deterministic static models are useful.

While the performance dynamic-probabilistic models are necessary, if the purpose is predictive maintenance, accidental safety, or system ageing management. That is beyond the local availability and reliability of a cluster of pipes (mains, connections, and distributions) or nodes (Diao et al., 2016; Giudicianni et al., 2018; Gutiérrez-Pérez et al., 2013; Herrera et al., 2016; Ponti et al., 2021; Shuang et al., 2014; Torres et al., 2017; Yazdani & Jeffrey, 2010).

Regarding the performance notion many indicators are of great use as will be mentioned later in the chapter. Especially, if one considers only the steady-state nominal operational mode of the WSN. However, the authors have chosen to use the supply disruption extension (*SDE*) level as the most meaningful indicator to monitor the performance of a WSN (Directive EU 2020/2184, 2020; Pietrucha-Urbanik et al., 2018, 2020, 2021; Tchórzewska-Cieślak et al., 2021; WHO, 2011). The reason is that systemic failures in main, con-

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nection, or distribution pipes do not result in similar supply disruptions in terms of occurrence frequency, supply down time, or disrupted zone extension (Dong et al., 2018; Kang et al., 2009; Meier & Barkdoll, 2000; Wagner et al., 1988; Zhou et al., 2019). In addition, systemic failures are of random nature. The randomness of failures results in a probabilistic-dynamic feature in the systems' performance.

If one develops a probabilistic-dynamic model describing directly the WSN connectivity, it would serve as lower-level layer to other higher-level models designed to determine other kinds of indicators such as: demand satisfaction indicators (Ciraane et al., 2022), robustness indicator (Hayelom & Ostfeld, 2022), cost-benefit indicator (Ganjidoost et al., 2021), energy balance indicator (Dziedzic & Karney, 2014). It would even serve to characterise the WSN in its steady-state nominal operational mode. It would serve to determine the resilience of the WSN, as well.

The primary-characteristics of the proposed model are therefore: probabilistic, dynamic, and directly related to node-to-node connectivity of the network.

Then, comes the choice of the supply disruption extension (*SDE*) level characteristics as performance indicators is fixed, the chapter proceeds to the treatment of the following issues:

- how to represent a continuous structure of pipes by a set of discontinuous nodes and links while preserving the same failure and repair characteristics,
- how to establish a systematic model that can produce the local functioning state of the network,
- how to establish a global performance function based of the local performance states of each part of the network.

The proposed modelling scheme is applied on a real WSN whose graph-structure has slightly been modified to preserve the didactic quality of the chapter and render the modelling scheme accessible at its first uses. Once the use of the scheme is mastered, its exploitation for real and complex WSN is straight forward.

Let's start by the physical-technical description of the WSS we have in hand.

2. Water Supply System description

The questioned WSN is sourced by a unique water treatment plant (WTP). In the 1990s, the facility was modernized, introducing preliminary ozonation of raw water.

The current emergency water supply potential to the city, considering all available water sources, are as follows: water storage capacity distributed in 11 equalizing reservoirs within the water supply network and public wells.

At present, the water treatment processes are the removal of large contaminants on the grates, water ozonation, coagulation, slow mixing, flocculation, sedimentation in horizontal sedimentation tanks (continuous sludge scraping), filtration through a sand bed (WTP_I station) and anthracite-sand (WTP_{II} station), indirect ozonation, filtration through a carbon bed, preliminary disinfection with UV and final disinfection with chlorine compounds (chlorine gas and chlorine dioxide) and the correction of the pH of the water as needed.

2.1. Pipes specifications

Water supply pipes are mostly made of plastic pipes. Polyvinyl chloride (PVC) pipes account for 29.4% and polyethylene (PE) – 48.0% of the total length of the water supply networks.

Pipes made of steel account for 3.5% of the length of all pipes, cast iron pipes account for almost 14.5%, and pipes made of asbestos-cement only 0.18%.

The mains of the network represent approximately 6% (19 km). The connections constitute approximately 31% of the network (104 km). The remaining part, approx. 63% of the networks, are distribution pipes (210 km).

In total, the water supply network administered by water company is 333 km long, with diameters vary from 25 to 1200 mm.

2.2. Network topology

The WSN is represented by a network with three types of nodes: main, connection, and distribution.

The circular main pipe is lumped in 5 main-nodes each supplies 4 connection-nodes, each connection supplies 5 distribution-nodes, see schematic representations in Figure 1.

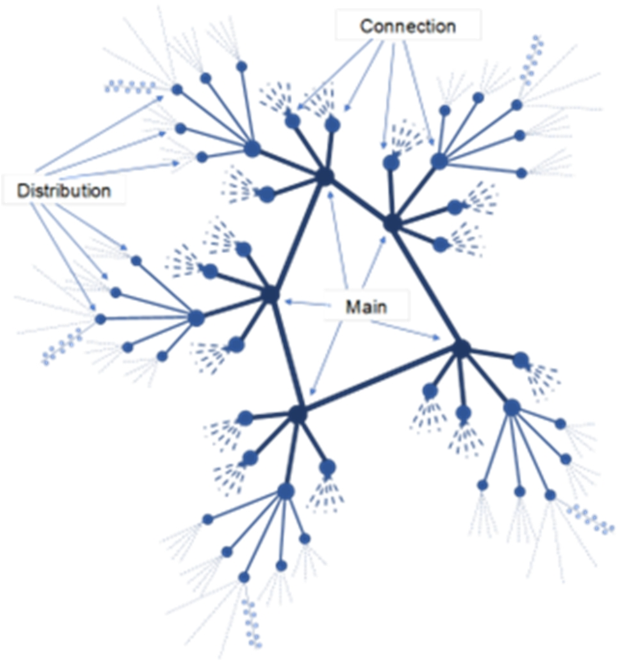

Figure 1. Schematic representation of WSN.

As we may expect, the WSN is represented by an open directional graph, often called graph neural networks (GNN). The quantitative analysis of GNN performance is generally not systematic by traditional graph analysis approaches (Di Nardo et al., 2018; Hamilton, 2020; Peng et al., 2022; Tzatchkov, et al. 2006). However, quantitative models may be developed for specific cases. It is our case in the chapter.

2.3. Repair and failure data

The data used in the chapter are issued from the water supply utility in charge of the operation of the WSS under investigation.

The repair data used in the modelling of the WSN are the mean repair time (MRT) of four failure modes (small crack, medium crack, large crack, clear cut) divided in three categories (localisation-administration, effective reparation, control-commissioning).

The repair data are given for the main, the connection and the distribution in Tables 1a-b-c, respectively.

The overall MRT for each failure-mode i and for each pipe-type T_{ij} is the cumulation of the administrative, the effective repair and the commissioning times for each failure-mode i and for each pipe-type j.

Table 1a. Mean repair time [h] of mains

MRT		j = 1 Admin.	j = 2 Eff. Rep.	j = 3 Cont. & Comm.
i = 1	Small crack	1.5	6.1	1.1
i = 2	Medium crack	1.0	4.2	2.9
i = 3	Large crack	0.5	5.0	1.0
i = 4	Clear cut	0.5	5.5	4.5

Table 1b. Mean repair time [h] of connection

MRT		j = 1 Admin.	j = 2 Eff. Rep.	j = 3 Cont. & Comm.
i = 1	Small crack	1.0	4.1	1.3
i = 2	Medium crack	1.5	3.3	2.3
i = 3	Large crack	1.0	5.1	1.8
i = 4	Clear cut	0.5	4.8	3.6

Table 1c. Mean repair time [h] of distribution

MRT		j = 1 Admin.	j = 2 Eff. Rep.	j = 3 Cont. & Comm.
i = 1	Small crack	2.5	3.2	1.5
i = 2	Medium crack	2.0	2.9	1.1
i = 3	Large crack	1.5	3.3	1.1
i = 4	Clear cut	1.5	3.5	1.7

The overall MRTs T_{ij} are used to determine the mean repair rates $\mu_{ij}(h^{-1})$ of each failure-mode i, for each pipe-type j (mains, distribution, connection), Table 2, such as:

$$\mu_{ij} = 1/T_{ij}. \quad (1)$$

Table 2. Repair rates μ_{ij} [h^{-1}]

Repair rates μ_{ij} [h^{-1}]		j = 1 Main 19 km	j = 2 Connection Total length 104 km	j = 3 Distribution 210 km
i = 1	S. crack	0.115	0.156	0.139
i = 2	M. crack	0.123	0.141	0.167
i = 3	L. crack	0.154	0.127	0.169
i = 4	Clear cut	0.095	0.112	0.149

As for the occurrence failure rates $\lambda_{ij}(h^{-1})$ associated to each failure-mode i, for each pipe-type j (mains, distribution, connection) are given in Table 3.

Table 3. Failure rates λ_{ij} [$km^{-1} \cdot h^{-1}$]

Failure rates λ_{ij} [$km^{-1} \cdot h^{-1}$]		$j = 1$	$j = 2$	$j = 3$
		Main	Connection	Distribution
			Total length	
		19 km	104 km	210 km
$i = 1$	S. crack	0.55	0.746	0.791
$i = 2$	M. crack	1.26	0.623	0.58
$i = 3$	L. crack	1.077	0.788	0.732
$i = 4$	Clear cut	0.301	0.751	0.791

It is worth, as well, to give the percentage share of each failure-mode occurrence in the total failure occurrence per each pipe-type, Table 4.
This indicator will be used in the clustering of the WSN into discontinuous point nodes rather than continuous kilometres.

Table 4. Failures breakout per failure-mode

Failure modes	Mains	Connections	Distribution
Small crack	17.3%	25.7%	27.3%
Medium crack	39.5%	21.4%	20.0%
Large crack	33.8%	27.1%	25.3%
Clear cut	9.4%	25.8%	27.3%

2.4. Availability modelling and segmentation

As it will be detailed in the following section, the water supply network will be modelled using a graph $\boldsymbol{G}(n,l)$ built up by: nodes and links. Considering the water supply network, these basic constituents will have two different functions. The nodes should allow to distribute water between some links. And the links should supply water to some couples of nodes. To be able to describe the performance of the nodes and the links, i.e., their availability to supply and distribute water, a model is proposed. The model is dynamic and describes the availability $a(t)$ of any functional entity including nodes and links as following:

$$\frac{d}{dt}a(t) = -\lambda a(t) + \mu u(t) \tag{2}$$

where, λ, μ, $a(t)$ and $u(t)$ are the failure rate, the repair rate, the availability, and the unavailability of the given functional entity, respectively.
One could certainly describe the unavailability rather than the availability as follow:

$$\frac{d}{dt}u(t) = \lambda a(t) - \mu u(t) \tag{3}$$

as both quantities are complementary, it is enough to determine only one of them. Reliability, safety, and system performance analysts prefer to determine the system unavailability rather than its availability, for obvious reasons, as unavailability is often and by so far the smallest quantity. This is also the authors' preference.
It is worth underlining that failure and repair rates are input data issued from the operational experience feedback of the given functional entities. The failure and repair rates are generally function of time. That will not be the case used in the chapter for two reasons. The first is to avoid unnecessary complexity that may hid the real interest of the proposed model and subsequently the didactic quality of the chapter. The second reason is due to the medium quality of the database we have. As, it does not allow the adjustment of a robust time-dependent model describing the failure and the repair rate of the nodes and the links. Thus, we use constant failure and repair rates.
Using the same reasoning scheme used to deduce Eq. (2) and (3), the reliability $r(t)$ of a functional entity is described by:

$$r(t) = e^{-\lambda t} \tag{4}$$

where, $r(t)$ is the probability that the functional entity does not fail within a time-interval t. Similarly, one may deduce the maintainability $m(t)$, described by:

$$m(t) = e^{-\mu t} \tag{5}$$

where, $m(t)$ is the probability that the functional entity is not repaired within a time-interval t.
Eqs. (2) to (5) will be applied on separate functional entities such as nodes and links.

3. Network modelling and data clustering

The WSN will approximately be represented by a graph including five identical main-nodes, each supplying water to four identical connection-nodes and each connection-node supplies water to five distribution-nodes, Figure 1.

3.1. Water Supply Network modelling

The WSN topology is modelled using an approximate graph. As mentioned above, the WSN is composed of three types of pipe-structures: main, connection, and distribution. Each of these structures will be represented by a type of nodes that condensate in one point the attributes corresponding to WSN structure and relevant for the WSN performance assessment. These attributes are exclusively the failure and the repair characteristics. Subsequently, the mains (19 km) will be represented by 5 identical main-nodes, the connections (104 km) by 20 identical connection-nodes, and the distribution (210 km) by 100 identical distribution-nodes. These 125 elementary point-structures will be distributed in identical structures, such as: each main-node supplies water to four connection-nodes and each connection-node supplies five distribution nodes, figure 1. Only the main-nodes are connected to form a ring, Figure 1.
The finest pipes that supply the water from the distribution-nodes to the end-users are not considered. Neither the pumping stations nor the electricity supply systems necessary for the monitoring and control equipment are considered.
The links between the nodes are considered as perfect, regarding performance standpoint. They do not fail. Hence, they require no repair.
Only nodes condensate the failure and the repair features on its corresponding pipe-types.
To note that the proposed graph is a mono-directional one. Water flows from main-nodes to connection-nodes and then to distribution-nodes.
To note also that main-nodes are supposed to be functionally independent. The loss of any main-node does not impact on the others.

3.2. Data clustering

The data given above, in section §2, are associated with continuous lengths of pipes: main, connection, and distribution. They should be clustered in such a way to fit with the node discontinuous structure of the graph representing the WSN. Thus, the total failures of the main, connection, and distribution pipes will be uniformly distributed over 5 main-nodes, 20 connection-nodes, and 100 distribution-nodes, respectively.
In Table 5, the representative mean failure rate (MFR), mean down time (MDT) and mean repair rate (MRR) per node are reproduced.
Although the MFR integrates the length of the pipes per category, the MRR does not consider the lengths. The MRR consider only the node type.
Once the topology of the WSN graph is fixed and the global failure-repair performances are conserved, one proceeds to modelling the WSN performance.

Table 5. MFR, MDT and MRR per node

	Mains	Connection	Distribution
Km	19.0	104.0	210.0
Nodes	5	20	100
	/node	/node	/node
MFR (h^{-1})	1.38E-03	1.73E-03	6.93E-04
MDT (h)	7.90E+00	7.60E+00	6.50E+00
MRR (h^{-1})	1.27E-01	1.32E-01	1.54E-01

4. Water Supply Network performance modelling

The performance of a WSN can best be described in terms of “water supply disruption level (extension, duration)” and their occurrence probabilities.
A model based on these 3-D classification, “supply disruption extension”, “supply disruption duration”, and “occurrence probability” is then proposed.
The supply disruption extensions are grouped in 4 classes dependent on the percentage of the non-supplied distribution-nodes:

- EX1 ($0\% \leq \text{Ex} < 20\%$),
- EX2 ($20\% \leq \text{Ex} < 40\%$),
- EX3 ($40\% \leq \text{Ex} < 60\%$),
- EX4 ($60\% \leq \text{Ex} \leq 100\%$).

The supply disruption durations (down time) are grouped in 5 classes:

- DT1 ($0\text{h} \leq \Delta < 2\text{h}$),
- DT2 ($2\text{h} \leq \Delta < 12\text{h}$),
- DT3 ($12\text{h} \leq \Delta < 24\text{h}$),
- DT4 ($24\text{h} \leq \Delta < 48\text{h}$),
- DT5 ($48\text{h} \leq \Delta$).

The WSN performance is assessed using two complementary metrics: the probability to be in a supply disruption class and the conditional probability to recover within a given delay.

4.1. Disruption extension modelling

Given that the WSN has 5 main-nodes, 4 connection-nodes per main, and 5 distribution-nodes per connection, so one may scan all the possible disruption states, using a state structure function $f(m,c,d)$ that determines the percentage of the non-supplied distribution nodes, such as:

$$f(m,c,d) = 20m + 5c + d \tag{6}$$

where:

$0 \leq m \leq 5,\ \ 0 \leq c \leq\ c_{\max} = (20 - 4m),$

$0 \leq d \leq d_{\max} = (100 - 20m - 5c),$

$0 \leq f(m,c,d)\ \leq 100.$

Given that m is the number of the unavailable maim-nodes, c is the number of the unavailable connection-nodes excluding those become unavailable due to the unavailability of the main-nodes ($4m$), and d is the number of the unavailable distribution nodes excluding those become unavailable due to the unavailability of the main-nodes and the connection-nodes ($20m + 5n$). The supply disruption extension (*SDE*) level will be then measured as:

$$SDE \leq\ f(m,c,d)/100 \tag{7}$$

where 100 represents the total number of the distribution-nodes.

To note that the SDE is a function of the number of the unavailable independent main-, connection-, and distribution-nodes, Eq. (6). Given that the unavailability of each of these constituents is governed by the differential equation in Eq. (3).

At any instant of time t ($t \in [0,\infty[$), the WSN has a non-zero probability to be in one of the states included in its probability space Ω. The probability space Ω contains 2416 disjoint states separated into:

- one perfect state, where all the nodes (125 nodes) are available. It is a zero-supply disruption state (0%), and,
- 2415 states sharing a supply disruption extension belongs to the SDE interval $]0\%, 100\%]$.

The exact structure function $f(m,c,d)$ of each state is well-determined and allows the determination of the probability $p_{m,c,d}(t)$ that the WSN will be at each of the functional states.

The probability to be in a state defined by the structure function $f(m,c,d)$ is given by:

$$p_{m,c,d}(t) = \binom{m+\bar{m}}{m} a^{\bar{m}}(t)u^{m}(t) \cdot \binom{c+\bar{c}}{c} a^{\bar{c}}(t)u^{c}(t) \cdot \binom{d+\bar{d}}{d} a^{\bar{d}}(t)u^{d}(t) \tag{8}$$

where:

$p_{m,c,d}(t)$: is the probability to be in the state defined by the state structure function $f(m,c,d)$.

$a(t)$ and $u(t)$ are, respectively, the availability and the unavailability of the node, Eqs. (1) and (2),

m, c, d: are respectively the number of the unavailable nodes from class main, connection, and distribution. Nodes are identical par class,

$\bar{m}, \bar{c}, \bar{d}$: are respectively the number of the available nodes from class main, connection, and distribution. Following the conditions in Eq. (5), then $\bar{m} = 5 - m$, $\bar{c} = c_{\max} - c$, and $\bar{d} = d_{\max} - d$.

4.2. Disruption duration modelling

Having identified the states of the functional space Ω, the structure function of each state $f(m,c,d)$, and the corresponding probability $p_{m,c,d}(t)$ of being in the state, one can proceed to determining the recovery probability within a given laps of time (Δ) from the supply disruption associated to a given state.

The recovery probability, within an interval (Δ), is dependent on the reparation time of the unavailable nodes.

For the supply disruption state described by the state structure function $f(m,c,d)$, one can calculate the non-recovery probability $s_{m,c,d}(\Delta)$ as:

$$s_{m,c,d}(\Delta) = p_{m,c,d}(t) \cdot \left(e^{-(m\mu_m + c\mu_c + d\mu_d)\Delta}\right) \tag{9}$$

where:

$s_{m,c,d}(\Delta)$: is the recovery probability from a state defined by the structure function $f(m,c,d)$,

$p_{m,c,d}(t)$: is the probability of being at that state at instant t,

$e^{-(m\mu_m+c\mu_c+d\mu_d)\Delta}$: is the probability of non-recovery of any of the unavailable nodes, given that all the nodes are identical per class. This is the hypothesis used in our case. The extension to nonidentical nodes in each class is straightforward.

5. Water Supply Network performance assessment

As mentioned above, the probability space associated with the case is built up of 2416 disjoint functional states. Each state is defined by its proper state structure function $f(m,c,d)$. Only one state is functionally perfect and described by the state structure function $f(0,0,0)$, i.e., none of the nodes are unavailable. The rest of states (2415) have been grouped in 4 categories according to their supply disruption level. We get then four sets of states, the first set contains the 50 states with supply disruption ratio in the range of $[0\% - 20\%[$, the second set of supply disruption ratio in the range $[20\% - 40\%[$ with 180 states, the third one of the range $[40\% - 60\%[$ with its 380 states, and finally the fourth set of the range $[60\% - 100\%]$ containing 1796 states. Some samples from the 2416 generated state structure functions in Table 6.

Given that the 2416 states are disjoint, then the probability of being in a group of states at time t is simply the sum of the probabilities over all the states in each group. The probability of being in each of these four categories are given in Table 7. Their time evolution profile is traced in Figure 2. We can see in Figure 2 that the state groups attend their asymptotic values almost after 48h from each resetting of the WSN to its perfect state $f(0,0,0)$. This 48h could be considered as a rapid dynamic behaviour considering that water supply utilities generally think in years.

Table 6. Samples of structure function

Class	m	c	d	$f(m,c,d)/100$
1	0	0	0	0%
	0	0	1	1%
	0	0	2	2%
	0	0	3	3%
	0	0	4	4%
	0	3	3	18%
	0	0	19	19%
	0	1	14	19%
	0	2	9	19%
50	0	3	4	19%
1	0	0	20	20%
	0	1	15	20%
	0	2	10	20%
	0	3	5	20%
	0	4	0	20%
	0	7	4	39%
	1	0	19	39%
	1	1	14	39%
	1	2	9	39%
180	1	3	4	39%
1	0	0	40	40%
	0	1	35	40%
	0	2	30	40%
	1	7	4	59%
	2	2	9	59%
380	2	3	4	59%
1	0	0	60	60%
	0	1	55	60%
	0	2	50	60%
	0	3	45	60%
	0	4	40	60%
	0	5	35	60%
	3	8	0	100%
	4	0	20	100%
	4	1	15	100%
	4	4	0	100%
1796	5	0	0	100%

Table 7. Probability to be in performance class

Class	$0h \leq \Delta < 2h$	$2h \leq \Delta < 12h$	$12h \leq \Delta < 24h$	$24h \leq \Delta < 48h$	$\Delta \geq 48h$
$0\% - 20\%$	9.43E-01	8.38E-01	8.12E-01	8.06E-01	8.06E-01
$20\% - 40\%$	5.60E-02	1.57E-01	1.81E-01	1.87E-01	1.87E-01
$40\% - 60\%$	5.01E-04	4.93E-03	6.93E-03	7.50E-03	7.53E-03
$60\% - 100\%$	1.81E-06	6.36E-05	1.10E-04	1.25E-04	1.26E-04

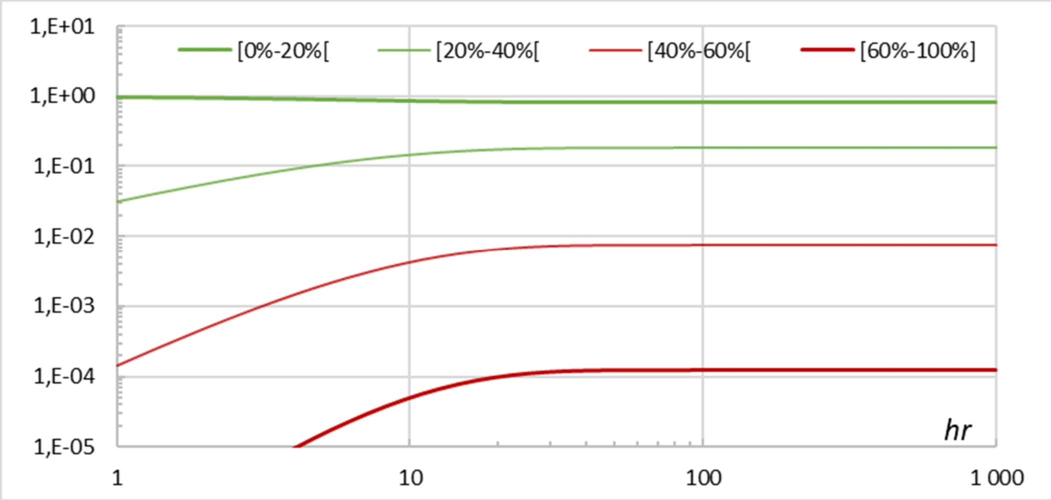

Figure 2. Time profile of state probability per group of supply disruption ratio.

Subsequently, we may consider that beyond 2 days after the resetting of the WSN to its perfect supply performance, the WSN is characterised by its asymptotic state probability vector as follows:

$$P(t \to \infty) = \begin{bmatrix} P_{([0\%-20\%[)} \\ P_{([20\%-40\%[)} \\ P_{([40\%-60\%[)} \\ P_{([60\%-100\%])} \end{bmatrix} \to \begin{bmatrix} 8.06E-01 \\ 1.87E-01 \\ 7.53E-03 \\ 1.26E-04 \end{bmatrix}.$$

Given the probability of being in a state or in a group of states is not enough to assess the performance of a WSN. It is often required to assess the recovery characteristic from a state or from a group of states. The required figure can be the conditional probability of non-recovery from a group of states within an interval Δ, knowing that the WSN is in this group of states $Q(\Delta)$. The non-recovery conditional probability is given in Table 8 using the same assessment grid as for the state group probability.

Table 8. Non recovery probability within Δ hours

Probability	$0h \le \Delta < 2h$	$2h \le \Delta < 12h$	$12h \le \Delta < 24h$	$24h \le \Delta < 48h$	$48h \le \Delta$
0% – 20%	1.63E-04	1.83E-04	1.89E-04	1.91E-04	1.63E-04
20% – 40%	2.16E-03	7.73E-04	6.71E-04	6.50E-04	2.16E-03
40% – 60%	5.13E-06	5.22E-07	3.71E-07	3.43E-07	5.13E-06
60% – 100%	1.21E-08	3.44E-10	1.99E-10	1.75E-10	1.21E-08

One may equally be interested in a longer disruption down times. This indicator does generally helps assessing the global effectiveness of the maintenance service. It represents the nonrecovery probability for periods higher than three days.

$$Q_{t\to 72h} \begin{bmatrix} [0\%-20\%[\\ [20\%-40\%[\\ [40\%-60\%[\\ [60\%-100\%] \end{bmatrix} = \begin{bmatrix} 7.329E-04 \\ 4.289E-03 \\ 6.364E-06 \\ 1.278E-08 \end{bmatrix}.$$

It is necessary to recall that it is assumed that after each resetting of the water supply service, the WSN becomes as good as before the supply disruption. No aging effects have been considered.

Considering, any ageing effect can still be modelled using the same model presented in the present case if ageing rates of each pipe-category were available.

6. Model characteristics and limits

If one should make a statement describing the proposed model, one would write "it is a dynamic-probabilistic and supply-disruption risk-oriented model". It determines the supply-disruption risk using three attributes: disruption likelihood, disruption extension, and disruption duration. The model base-layer encompasses the graph representation of the WSN and the representation of nodes by their failure/repair dynamics. The model is predictive, dynamic, and probabilistic. The performance indicator estimated by the model, the *SDE,* is directly related to the global connectivity state of the WSN in case of systemic failures.

Many other performance indicators are already developed and used by water supply engineers and utilities to measure the performance of the WSN, such as:

- biological stability of water indices (Liu et al. 2017; Papciak et al. 2018; Van Der Kooij, 2000),
- chemical stability of water indices (Li et al., 2022; Tchórzewska-Cieślak et al., 2019),
- the acceptance risk index (Voogd et al., 2022; Tchórzewska-Cieślak et al., 2020),
- the cost-benefit indicator (Ganjidoost et al., 2021) and pollution water index (Mian et al., 2023),
- the demand satisfaction indicators (Ciraane et al., 2022),
- the energy balance indicator (Dziedzic & Karney, 2014),
- the energy efficiency performance indices (Alegre et al., 2016),
- the energy sustainability index (Zaman et al., 2021),
- the failure rate (Kwietniewski et al., 1993),
- the minimum night flow indicator (Eugine, 2017; Farah & Shahrour, 2017), top-down water balance, bursts and background estimates (AL-Washali et al., 2016; Farley & Trow, 2007),
- the quality of the water-supply service approach (Pietrucha-Urbanik & Rak, 2020),
- the risk indices for drinking water (Rak & Pietrucha-Urbanik, 2019),
- the robustness indicator (Hayelom & Ostfeld, 2022),
- the water losses indicators (Ashton & Hope, 2001; Lambert & Hirner, 2000; Lambert et al., 1999; Puust et al., 2010).

Some other tendencies still exploit the graph spectral techniques (Di Nardo et al., 2018) rather than the direct connectivity models such as the connectivity matrix. The graph spectral techniques can't by essence produce predictive dynamic models.

The above sample of indicators is representative of different kind of models one may come over in the literature. They are static, deterministic, and considering the network in its steady-state nominal operational mode.

Regarding the *SDE* indicator proposed in this work, it does not describe the performance of the WSN in its nominal operational mode, except if the operator is interested in determining the fluctuations in the network performance due to the existing systemic failures.

The proposed model describes the WSN degraded performance caused by the occurrence of systemic failures.

7. Conclusion

Performance modelling and assessment of Water Supply System (WSS) is a critical activity in system management process. It requires developing performance indicators necessary for the management and the optimization of the system operation, maintenance, safety, and resources use.

Some pertinent performance indicators can be developed and used to support decision making in WSS management. The supply disruption extension (*SDE*) level characteristic indicators are some of them.

Once the targeted performance indicators are fixed, the authors treat three issues: clustering a continuous structure of pipes in a pointwise structure of a graph representing the WSN topology, clustering the relevant failure-repair data of the physical network to fit with the graph pointwise nature, and finally developing a probabilistic-dynamic model to assess the WSN performance.

The WSN global performance is thus assessed by two measures: the probability at instant t to be in each functional state or in each set of functional states, and the non-recovery probability within a time interval Δ from each degraded state or from each set of degraded states.

The failure-repair data used in the chapter are provided by the WSN regional operator. The clustering of the data to fit with the graph representation of the WSN is carried out under the condition of conserving the global failure-repair characteristics of the WSN.

The time instant t is measured considering the last resetting of the WSN after a supply disruption. After each resetting, the WSN is considered as good as before.

No ageing effects were considered. Subsequently, failure and repair rates are constants. A hypothesis that fits with the statistical quality of the operational data we could obtain.

The acceptability or not of the values of these probabilistic figures is a managerial decision. A decision that is constrained by many other external factors beyond the pure functional characteristics of the WSN, such as: the normative codes, customers satisfaction, available resources to improve the WSS performance, and many other national constraints of higher strategic levels.

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