ABSTRACT. Background: A key element of the evaluation of warehouse operation is the average order-picking time. In warehouses where the order-picking process is carried out according to the "picker-to-part" rule the order-picking time is usually proportional to the distance covered by the picker while picking items. This distance can be estimated by simulations or using mathematical equations. In the paper only the best described in the literature one-block rectangular warehouses are considered.

Material and methods: For the one-block rectangular warehouses there are well known five routing heuristics. In the paper the author considers the return heuristic in two variants. The paper presents well known Hall's and De Koster's equations for the average distance traveled by the picker while completing items from one pick list. The author presents own proposals for calculating the expected distance.

Results: the results calculated by the use of mathematical equations (the formulas of Hall, De Koster and own propositions) were compared with the average values obtained using computer simulations. For the most cases the average error does not exceed 1% (except for Hall's equations). To carry out simulation the computer software Warehouse Real-Time Simulator was used.

Conclusions: the order-picking time is a function of many variables and its optimization is not easy. It can be done in two stages: firstly using mathematical equations the set of the potentially best variants is established, next the results are verified using simulations. The results calculated by the use of equations are not precise, but possible to achieve immediately. The simulations are more time-consuming, but allow to analyze the order-picking process more accurately.

Key words: order-picking, warehousing, heuristics, simulations.
heuristic will be presented. The equations discovered by the author and the equations known from literature will be compared with the results obtained by the use of the simulation.

**ORDER-PICKING**

The most laborious and time-consuming warehouse process is order-picking (it generates approximately 55% of warehouse operating costs), which includes retrieving items from storage locations in response to a specific customer request [De Koster, Le-Duc and Roodbergen, 2007]. In more detail about the functions of order-picking and about the possibilities of optimization of this process in polish literature treat Krawczyk and Jakubiak [2011], Kłodawski and Jacyna [2011], Kłodawski [2012] oraz Kłodawski and Zak [2013].

The problem of optimization the average (expected) order-picking time for the specific warehouse parameters, the size of pick lists and routing heuristic is very important at the design stage of warehouses. The task is achieved usually by the use of time-consuming simulations. The scientists often assume that the picker's travel time is an increasing function of travel distance [De Koster, Le-Duc and Roodbergen, 2007]. Then the issue of minimizing the order-picking time can be solved by the search of the shortest route. The expected distance traveled by the picker can be determined in two ways: by the use of simulations or mathematical equations. For the first time as the statistical problem this task was treated by Kunder and Gudehus [1975]. Although written in German, the paper is very often cited to this day. The issue was further developed, among others, by Hall [1993] and De Koster and Van Der Poort [1998]. A more advanced analysis, taking into account the ABC classification based on items turnover was carried out by Jarvis and McDowell [1991] and Le-Duc and De Koster [2005].

The route covered by the picker can be determined by one of the following heuristics: s-shape, midpoint, return, largest gap and combined (the Polish translations of the heuristics names can be found in the paper of Jacyna and Kłodawski [2012]). There is also a very fast Ratliff and Rosenthal algorithm for determining the optimal route, although for different reasons, it is not used in practice [De Koster, Le-Duc and Roodbergen, 2007].

The efficiency of the order-picking process is directly affected by the appropriate storage location assignment. Jakubiak [2013] showed that much more important than the method of determining the picker's route, is to optimize the assignment of items to slots. The problem of dynamic allocation of items in the warehouse was defined by Lewczuk [2012]. These issues, although very important, will not be the subject to further analysis in this paper. The research will be limited to the return heuristic and the attempt to the quick order-picking time estimation (through the distance) using mathematical equations. In the paper two ways of order-picking using return heuristics will be considered: the first one where all items are picked in one cycle and the second one where each item (different index) is collected in a separate cycle (figure 1). The study will take into account two possible locations for the Pick-up / Drop-off Point. The equations for the order-picking time taking into account the technical characteristics of devices that support this process (forklift trucks) can be found in the paper of Fijałkowski [2003].

In the next two chapters the equations for the average order-picking time are presented: the Hall's and De Koster's conceptions and the proposal of the author. The following notation was adopted:

- $d_h$ - average distance traveled by the picker in the main aisle,
- $d_v$ - average distance traveled by the picker in the picking aisles,
- $D$ - average total distance traveled by the picker while completing one pick list: $D=d_h+d_v$,
- $d_1$ - distance between the entrances to the two adjacent picking aisles (figure 2),
- $d_2$ - length of the picking aisle (figure 2),
- $n$ - number of different items (indexes) on the pick list,
- $N$ - total number of picking aisles,
- $j$ - number of picking aisle,
- $m$ - number of slots in one side of the picking aisle.
Fig. 1. Two versions of the picker's routing for the return heuristic
Rys. 1. Dwie wersje heurystyki return (wszystkie towary pobierane są w jednym cyklu / każdy wyrób pobierany jest w osobnym cyklu)

Fig. 2. The possible locations of the Pick-up / Drop-off points and the basic distances in the warehouse
Rys. 2. Możliwe lokalizacje punktu przyjęcia i wydania towarów oraz podstawowe odległości w magazynie
ESTIMATION OF THE AVERAGE DISTANCE TRAVELED BY THE PICKER IN THE MAIN AISLE

The determination of the average distance traveled by the picker in the main aisle while each item is taken in a separate cycle is relatively simple. The picker after taking each item returns to the Pick-up / Drop-off point. For each item the distance in the main aisle is traveled twice: from the PD point to the appropriate slot and with the item: from the slot to the PD. For the case in which the Pick-up / Drop-off point is located in front of the first picking aisle the average distance can be calculated using equation:

\[ d_h = n \cdot (N - 1) \cdot d_i \]  \hspace{1cm} (1)

If the PD point if located in front of the middle aisle (when the number of picking aisles is odd) or in front of one of the two middle aisles (when the number of picking aisles is even), the average distance traveled by the picker in the main aisle can be calculated as follows:

\[ d_h = \begin{cases} 
\frac{n \cdot N}{2} \cdot d_i & \text{when } N \text{ is even,} \\
\frac{n \cdot (N - 1) + (N - 1)}{2N} \cdot d_i & \text{when } N \text{ is odd.} 
\end{cases} \]  \hspace{1cm} (2)

It is somewhat more complicated to calculate the average distance traveled by the picker in the case where in one cycle more than one different items (indexes) are completed. Hall [1993] noticed that one can use the uniform distribution to estimate this value. If we assume that the length of the main aisle is equal 1, the items from pick list can be treated in some approximation as a realization of a random variable with uniform distribution on the interval <0,1>. Such assumptions are acceptable in the case where the demand for each stored item is the same (more precisely: all items occur on pick lists with the same frequency). Then apply the well known equations for the expected value of the minimum and maximum of n numbers randomly generated from the uniform distribution:

\[ E(\min\{x_1, x_2, \ldots, x_n\}) = \frac{1}{n+1}, \]
\[ E(\max\{x_1, x_2, \ldots, x_n\}) = \frac{n}{n+1}. \]

In the case where the Pick-up / Drop-off point is located in front of the middle picking aisle, the average distance traveled by the picker can be calculated by subtracting the expected location of the item from pick list which is the nearest to the left wall from the expected location of the needed item farthest from the left wall in the warehouse:

\[ d_h = 2 \cdot (N - 1) \cdot d_i \cdot (E(\max\{x_1, x_2, \ldots, x_n\}) - E(\min\{x_1, x_2, \ldots, x_n\})) \]  \hspace{1cm} (3)
\[ = 2 \cdot (N - 1) \cdot d_i \cdot \left( \frac{n-1}{n+1} \right). \]

This equation will give correct results only when the items from each pick list will be placed on both sides of the PD point. Hall claims that it applies only when the number of items n on pick lists is big enough (more than 5) - figure 3 shows that it should be even up to twice greater. Otherwise, Hall shows how to modify the formula.

If the Pick-up / Drop-off point is located in the front of first picking aisle, then \( E(\min\{x_1, x_2, \ldots, x_n\}) = 0 \), and the modified Hall's equation will take the form:

\[ d_h = 2 \cdot (N - 1) \cdot d_i \cdot \left( \frac{n}{n+1} \right). \]  \hspace{1cm} (4)
The task of determining the distance traveled by the picker in the main aisle can be solved in another way; by designating the probabilities of passing by the picker next to the entrances to the following picking aisles (it is not important whether the picker enters the picking aisle or not):

\[
P_j = \begin{cases} 
1 - \left( \frac{N-j}{N} \right)^n & \text{for } j < a, \\
1 & \text{for } j = a, \\
1 - \left( \frac{j-1}{N} \right)^n & \text{for } j > a,
\end{cases}
\]

where:
- \( P_j \) - probability that the picker will pass next to the entrance of \( j \) picking aisle,
- \( a \) - number of picking aisle with the PD point.

The average distance traveled by the picker in the main aisle can than be calculated using the equation:

\[
d_a = 2 \left( \sum_{j=1}^{N} P_j - 1 \right). \tag{5}
\]

When the PD point is located in front of the first picking aisle: \( a=1 \). Then the equation (5) can be reduced to form:

\[
d_a = 2 \left( \sum_{j=1}^{N} \left(1 - \left( \frac{j-1}{N} \right)^n \right) - 1 \right). \tag{6}
\]

When the PD point is located in front of the middle picking aisle:

\[
a = \begin{cases} 
\frac{N}{2} & \text{when } N \text{ is even}, \\
\frac{N+1}{2} & \text{when } N \text{ is odd}.
\end{cases}
\]

**ESTIMATION OF THE AVERAGE DISTANCE TRAVELED BY THE PICKER IN THE PICKING AISLES**

The distance traveled by the picker in the picking aisles is - in contrast to the distance crossed in the main aisle - independent from the location of the Pick-up / Drop-off point. For the case in which in one cycle the picker picks only one item (one index), the average position of the needed item in the picking aisle is equal to half the length of the aisle. After picking an item the picker has to return to the PD point, so the average total distance is equal (twice a half) the length of the picking aisle:

\[
d_v = n \cdot (m+1) \cdot \frac{d_a}{m}. \tag{7}
\]
The picker has to enter the picking aisle, so a small value has to be added to the distance (in the equation (7) in the expression in the brackets it was assumed that the distance from main aisle to the first slot in the picking aisle is equal to the width of a slot, that is why the number of slots was increased by 1). The equation (7) measures the average distance traveled by the picker in the picking aisles while completing one pick list, so the result has to be multiplied by the number of picked items n.

For the case in which in one cycle more different items are collected, the problem is no longer so simple. According to the conception of Kunder and Gudehus [1975] expanded later by Hall [1993] as well as Le-Duc and De Koster [2005] the expected value of the distance traveled by the picker in the picking aisles is equal to the sum of the products of expected distance crossed in each picking aisle and the corresponding probabilities. For the return heuristic the distance traveled by the picker in one picking aisle depends on the number of items to be picked in this aisle and the length of the aisle. If the probabilities of occurrence on pick lists for all items are the same, then for calculating the expected distance in the picking aisles - like in the case of estimation the distance crossed in the main aisle - one can use the uniform distribution. In each picking aisle the picker has to cover the distance to the farthest slot with needed item and then return to the main aisle. That is why the expected value of the maximum of n numbers generated from uniform distribution has to be multiplied by 2 (and of course by the length of the aisle). As the calculation of the probability distribution P(x) may cause some computational problems, the equation (8) can be simplified. The expected value of the number of picking aisles visited by the picker is equal to:

\[ E(N_p) = N \cdot P(x > 0) = N \cdot \left(1 - \left(\frac{N-1}{N}\right)^x\right). \]

The average number of items picked from different slots if one picking aisle is equal to \( \frac{n}{E(N_p)} \). By making simple substitutions, the average distance traveled by the picker in picking aisles during completing one pick list can be calculated as follows:

\[ d_e = 2 \cdot \frac{n}{E(N_p)} \cdot d_s \cdot E(N_p) = \frac{n \cdot d_s \cdot E(N_p)}{E(N_p) + 1}. \]

\[ 2 \cdot \frac{n \cdot d_s}{N \cdot \left[1 - \left(\frac{N-1}{N}\right)^x\right]^{x+1}} \]

### COMPARING EQUATIONS WITH SIMULATIONS

To check the effectiveness of the equations presented in previous chapters, a lot of experiments was performed. Using formulas the average distance traveled by the picker in the main aisle and in the picking aisles was calculated for different: number of picking aisles, number of slots in one picking aisle and the size of pick list. The results were compared with average values obtained by the use of simulation tool Warehouse Real-Time Simulator [Tarczyński, 2013]. The accuracy of equations was measured using error function:

\[ E = \frac{|d - d_{sym}|}{d_{sym}} \cdot 100\%. \]

where:
- d - value calculated by the use of appropriate equation (d=d_{h}, \ Vd=d_{s}),
- d_{sym} - average value obtained by simulations.
Tables 1 and 2 show obtained values of error function. The average error for all equations was smaller than 10%. The relatively worst results were obtained for Hall’s conception - especially the original formula (3) gave the average value of the error function equal to 8.22%, and the maximum 15.27%. The value of the error function decreases with increasing: the number of picking aisles in the warehouse and the number of different items of pick lists. For the other analyzed methods of estimating the average distance traveled in the main aisle the error value is smaller than 1% (usually strongly). For the distance covered by the picker in the picking aisles best results were obtained by the use of De Koster's formula (8). For the equation (9), which is easier to practical implementation the average error is a little bigger, but still acceptable (smaller than 2%).

**CONCLUSIONS**

The order-picking time is usually proportional to the distance traveled by the picker - then in order to compare different variants of warehouse functioning and work organization one can calculate instead of the order-picking time - the average distance covered while completing one pick list. The mean values of the distance for different warehouse parameters and the size of pick list can be designated be the use of simulation tools or mathematical equations. Simulations give the accurate results and allow to analyze the order-picking process in a more complete way (among others instead of average values one gets a full probability distributions), but simulations are usually quite time-consuming. The results obtained by the use of mathematical equations are not as accurate, however, they are possible to achieve immediately.

The average distance traveled by the picker while completing one pick-list is a function of routing heuristic, the size of pick list and warehouse parameters (the number and length of picking aisles, the number of slots in each picking aisle, the location of Pick-up / Drop-off point). The problem of designating the
optimal values of the number and length of picking aisles, when only the number of slots in the warehouse is constant can be solved in two stages. Firstly, using mathematical equations the acceptable configurations are determined. Afterwards, by means of more time-consuming simulations the results are verified, the more precise outcomes are calculated and the best variant is chosen. The performed experiments show that it suffices to consider in the second stage only variants not worse than the best one (after the first stage) by more than 10%.

In the paper only one routing heuristic was analyzed and it was assumed that the demand for all stored items is the same. The problem will significantly complicate when the research will cover more heuristics, and the items will be divided according to ABC classification on groups with different rotation ratio. Then the proposed two-stage approach will be probably the only possible way to optimize the order-picking time.

REFERENCES


Krawczyk S., Jakubiak M., 2011, Rola komisjonowania w sterowaniu przepływami produktów, Logistyka 4 [The role of picking in the management of goods’ flow], 475-486.


Lewczuk K., 2012, Selected aspects of organizing order-picking process with dynamic material to location assignment, Proc. 2nd Carpathian Logistics Congress.

SZACOWANIE CZASÓW KOMPLETACJI ZAMÓWIEŃ DLA HEURYSTYKI RETURN - WZORY I SYMULACJE

STRESZCZENIE. Wstęp: Kluczowym elementem oceny funkcjonowania magazynu jest średni czas kompletacji zamówień. W magazynach, w których kompletacja odbywa się wg zasady "człowiek do towaru" czas kompletacji zazwyczaj jest proporcjonalny do dystansu pokonanego przez magazyniera, który może być oszacowany za pomocą symulacji lub z wykorzystaniem wzorów matematycznych. W artykule rozpatrywane są najlepiej opisane w literaturze magazyny prostokątne jednoblokowe.


Słowa kluczowe: kompletacja zamówień, magazynowanie, heurystyki, symulacje.

DIE BEWERTUNG VON KOMMISSIONIERUNGZEITEN FÜR DIE RETURN-HEURISTIK - FORMELN UND SIMULATIONEN


Codewörter: Kommissionierung, Lagerung, Heuristiken, Simulationen

Grzegorz Tarczyński
Operations Research Department
Wroclaw University of Economics
Komandorska 118/120, 53-345 Wroclaw, Poland
e-mail: grzegorz.tarczynski@ue.wroc.pl