Didactical situations in building children’s ideas about mathematical concepts in preschool education

Abstract: If we want to understand how children in preschool period build their ideas of mathematical concepts, we must explore the educational environment, i.e. situations that are organized in kindergarten, and the role of children and the educator in them. This paper focuses on identification of the phenomena that accompany the process of “evolution” of mathematical concepts in a preschool child’s cognition in the environment of institutional preschool education. The basic methodological approach to the research was the grounded theory. Using axial coding the various subcategories were identified within the frame of the so-called paradigmatic model according to Strauss and Corbin (1999). On their basis a model of a didactical situation in the environment of a kindergarten was developed. The theoretical framework was Brousseau’s Theory of Didactical Situations.

1 Introduction

Preschool education makes a constituent of the system of education in the Czech Republic. The approaches to it are currently given by the Framework Education Programme for Preschool Education (Smolíková, 2004). This document defines the content, the teaching methods, the target competences and thus it influences the nature of preschool teachers’ activities. The teachers are
quite fast to start “handing over” mathematical concepts, quite a lot of time attention is paid to work with worksheets, much less attention is paid to what the child actually learns. The question that needs to be answered is what is really going on in pre-mathematical education in kindergartens.

If we approach mathematical education at pre-school level in the social-constructivist perspective, a game and exploration may awaken the child’s ability to give objects and related activities mathematical meanings. Simultaneously they open space for communication, creation and enhancement of knowledge. We can suppose that this will create a specific interplay between the teacher, child and mathematical content, specific didactical situation. In 2010, within the work on my PhD thesis I started to explore situations that can be observed when intentionally building mathematical ideas of preschool children in kindergartens.

2 Theoretical background

The theoretical background for these considerations is the Theory of Didactical Situations (TDS) (Brousseau, 1997, in Czech Brousseau, 2012). Brousseau and his collaborators have conducted a number of researches concerning mathematics in primary and lower secondary education. However, the potential of TDS as a theoretical framework for research in preschool education has not been explored.

According to Brousseau a situation is a system entered by a teacher, a child, the environment, rules and regulations needed for discovery of some mathematical knowledge. The central concept of the Theory of Didactical Situations (Brousseau, 1997) is a didactical situation (see Fig. 1). This is defined as a system in which interaction between a pupil/group of pupils, teacher and mathematical knowledge takes place. A teacher looks for suitable situations and environments in which they organize a plan of activities whose aim is modification, discovery or creation of some knowledge (mathematical concept or idea) of a child.

An important component of didactical situations is pupils’ individual work in the so called a-didactical situation. If an a-didactical situation is to happen, it must be preceded by successful devolution (the process in which the teacher hands over part of their authority connected to guiding the activity to the pupils) and followed by institutionalization (the process in which the teacher helps pupils to integrate the discovered knowledge into their system of knowledge so that it can be applied in other activities).
An a-didactical situation according to Brousseau (1998) has three stages: action (its outcome is the expected (implicit) model, strategy, initial tactics), formulation (formulation of conditions under which the strategy is going to work), validation (explores the validity of the strategy: it works, it does not work). The same stages were identified in research of other authors (Složil, 2005; Novotná, Hošpesová, 2013).

A didactical situation is illustrated by the scheme in Fig. 1.

![Figure 1. The scheme of a didactical situation; from (Brousseau, 1997).](image)

## 3 Methodology

We based our analysis on the written narratives obtained from a group of kindergarten teachers who improved their qualification studying the bachelor program in the University of South Bohemia. The data come from seminar works entitled *Didactical situations in emergence of mathematical ideas*, which was compulsory for them. The students’ task was to:

1. create a situation in which preschool children could be expected to be using, based on their experience, mathematical concepts;
2. solve the task formulated in this situation with a group of at least five children;
3. record (audio or video) the whole communication with children;
4. intercept children’s reactions, problems they faced while solving, surprising reactions in relation to their own (teacher’s) interventions into the process of discovery.
In the analysis I mainly focused on the process of didactical situations and I tried to identify the Brousseau’s phases, while noting regularly repeating sequences that would create a new theory. I conducted the analysis in several stages. In the first stage, only those situations were selected which contained a transcript of the dialogue between the teacher and children. I could thus come to own conclusions. This requirement was fulfilled by 18 of the teachers. The conclusions were formulated on the basis of analyses of the teachers’ narrative\(^1\) of the situation. These analyses were framed by the grounded theory, whose specific techniques of coding (open, axial and selective) were used.

Data analysis started by open coding. The coding unit was one transcription of an interview which was coded line by line. I started by close reading of all the transcriptions and primarily focused on implementation of a-didactical situations and then identification of their stages; later my reading focused on all the important aspects related to the explored phenomenon with the aim of eliminating all the unrelated data. Having finished this, the situations were read again with the aim of pinpointing passages with similar meanings, naming them and relating them to more abstract units – categories. In the following reading I again compared the data, sought new concepts and categories, situations as wholes and substitutability and frequency of different categories and concepts in the dialogues. Having become theoretically more sensitive, new relations among the defined categories started emerging. Open coding was conducted using specialised computer programme Atlas.ti. Open coding resulted in the creation of a specific number of differently saturated and interrelated codes.

Gradually logical structure of relationships started from the data began to emerge, first in the form of relations between codes, and later for larger semantic units (categories). Having finished open coding, all data were arranged in a new way using connections among categories within a paradigmatic model (axial coding). This was based on the basic form of a paradigmatic model of Strauss and Corbin (1999, p. 72), that, in a simplified form, looks as follows:

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\text{CAUSAL CONDITIONS } \Rightarrow \text{PHENOMENON } \Rightarrow \text{CONTEXTUAL CONDITIONS } \Rightarrow \text{INTERVENING CONDITIONS } \Rightarrow \text{ACTION/INTERACTIONAL STRATEGIES } \Rightarrow \text{CONSEQUENCES.}
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Based on this I went through all the codes and extracts and intentionally defined relationships between conditions, interaction and consequences. At this

\(^1\)One must bear in mind that teacher’s comments were not necessarily equivalent to the real situation. Still I am convinced that they were the teachers’ ideas of what “correctly” implemented situation should be like, of what kind of support they should provide to the children while learning.
stage the coding focused on a narrower definition of a category (phenomenon) by the conditions that had caused it, the context (its particular set of properties) in which it was set, by the strategy of action and interaction that helped to cope, master, carry out, and the consequences of this strategy.

After this the analysed data underwent the process of selective coding, i.e. some of the data were left aside and the relevance of the results was related to the meaning of the main analytical story of the study.

In the following text I will use the different subcategories to illustrate the form of the paradigmatic model whose use was of crucial importance for conceptualization of the results, creation of a new theory and proposal of a model of a didactical situation in the environment of a kindergarten.

4 Results

The basic framework was created by a category *types of didactical situations*, which was formed by combining the categories *teacher intervention* and *ways of solving by children* which showed correlation. Through these categories, which influenced the course of the whole situation, I identified the phenomenon of *didactical situation in mathematics*. The main causal conditions of *didactical situation in mathematics* were categories *teachers’ believe* and *stimulating learning environment*. The category *children’s experiences* acted as an intervening variable. Creating and refining the mathematical concepts in children usually took place in the context of spontaneous children’s game influenced by the category *management of learning situation*.

All categories of paradigmatic model in the thesis were supported by relevant examples from the narratives of participants (Semerádová, 2015, pp. 100-148). In this paper I will give several illustrations due to the limited space.

4.1 Didactical situation in mathematics as a phenomenon

Let us begin by describing the basic phenomenon that this stage of coding was concerned with, i.e. the phenomenon of a *didactical situation in mathematics*. Based on data analyses, several variants of didactical situations with respect to particular characteristics that had emerged in the process of open coding were developed. These were especially codes representing the teacher’s incentives resulting in children’s activity and the types of children’s reactions. The characteristics I relied on when describing *didactical situations in mathematics* in kindergartens and which played a key role were the following:
• teacher’s intervention,
• ways of solving by children.

Teacher’s intervention

Based on a detailed data analysis the teacher’s intervention could be classified into the following categories with respect to the way of “managing” the situation:

• an incentive to a child’s activity,
  Illustration
  Jaroslava: Boys, watching and hearing you I realized that you have come to something very important.
• a stimulus (usually in the form of a question) directing considerations,
  Illustration
  Ivana: Boys, how would you fair share the pieces of the kit?
• intervention in the form of a particular question with more or less straightforward answer,
  Illustration
  Tereza addressed a group of children who played with tractors and trucks: How many of you went out to the field? Are there the same amount of tractors and trucks?

Apart from the different categories of intervention I could also identify the level of a teacher’s intervention, which determines the proportion of children’s active influence on the situation.

Three levels of intervention could be observed:

• intervention responding to children’s needs,
• intervention guided by the teacher’s aim,
• continuous teacher’s intervention.

Ways of solving by children

The category Ways of solving by children was divided into:

1. procedures in the form of activity, e.g. manipulation with objects, demonstration, cooperation,

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2We use the nicknames instead of real names of the teacher who wrote the narratives.
2. procedures in the form of formulation, e.g. a more detailed explanation to another child, discussion among the children about the problem or involvement of the main initiator,

3. procedures of validation or explanation of a solution, e.g. checking the activity with the model, checking correctness of the mathematical term or analysis of a case.

Types of didactical situations

On the basis of interaction between the teacher’s intervention influencing the situation and children’s ways of solving the situation, the following three didactical situations were defined:

- didactical situations managed by children,
- didactical situations guided by the teacher,
- didactical situations directed by the teacher.

Let us now look at a brief characteristic of these types of didactical situations.

1. Didactical situations managed by children

These situations emerged both from children’s natural, spontaneous work and from intentional activity pre-planned by the teacher. Spontaneous, natural situations (initiated by preschool children) stemmed either out of spontaneous game activities, or from regular daily activities in the kindergarten (while eating, on a walk, when clearing away toys etc.). In both cases the course of the situation was left up to the children, the teacher’s intervention was minimal. Actors of the situation were most often children aged 5–6(7). The teachers reported these situations mostly in groups of up to five children. In this type of situation, thanks to theoretical definition of an a-didactical situation the teachers were able to see the potential that the situation bore. This could also be read in the testimonies the teachers presented in their seminar works.

The category teacher’s intervention was represented in these situations by challenges, incentives or stimulating questions that gave the children the space for their own thinking, activity, initiative. Teachers only reacted in case the preschool children needed it. All the three procedures from the category ways of solving by children were present – procedures in the form of activity, in the form of formulation and in the form of validation or explanation of a solution.

Illustration

Teacher Marie described a situation in which children complemented napkins in the stands on the tables according to the number of
children who had to sit at the table. The teacher only observed the situation. She recorded the following dialogue at the table with 6 seats occupied by 5 children (there were 2 napkins in the stand.

Kate prepares 6 napkins.

Niky: Kate, not so many napkins, There are 2, give me... 4...

No. Today, me, you, Jonas, Sam and Amy are sitting at this table. 2 napkins for 2 children are here. I need napkins for 3 children.

Give me 3.

Kate: How is it that only 3?

Niky pulls from the rack 2 napkins and put them on the table:

These are for Jonas and Sam. I need for Amy, you and me.

Katy: But there are 6 seats, here.

Niky: It does not matter. One remains empty.

A sample described by teacher Marie came quite naturally during snack. She entered into the situation only one invited by children. In conclusion, she summed up the situation.

2. Didactical situations guided by the teacher

This type of situations was to be come across in spontaneous game activities. This was an unplanned situation that developed naturally. The actors of this situation were children aged 5 – 6 (7), the maximum number of children involved never exceeded 7. The teacher entered the play through intervention (most often in the form of guiding questions, less often in the form of stimulating questions that gradually turned into guiding questions), whose goal was either to build on some mathematical activity and develop it, or introduce new mathematical knowledge. However, the teacher’s intervention usually interrupted the children’s activity and prevented its further development.

In this case, the situation did not “grow out” of the children’s needs while playing, the teacher “forcibly” diverted their attention to some mathematical knowledge or concept. These situations showed that the teacher was trying to follow the stages of didactical situations (according to Brousseau), they initiated the different stages by questions. In these situations, the category ways of solving by children was represented by procedures in the form of activity or formulation and only very rarely with procedures in the form of validation or explanation. In most cases this reaction was induced by the teacher’s question or intervention.
Illustration

Teacher Eva described the situation when several boys built buildings from bricks. Jan (one of the boys) warns the others that they must prepare only cubes and not blocks. The teacher realized that they do not know correct names and entered the play.

Teacher: Boys, do you know how to properly called this brick? (She hands a cube.)

Boys: No.

Teacher: What shape has one of her sides?

Jan: A square.

Teacher: Yes, that’s right, it’s a square. All sides are equal squares.

Adam: And what is the name of this shape? Is it cube, too?

Teacher: What do you think, boys?

Patrick: No, you did not hear the teacher? The cube has sides like a square. This cube is different.

3. Didactical situations directed by the teacher

These were pre-planned situations. The teacher had intentionally selected the mathematical activity to be practiced with the preschool children. In these situations, it was usually the whole class that got involved. The minimum number of children was 15, the children were aged 5 – 6 (7). This type of situation is characterized by continuous teacher’s intervention in the form of specific questions aiming at a given goal. No procedures from the category ways of solving by children were represented as all were mere reactions to the teacher’s question. The situation was directed by the questions through which the teacher tried to practice a particular item of mathematical knowledge. The questions were posed in such a way that they could usually be answered easily, often just with one word. If this discussion worked and the children were reacting, the teacher made a record of the situation, as it was dealing with mathematical concepts.

Illustration

Teacher Miloslava described following dialogue with children:

Teacher: Children, How do I know that I have enough worksheets for each of you?

Jirka: We count them. (We counted 18 sheets.)

Teacher: How we will continue?
Petra: We will count children. (We counted 20 children.)
Teacher: Have I enough sheets for each of you?
The children hesitate.
Teacher: How could you find out if everyone gets one worksheet and do not count?
Children (impatiently): Ms. teacher, give them to us. (The teacher hand out the worksheets. Viktorka and George do not receive any.)
Cathy: Two missing!
Teacher: Well, you see. What is more?
Children spontaneously: Children!
Teacher: How much more?
Children: Two.

It can be said that the types of didactical situations in fact headed towards some kind of stability according to different recurrent patterns. The most frequent type of a didactical situation was the situation when children were solving some mathematical problem mostly on their own. The key category was the category connected with the teacher. This category affected the degree of independence of children in the activities as well as organization and course of the situation.

Each type is connected to specific phenomena on the level of other subcategories (stimulating environment, the teacher’s beliefs, children’s experience and also management of the didactical situation by the teacher).

4.2 Stimulating environment as a causal condition of a didactical situation in mathematics

Let us now focus what actually caused this phenomenon. I tried to pinpoint the most important causal conditions: to find out why creation or elaboration of children’s ideas of mathematical concepts was to be come across only in certain types of didactical situations and to identify the factors that helped and inhibited creation of children’s ideas of mathematical concepts.

The basic assumption in this subcategory is that when building mathematical ideas, a child must be active. Mathematical content emerges more or less spontaneously.

The following two factors were identified at the main causes for emergence and course of the phenomenon (didactical situation in mathematics):

- teacher’s beliefs,
- stimulating environment.
Teacher’s beliefs

The category teacher’s beliefs was primarily analysed on the basis of reflection of the situation that the teachers were asked to present in their seminar work. It turned out that it was important to understand the a-didactical situation and its phases (organization of knowledge and information, influence on children’s action and behaviour). Teachers who understood a-didactical situation and attempted to realize it, they are interested in new information, prioritized the needs of individual children, tried to build on the experiences of children and created space for their activity.

Other teachers have adopted the theory as an additional option for creating mathematical knowledge; they did not take too much into account the needs of individual children; they often intervened (in the form of questions) in the situation to direct it towards practicing mathematical concepts. Their aim was to create a situation in which there was either practicing mathematical concept, but which was organized by a teacher or a controlled dialogue (teacher), a mathematical concept. Based on their reflections, it is possible to state that access to the teachers realized the situation was influenced by knowledge of the theoretical definition of a didactic situation.

Stimulating environment

The analysis showed that building and development of children’s ideas of mathematical concepts most often occurred during spontaneous game activities. The success of a game largely depended on a stimulating environment.

The type of didactical situation presented in this study that most closely corresponds to thus conceived environment is the first type of a didactical situation, e.g. didactical situation managed by children. The stimulus to this situation is any problem the preschool children come across during the day. It is also related to the teacher’s considerations which interlink activities targeting at posing questions naturally arising from the situation, solving problems and explaining the results. Prerequisite to this is the teacher’s sufficient knowledge of the subject and pedagogical expertise.

The data show that the extent of how stimulating the environments are differs from one kindergarten to another. The category stimulating environment was further divided into two subcategories with respect to the factors that either support or inhibit building and development of mathematical concepts. Both of these categories were more or less in the teacher’s hands, i.e. teachers’ role seems to be of key importance. The factors with positive impact on the stimulating nature of the environment in kindergartens were classified and labelled as facilitators of a stimulating environment. On the contrary the
factors with negative impact on the stimulating nature of the environment in kindergartens were placed in the subcategory *inhibitors of a stimulating environment*.

**Facilitators of a stimulating environment**

The factors with positive impact on the quality of didactical situations were for example:

- material environment,
- material aids,
- topic games,
- natural materials,
- stimulating atmosphere.

**Inhibitors of a stimulating environment**

The following factors were identified as having negative impact on development and course of a didactical situation:

- number of preschool children in a class,
- classes composed of preschool children of different ages,
- number of teachers present.

The data showed that if the teacher was looking after more children (15 and more) and moreover if the children were not of the same age, it was much more difficult to induce a situation in which children’s ideas of mathematical concepts were created or developed.

### 4.3 Children’s experience as an intervening variable

The previous text summarizes the main causal conditions that are affected mostly by the teacher. However, this does not mean that preschool children themselves would not be able to create a situation in which:

- they would communicate about (pre)mathematical concepts gained outside of the kindergarten\(^3\),
- they would discover a particular mathematical connection or regularity.

\(^3\)Most of this experience comes from an older sibling or parents.
The basic initial vision was that preschool children should be building ideas of mathematical concepts on their own, if possible without any visible intervention of the teacher. However, this largely depends on experience they already have. This would especially be such experience that would allow children to solve problems stemming from misunderstanding, ambiguity, wrong formulation of the solution or a particular mistake. Missing experience caused by low age or level of cognitive development would become the incentive for solving a situation. In this study, children’s experience was handled with respect to their age. The data showed that didactical situations resulting in creation and development of mathematical concepts and managed by the children occurred exclusively at the age of 5 - 6 (7).

The category *children’s experience* filled most of the situation but was not the cause of emergence a didactical situation managed by children. Therefore children’s experience was not decisive. This category is an intervening variable.

4.4 Spontaneous children’s game as context

The following text focuses on the context of the whole problem, i.e. on children’s game activities. In the conception of TDS, an a-didactical situation is conceived as a game; and game is something very familiar and natural to children at the age of 3 to 6 (7). By a game activity I mean any spontaneous activity “determined by the level of one’s own experience in which a child discovers the world and different links in this world” (Opravilová, 2004, p. 7). All this takes place within children’s activity which is influenced by the teacher’s action to a greater or lesser degree. The teacher’s action can be both unconscious, unintentional, unplanned, natural, arisen within the course of the children’s playing, or conscious, intentional, “bought about artificially”, pre-planned and primarily pedagogical-led (for more details see the subcategory managing didactical situation).

Data analysis brought the following conclusion: prerequisite to spontaneous activity was a stimulating environment and various incentives. These incentives and conditions for a game activity were created, planned and offered by the teacher (not always explicitly). When selecting an activity, it was important whether the activity was taking place in the classroom or outside as the preschool children had to adapt their activity to it. Spontaneous, natural activities based on children’s experience described by the teachers emerged spontaneously anywhere and any time depending on the time and space provided by the teacher.

An analysis of teachers’ comments resulted in definition of three main factors of major impact on the course of spontaneous children’s game activity.
These are age, the number of children involved and the role of the initiator of the situation, very often connected to gender of the child. The most efficient alternative leading to didactical situations aiming at creation or elaboration of a mathematical concept managed by children was the alternative of a group of children consisting of boys aged 5 – 6 (7) at the maximum number of 5 children. The question is whether this was really the case or whether this role was assigned to boys by the teachers as boys appeared to them more creative. This conclusion may also be formulated as follows: the selection of boys in the situations was influenced by the teachers’ expectations. This asks for further empirical research, e.g. focusing on gender roles in didactical situations.

4.5 Managing didactical situation as a strategy of action and interaction

The category with a decisive impact on didactical situation in mathematics was the category managing didactical situation, i.e. intentional and deliberate activity of the teacher. Managing didactical situation may be presented by teachers’ approaches and strategies. The following approaches were used for these ends:

- approach based on children’s experience,
- approach based on a priori analysis,
- approach based on instructions,
- directed approach.

Their goal was to create optimal conditions in which various forms of intervention should result in children’s discovery of a particular mathematical regularity or to creation or development of a mathematical concept. Each approach had the same teaching goal. Data analysis shows that teachers most often used the approach based on children’s experience that seemed to be most efficient from the point of view of the objectives.

The educational strategies of the teachers differed especially with respect to how much the preschool children were let to implement the activity that involved mathematical concepts and relations on their own. The following strategies were identified with respect to the role the teachers played in the situation:

- role of an observer,
- role of an initiator,
- role of an operator,
- role of a conductor.
The most frequent strategies were those where the teacher was in the role of an observer or initiator and the main actors of the whole situation were children. In the remaining two cases (strategies of teachers in the roles of operators and conductors), the situations were directed and sometimes even implemented by the teachers.

The data in the end proved that one explanatory model would be sufficient to show both the structure of the analysed phenomenon and its development in time (see Semerádová, 2015, p. 150).

5 Conclusion

The creation of mathematical knowledge in kindergarten is causally affected primarily by management of didactical situations, and by spontaneous child’s play. Management of didactical situation can be presented via the approaches and strategies of the teacher. Teacher’s beliefs have a major influence on her/his negotiations in the classroom. The most effective can be considered teachers’ approaches based on the experience of children and on the analysis a priori. These practices are characterized by the following:

- they take into account the experience of children, which builds on the progress of the situation,
- provide the space for children to be active and take the initiative.

Based on the analysis I propose the following model of a didactical situation in the environment of a kindergarten (see scheme 1).
To conclude the paper it must be stressed that it seems most opportune when this building and development of mathematical concepts naturally evolves from children’s experience, when teachers assist this “emergence” of mathematical concepts by adequate reactions and the environment for creation and development of mathematical knowledge and concepts are didactical situations.

References


Sytuacje dydaktyczne sprzyjające budowaniu pojęć matematycznych w edukacji przedszkolnej

S t r e s z c z e n i e

Jeżeli chcemy zrozumieć, w jaki sposób dzieci przedszkolne budują swoje rozumienie pojęć matematycznych, musimy zbadać ich środowisko edukacyjne, to znaczy wyróżnić sytuacje, które są organizowane w przedszkolu oraz określić role pełnione w nich przez dzieci i nauczycieli. W tym artykule skupiono
uwagę na zidentyfikowaniu zjawisk towarzyszących procesowi „ewolucji” pojęć matematycznych w dziecięcych umysłach, w środowisku zinstytucjonalizowanej edukacji przedszkolnej. Teoretyczną podstawę do prowadzonych badań stanowiła „grounded theory”. Zostało wyróżnionych kilka różnych podkategorií funkcjonujących w ramach tzw. modelu paradygmatycznego, opisanego przez Straussa i Corbina (1999). Na tej podstawie został określony model sytuacji dydaktycznej w środowisku przedszkolnym, bazujący na Teorii Sytuacji Dydaktycznych Brousseau.