A Tauberian Theorem for Weighted Means in Non-Archimedean Fields - Revisited and Revised

Abstract. In this note, $K$ denotes a complete, non-trivially valued, non-archimedean field. We correct a Tauberian theorem for weighted means in $K$ proved earlier in [1].

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Throughout this short note, $K$ denotes a complete, non-trivially valued, non-archimedean field. Sequences and series have entries in $K$. The present note is a sequel to [1]. In the proof of the main result, i.e., Theorem 4 of [1], there is an error towards the end of the proof. It seems that (2.1) is not sufficient to prove Theorem 4 of [1]. We have the following revised form of Theorem 4 of [1]. We suppose that $(\bar{N}, p_n)$ is regular.

Theorem 4 (revised form) If $\sum_{k=0}^{\infty} a_k$ is $(\bar{N}, p_n)$ summable to $s$ and if

\begin{equation}
 a_n = O \left( \frac{p_n}{P_n^2} \right), \quad n \to \infty;
\end{equation}

and

\begin{equation}
 a_n \to \ell, \quad n \to \infty,
\end{equation}

then $\sum_{k=0}^{\infty} a_k$ converges to $s$. 

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PROOF We can suppose that $s = 0$. We now claim that $\ell = 0$. Suppose not. Choose $\epsilon > 0$ such that $\epsilon < |\ell|$. We can now choose a positive integer $N$ such that

\[(3) \quad |s_n| \left| \frac{p_n}{P_n} \right| < \epsilon, \quad n \geq N,\]

using Theorem 3 of [1] and

\[(4) \quad |a_n - \ell| < \epsilon, \quad n \geq N,\]

using (2). Now,

\[|a_n| = |(a_n - \ell) + \ell| = \max(|a_n - \ell|, |\ell|)\]
\[\quad = |\ell|, \quad n \geq N.\]

Also,

\[|a_n| \leq M \left| \frac{p_n}{P_n} \right|, \quad M > 0,\]

in view of (1). Thus, for $n \geq N$,

\[|\ell| \leq M \left| \frac{p_n}{P_n} \right|,\]

i.e., \[\left| \frac{p_n}{P_n} \right| \geq \frac{|\ell|}{M}.\]

Using (3), for $n \geq N$,

\[\epsilon > |s_n| \left| \frac{p_n}{P_n} \right| \geq |s_n| \frac{|\ell|}{M},\]

i.e., \[|s_n| < \epsilon M |\ell|, \quad n \geq N.\]

In other words, $s_n \to 0$, $n \to \infty$ so that $a_n \to 0$, $n \to \infty$. Thus $\ell = 0$, which is a contradiction. This contradiction leads to the fact that $\ell = 0$. Consequently $\sum_{k=0}^{\infty} a_k$ converges. Since $(\bar{N}, p_n)$ is regular, $\sum_{k=0}^{\infty} a_k$ converges to 0, completing the proof. ■

REFERENCES