CERTAIN TYPES OF AFFINE MOTION
IN A FINSLER MANIFOLD. III

BY

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1. Introduction. In a series of papers [7]–[9] the author discussed the necessary and sufficient conditions for contra, concurrent, special concircular, recurrent, concircular, torse forming and birecurrent vector fields to generate an affine motion in a Finsler manifold. The present paper, which is the last one of the above series, deals with the same problem for a vector field $v^i(x')$ whose Berwald's covariant derivative $\mathfrak{M}_k v^i$ is recurrent, i.e., $\mathfrak{M}_j \mathfrak{M}_k v^i = u_j \mathfrak{M}_k v^i$, $u_j$ being a non-zero covariant vector field. This paper also presents an elegant generalization of theorems due to the author [4], Kumar [1], Misra and Meher [2] and Takano [11].

2. Preliminaries. Let $F_n(F, g, G)$ be an $n$-dimensional Finsler manifold of class at least $C^7$ equipped with a metric function $F$ (1) satisfying the required conditions [10], the corresponding symmetric metric tensor $g$ and the Berwald's connection $G$. The coefficients of Berwald's connection $G$, denoted by $G^i_{jk}$, satisfy

$$G^i_{jk} = G^i_{kj}, \quad b) \ G^i_{jk} \ x^k = G^i_j, \quad c) \ \dot{\partial}_k G^i_j = G^i_{jk},$$

where $\dot{\partial}_k$ means the partial derivative with respect to $\dot{x}^k$. The partial derivatives $\dot{\partial}_k G^i_{jk}$ of the connection coefficients $G^i_{jk}$ constitute a tensor whose components are denoted by $G^i_{jkh}$. This tensor is symmetric in its lower indices and satisfies

$$G^i_{jkh} x^k = G^i_{khj} x^k = G^i_{hjk} x^k = 0.$$

The covariant derivative $\mathfrak{M}_k T^i_j$ of an arbitrary tensor $T^i_j$ for the connection $G$ is given by

$$\mathfrak{M}_k T^i_j = \dot{\partial}_k T^i_j - (\dot{\partial}_i, T^i_j) G^i_k + T^i_j G^i_{rk} - T^i_r G^i_{jk},$$

(1) Unless otherwise stated, all the geometric objects used in the paper are supposed to be functions of line elements $(x', x^i)$. The indices $i, j, k, \ldots$ take positive integral values from 1 to $n.$
where \( \hat{\partial}_x \equiv \partial / \partial \hat{x}^k \). The operator \( \mathfrak{B}_k \) commutes with the operator \( \hat{\partial}_k \) and itself according to

\[
(2.4) \quad \hat{\partial}_j \mathfrak{B}_k \, T^i_k - \mathfrak{B}_k \, \hat{\partial}_j \, T^i_k = T^i_k \, G^i_{jkr} - T^i_r \, G^r_{jkh},
\]

\[
(2.5) \quad \mathfrak{B}_j \mathfrak{B}_k \, T^i - \mathfrak{B}_k \mathfrak{B}_j \, T^i = T^i_k \, H^i_{jkr} - T^i_r \, H^r_{jkh} - (\hat{\partial}_r \, T^i_k) \, H^i_{jr},
\]

where \( H^i_{jkh} \) constitute Berwald's curvature tensor. This tensor is skew-symmetric in first two lower indices, and positively homogeneous of degree zero in \( \hat{x}^i \). It should be noted that \( H^i_{jkh} \) coincides with \( H^i_{kaj} \) of Rund [10]. The tensor \( H^i_{jk} \) appearing in (2.5) is connected with the curvature tensor by

\[
(2.6) \quad (a) \, H^i_{jkh} \, \hat{x}^k = H^i_{jk}, \quad (b) \, \hat{\partial}_k \, H^i_{jk} = H^i_{jkh}.
\]

This tensor is related with the deviation tensor \( H^i_j \) by

\[
(2.7) \quad (a) \, H^i_{jk} \, \hat{x}^k = H^i_j, \quad (b) \, \frac{1}{2} (\hat{\partial}_k \, H^i_{jk} - \hat{\partial}_j \, H^i_{lk}) = H^i_{jk}.
\]

The associate vector \( y_i \) of \( \hat{x} \) satisfies the relations (see [6] and [10])

\[
(2.8) \quad (a) \, y_i \, \hat{x}^i = F^2, \quad (b) \, y_i \, H^i_{jk} = 0, \quad (c) \, g_{ik} \, H^i_{mjk} + y_i \, H^i_{mjk} = 0,
\]

where \( g_{ij} \) are components of the metric tensor \( g \).

Let us consider the infinitesimal transformation

\[
(2.9) \quad \hat{x}^i = x^i + \varepsilon v^i(x^i)
\]

generated by a vector \( v^i(x^i) \), \( \varepsilon \) being an infinitesimal constant. The Lie derivatives of an arbitrary tensor \( T^j_k \) and the connection coefficients \( G^i_{jk} \) with respect to (2.9) are given by (see [12])

\[
(2.10) \quad \mathfrak{L} T^j_k = v^r \mathfrak{B}_r T^j_k - T^j_k \mathfrak{B}_r v^r + T^j_r \mathfrak{B}_j v^r - (\hat{\partial}_r \, T^j_k) \mathfrak{B}_r v^r,
\]

\[
(2.11) \quad \mathfrak{L} G^i_{jk} = \mathfrak{B}_j \mathfrak{B}_k v^i + H^i_{mjk} v^m + G^i_{jkr} \mathfrak{B}_s v^s \hat{x}^r.
\]

The operator \( \mathfrak{L} \) commutes with the operators \( \mathfrak{B}_k \) and \( \hat{\partial}_k \) according to

\[
(2.12) \quad (\mathfrak{L} \mathfrak{B}_k - \mathfrak{B}_k \, \mathfrak{L}) \, T^j_k = T^j_k \, \mathfrak{L} G^i_{rk} - T^j_r \, \mathfrak{L} G^i_{kj} - (\hat{\partial}_r \, T^j_k) \, \mathfrak{L} G^i_k,
\]

\[
(2.13) \quad (\hat{\partial}_k \, \mathfrak{L} - \mathfrak{L} \hat{\partial}_k) \, \Omega = 0,
\]

where \( \Omega \) is a vector, tensor or connection coefficient. The necessary and sufficient condition for the vector \( v^i(x^i) \) to generate an affine motion is given by (see [12])

\[
(2.14) \quad \mathfrak{L} G^i_{jk} = 0.
\]

3. Affine motion in a Finsler manifold.

**Theorem 3.1.** A vector field \( v^i(x^i) \) which satisfies any two of the following conditions must satisfy the third one:

(A) \[ v^m H^i_{mjk} = w_j \mathfrak{B}_k v^i \]
(B) \[ \mathcal{B}_j \mathcal{B}_k v^j = u_j \mathcal{B}_k v^j, \]

(C) \[
\begin{align*}
(i) & \quad \mathcal{L}G^i_{jk} = 0, \\
(ii) & \quad G^i_{jk} \mathcal{B}_s v^j \dot{x}^s = 0,
\end{align*}
\]

where \( u_j \) and \( w_j \) are non-zero covariant vector fields.

**Proof.** Let us consider a vector field \( v^j(x^j) \) which satisfies (A) and (B). The Lie derivative of \( G^i_{jk} \) with respect to an infinitesimal transformation generated by the vector field \( v^j(x^j) \) is given by (2.11), which, in view of (A) and (B), may be written as

\[
\mathcal{L}G^i_{jk} = (u_j + w_j) \mathcal{B}_k v^j + G^i_{jk} \mathcal{B}_s v^j \dot{x}^s.
\]

Since \( G^i_{jk} \) and \( G^i_{jk} \) are symmetric with respect to the indices \( j \) and \( k \) in (3.1), \( (u_j + w_j) \mathcal{B}_k v^j \) must be symmetric, i.e.,

\[
(u_j + w_j) \mathcal{B}_k v^j = (u_k + w_k) \mathcal{B}_j v^j.
\]

This equation implies at least one of the following:

\[
\begin{align*}
(a) & \quad \mathcal{B}_k v^j = 0, \quad (b) \quad u_j + w_j = 0, \quad (c) \quad \mathcal{B}_k v^j = (u_k + w_k) X^j
\end{align*}
\]

for some non-zero vector field \( X^j \). If (3.3a) holds, equation (3.1) reduces to \( \mathcal{L}G^i_{jk} = 0 \). Also, in this case, \( G^i_{jk} \mathcal{B}_s v^j \dot{x}^s = 0 \) holds identically. Thus, condition (C) holds for the vector \( v^j(x^j) \). If (3.3b) holds, equation (3.1) reduces to

\[
\mathcal{L}G^i_{jk} = G^i_{jk} \mathcal{B}_s v^j \dot{x}^s.
\]

Transvecting (3.4) by \( \dot{x}^k \), and using (2.1b) and (2.2), we have

\[
\mathcal{L}G^i_{jk} = 0.
\]

Differentiating (3.5) partially with respect to \( \dot{x}^k \), utilizing the commutation formula (2.13) and using equation (2.1c), we have \( \mathcal{L}G^i_{jk} = 0 \), and hence (3.4) gives \( G^i_{jk} \mathcal{B}_s v^j \dot{x}^s = 0 \). Thus, condition (C) holds. In case of (3.3c), condition (A) becomes

\[
v^m H^i_{mjk} = w_j (u_k + w_k) X^i.
\]

Transvecting (3.6) by \( \dot{x}^k \) and using (2.6a), we have

\[
v^m H^i_{m} = w_j (u_k + w_k) \dot{x}^k X^i.
\]

Transvecting (3.7) by \( y_i \) and using (2.8b), we have at least one of the following:

\[
\begin{align*}
(a) & \quad (u_k + w_k) \dot{x}^k = 0, \quad (b) \quad y_i X^i = 0,
\end{align*}
\]

since \( w_j \neq 0 \). In case of (3.8a), equation (3.7) gives \( v^m H^i_{m} = 0 \), which after partial differentiation with respect to \( \dot{x}^k \) implies \( v^m H^i_{mjk} = 0 \). Also from (3.8b) and (3.6) we have \( v^m y_i H^i_{mjk} = 0 \). Transvecting (2.8c) by \( v^m \) and using \( v^m y_i H^i_{mjk} = 0 \), we have \( g_{ik} H^i_{m} v^m = 0 \). Transvecting \( g_{ik} H^i_{m} v^m = 0 \) by \( g^{kl} \) and
using $g^l_{jk} = \delta^l_1$, we get $H^l_{mjk}v^m = 0$, which implies $v^mH^l_{mjk} = 0$. Thus, both the conditions given by (3.8a) and (3.8b) imply $v^mH^l_{mjk} = 0$ separately. In view of this equation and $w_j \neq 0$, condition (A) gives $A_k v^j = 0$, which is nothing but (3.3a), a condition already discussed. Thus, we conclude that each of the conditions given by (3.3) implies (C). Therefore, (A) and (B) imply (C).

Let us consider a vector field $v^i(x^l)$ which satisfies (A) and (C). In this case, equation (2.11) may be written as

$$A_j A_k v^j = u_j A_k v^j,$$

where we have put $-w_j = u_j$. Thus, (B) holds for the vector field $v^i(x^l)$.

Again, if a vector field $v^i(x^l)$ satisfies (B) and (C), equation (2.11) gives (A), where $w_j = -u_j$. This completes the proof.

Let us consider a vector field $v^i(x^l)$ satisfying condition (B). Since any contra vector field $v^i(x^l)$ satisfies (B) trivially, we shall exclude this case from our discussion. Differentiating (B) partially with respect to $x^k$ and utilizing the commutation formula exhibited by (2.4), we have

(3.9) \[ A_j (G_{kk} v^j) + G_{jk} A_k v^j - G_{jk} A_j v^j = (\dot{\delta}_h u_j) A_k v^j + u_j G_{kk} v^j. \]

Transvecting (3.9) by $\dot{x}^k$ and using (2.2), we get

(3.10) \[ G_{jk} \dot{x}^k A_k v^j = (\dot{\delta}_h u_j) \dot{x}^k A_k v^j. \]

If condition (Cii) holds, (3.10) gives at least one of the following:

(3.11) \begin{align*}
(a) & \quad \dot{\delta}_h u_j = 0, \\
(b) & \quad \dot{x}^k A_k v^j = 0.
\end{align*}

If (3.11b) holds, its partial differentiation with respect to $\dot{x}^k$ and the use of (2.2) imply $A_k v^j = 0$, a trivial case. Hence, for a non-trivial case, (3.11a) must hold, i.e., the covariant vector field $u_k$ is independent of $\dot{x}^l$. Conversely, if the vector field $u_k$ is independent of $\dot{x}^l$, equation (3.10) gives (Cii). Thus, we conclude

**Theorem 3.2.** If a vector field $v^i(x^l)$ satisfies condition (B), then the necessary and sufficient condition for the covariant vector field $u_k$ to be independent of $\dot{x}^l$ is given by (Cii).

Let us consider a vector field $v^i(x^l)$ which satisfies (B) and the vector field $u_k$ which is independent of $\dot{x}^l$. In view of Theorems 3.1 and 3.2, we may conclude that conditions (A) and (C) are equivalent. Since (Ci) is the necessary and sufficient condition for the vector field $v^i(x^l)$ to generate an affine motion, we have

**Theorem 3.3.** Condition (A) is necessary and sufficient for a vector field $v^i(x^l)$ satisfying condition (B) where $u_k$ is independent of $\dot{x}^l$ to generate an affine motion.
4. Affine motion in special Finsler manifolds. Takano [11] considered a non-Riemannian manifold of recurrent curvature and proved that a vector field \( v^i \) generates an affine motion in a non-Riemannian manifold of recurrent curvature, characterized by

\[
V_m B_{jkh}^i = \lambda_m B_{jkh}^i,
\]

if

\[
\varepsilon \lambda_m = 0,
\]

\[
LB_{jkh}^i = A_{kh} V_j v^j,
\]

\[
L = \lambda_m v^m \neq \text{const},
\]

\[
A_{kl} = 2V_{[l} \lambda_{k]},
\]

where \( V_m \), \( B_{jkh}^i \) and \( \lambda_m \) are the operator for covariant differentiation, the curvature tensor and the recurrence vector, respectively.

Kumar [1], Misra and Meher [2] extended this theorem to Finsler manifolds of recurrent curvature and proved that a vector field \( v^i \) generates an affine motion in a recurrent Finsler manifold, characterized by

\[
\mathcal{B}_m H_{jkh}^i = \lambda_m H_{jkh}^i,
\]

if

\[
LH_{jkh}^i = A_{jk} \mathcal{B}_h v^j,
\]

\[
\varepsilon \lambda_m = 0,
\]

\[
L = \lambda_m v^m \neq \text{const},
\]

\[
A_{jk} = 2\mathcal{A}_{[j} \lambda_{k]},
\]

\[
G_{jk}^i = 0.
\]

The author [4] generalized this result by relaxing the condition (4.11). Now, we propose an elegant generalization of this theorem in the following form:

**Theorem 4.1.** A vector field \( v^i (x^j) \) generates an affine motion in a recurrent Finsler manifold if \( \alpha H_{jkh}^i = a_{jk} \mathcal{B}_h v^j \), \( \alpha \) and \( a_{jk} \) being any non-zero scalar and tensor fields, respectively.

**Proof.** Let us consider a vector field \( v^i (x^j) \) satisfying \( \alpha H_{jkh}^i = a_{jk} \mathcal{B}_h v^j \) in a recurrent Finsler manifold characterized by (4.6). Dividing the equation \( \alpha H_{jkh}^i = a_{jk} \mathcal{B}_h v^j \) by \( \alpha \) and putting \( a_{jk}/\alpha = \bar{a}_{jk} \), we have

\[
H_{jkh}^i = \bar{a}_{jk} \mathcal{B}_h v^j.
\]

Differentiating (4.12) covariantly with respect to \( x^m \) and using (4.6), we have

\[
(\lambda_m \bar{a}_{jk} - \mathcal{B}_m \bar{a}_{jk}) \mathcal{B}_h v^j = \bar{a}_{jk} \mathcal{B}_m \mathcal{B}_h v^j.
\]
Since the tensor field $\bar{a}_{jk}$ is non-vanishing, we may choose a tensor field $f^{jk}$ such that $\bar{a}_{jk} f^{jk} = 1$. Transvecting (4.13) by $f^{jk}$, we have condition (B), where

$$u_m = (\lambda_m \bar{a}_{jk} - \mathcal{R}_m \bar{a}_{jk}) f^{jk}.\label{eq:4.14}$$

Transvecting (4.12) by $v^j$, we get condition (A), where $w_k = \bar{a}_{jk} v^j$. In view of (2.11) and conditions (A) and (B), the Lie derivative of $G_{jk}^i$ with respect to an infinitesimal transformation generated by the vector field $v^j(x^i)$ is given by

$$\mathcal{L} G_{jk}^i = (u_j + w_j) \mathcal{R}_k v^j + G_{jk,kr} \mathcal{R}_s v^r \dot{x}^s.\label{eq:4.15}$$

Since $G_{jk}^i$ and $G_{jkr}^i$ are symmetric in the indices $j$ and $k$, equation (4.14) shows the symmetry of $(u_j + w_j) \mathcal{R}_k v^j$ in $j$ and $k$. Hence

$$u_j + w_j \mathcal{R}_k v^j = (u_k + w_k) \mathcal{R}_j v^j,\label{eq:4.16}$$

which implies at least one of the following:

(a) $\mathcal{R}_k v^j = 0$,  
(b) $u_j + w_j = 0$,  
(c) $\mathcal{R}_k v^j = (u_k + w_k) X^j$,

where $X^j$ is some non-zero vector field. (4.16a) cannot hold as it, in view of (4.12), leads to $H_{jkh}^i = 0$, a contradiction to the fact that the curvature tensor $H_{jkh}^i$ of a recurrent Finsler manifold is non-vanishing. If (4.16c) holds, (4.12) may be written as

$$H_{jkh}^i = f_{jkh} v^j, \quad \text{where} \quad f_{jkh} = \bar{a}_{jk} (u_h + w_h).$$

Thus, the curvature tensor $H_{jkh}^i$ is written as the product of a tensor and a vector. This contradicts the Theorem of [6] which states that the curvature tensor of a non-flat Finsler manifold cannot be written as the product of a tensor and a vector. Therefore, (4.16c) does not hold. Hence we have (4.16b). This reduces (4.14) to

$$\mathcal{L} G_{jk}^i = G_{jk,kr} \mathcal{R}_s v^r \dot{x}^s.\label{eq:4.17}$$

Transvecting (4.17) by $\dot{x}^k$ and using (2.1b) and (2.2), we have $\mathcal{L} G_{jk}^i = 0$. Differentiating $\mathcal{L} G_{jk}^i = 0$ partially with respect to $\dot{x}^k$ and using the commutation formula (2.13), we get $\mathcal{L} \dot{x}^k G_{jk}^i = 0$, which, in view of (2.1c), gives $\mathcal{L} G_{jk}^i = 0$. Thus, the vector field $v^j(x^i)$ generates an affine motion.

Proceeding in a similar way, we may prove

**Theorem 4.2.** A vector field $v^j(x^i)$ generates an affine motion in a symmetric Finsler manifold, characterized by $\mathcal{R}_m H_{jkh}^i = 0$, if $\alpha H_{jkh}^i = a_{jk} \mathcal{R}_h v^j$, where $\alpha$ and $a_{jk}$ are non-zero scalar and tensor fields, respectively.

Let us consider a vector field $v^j(x^i)$ satisfying $\alpha H_{jkh}^i = a_{jk} \mathcal{R}_h v^j$ in a recurrent Finsler manifold. According to Theorem 4.1, $v^j(x^i)$ generates an affine motion. Since every affine motion is a curvature collineation, we have
\( \mathcal{L}H^i_{jkh} = 0 \), which, in view of (2.10) and (4.6), can be written as

\[
(4.18) \quad L H^i_{jkh} - H^i_{jkh} \partial_v v^i + H^i_{rkh} \partial_j v^j + H^i_{jrk} H^j_{ih} \partial_h v^r + (\partial_r H^i_{jkh}) \partial_k v^r \dot{x}^i = 0,
\]

where \( L = \lambda \circ \partial_v \). Transvecting (4.18) by \( a_{lm} \) and using \( \alpha H^i_{jkh} = a_{jk} \partial_h v^i \), we have

\[
(4.19) \quad L a_{lm} H^i_{jkh} - \alpha \partial_v H^i_{jkh} - H^i_{mr} H^r_{lmj} - H^i_{mjr} H^r_{lkm} - H^i_{mkr} H^r_{lhm} - (\partial_r H^i_{jkh}) H^r_{lmj} = 0.
\]

Differentiating (4.6) covariantly with respect to \( x^i \) and taking skew-symmetric part with respect to the indices \( l \) and \( m \), we get

\[
2 \partial_{\{l} \partial_{m\}} H^i_{jkh} = 2 (\partial_{\{l} \lambda_{m\}}) H^i_{jkh},
\]

which, in view of the commutation formula exhibited by (2.5) and (4.10), gives

\[
(4.20) \quad H^i_{jkh} H^l_{mjr} - H^i_{brk} H^b_{lkm} - H^i_{bkr} H^b_{lkm} - (\partial_r H^i_{jkh}) H^r_{lmj} = A_{lm} H^i_{jkh}.
\]

From (4.19) and (4.20) we get

\[
(4.21) \quad L a_{lm} = \alpha A_{lm},
\]

since \( H^i_{jkh} \neq 0 \). Since \( a_{lm} \) and \( \alpha \) are non-zero tensor and scalar fields, respectively, it is obvious from (4.21) that the vanishing of \( L \) implies and is implied by the vanishing of the tensor \( A_{lm} \). Thus, we have

**Theorem 4.3.** If a recurrent Finsler manifold admits a vector field \( v^i(x^i) \) satisfying \( \alpha H^i_{jkh} = a_{jk} \partial_h v^i \), then the vector field \( v^i \) is orthogonal to the recurrence vector \( \lambda_m \) if and only if the tensor field \( A_{lm} \) vanishes identically.

Let us consider a vector field \( v^i(x^i) \) satisfying \( \alpha H^i_{jkh} = a_{jk} \partial_h v^i \). This vector field, in view of Theorem 4.1, generates an affine motion. Since the recurrence vector is a Lie invariant under an affine motion, we have \( \mathcal{L} \lambda_m = 0 \), which, in view of (2.10) and (4.10), may be written as

\[
(4.22) \quad v^i A_{rm} + \partial_m L = 0,
\]

the recurrence vector \( \lambda_m \) being independent of \( \dot{x}^i \) (see [5]). From (4.22) it is clear that the covariant derivative of the scalar \( L \) vanishes if and only if \( v^i A_{rm} = 0 \). We claim that \( v^i A_{rm} = 0 \) if and only if \( A_{rm} = 0 \). Suppose \( v^i A_{rm} = 0 \). If \( A_{rm} \neq 0 \), then the tensor \( A_{jk} \) satisfies (see [3])

\[
\lambda_m A_{jk} + \lambda_j A_{km} + \lambda_k A_{mj} = 0,
\]

which after transvection by \( v^m \) and then summing for \( m \), takes the form

\[
(4.23) \quad L A_{jk} + \lambda_j A_{km} v^m + \lambda_k A_{mj} v^m = 0.
\]
In view of \( \nu' A_{rm} = 0 \) and skew-symmetry of \( A_{jk} \), (4.23) gives \( LA_{jk} = 0 \), whence \( L = 0 \), which contradicts Theorem 4.3. Thus, \( \nu' A_{rm} = 0 \) implies \( A_{jk} = 0 \). Conversely, if \( A_{jk} = 0 \), we have \( \nu' A_{rm} = 0 \) identically. Hence the conditions \( A_{jk} = 0 \) and \( \nu' A_{rm} = 0 \) are equivalent. Thus we may conclude

**Theorem 4.4.** For a vector field \( \nu'(x') \) satisfying \( \alpha H_{jkh} = a_{jk} \mathcal{B}_h \nu^i \) in a recurrent Finsler manifold, the following conditions are equivalent:

(i) \( \mathcal{B}_m L = 0 \),
(ii) \( \nu' A_{rm} = 0 \),
(iii) \( A_{jk} = 0 \).

If \( L \neq 0 \), we have \( A_{jk} \neq 0 \) and both satisfy (4.21). Putting the value of \( a_{lm} \) from (4.21) in \( \alpha H_{jkh} = a_{jk} \mathcal{B}_h \nu^i \) and bearing in mind that \( \alpha \neq 0 \), we have (4.7). Hence, in this case, Theorem 4.1 takes the form

**Theorem 4.5.** If a vector field \( \nu' \) which is not orthogonal to the recurrence vector \( (L \neq 0) \) satisfies \( LH_{jkh} = A_{jk} \mathcal{B}_h \nu^i \), then it generates an affine motion in the recurrent Finsler manifold.

**References**


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