ADDENDUM TO
"ČECH AND STEENROD HOMOTOPY THEORY
WITH APPLICATIONS TO GEOMETRIC TOPOLOGY"

BY

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The proof of Lemma (3.3.32) of Edwards and the author's paper [2] is based on the earlier Lemma (3.3.31) in [2] which is incorrect. In this note* we give a new proof of Lemma (3.3.32) in [2], which allows us to remove one of the conditions in [2], i.e.

CONDITION N3. Every object in C is cofibrant or every object is fibrant, for pro-C to be a closed model category.

Thus, the new proof shows that pro-maps (C) are a closed model category for C being simplicial sets, topological spaces, simplicial groups, etc.

PROPOSITION (Lemma (3.3.32), [2]). Let \( f: X \to Y \) be a trivial fibration and let \( g: Y \to Z \) be a trivial cofibration. Then the composite \( g \circ f \) is a weak equivalence.

Proof. By Proposition (3.3.36) in [2] there is a levelwise trivial cofibration \( g': Y' \to Z' \) (in some \( C' \)) isomorphic to \( g \) (in maps pro-C). By Propositions (3.3.15) and (3.3.26) in [2] (Quillen's [4] Axiom M6 for pro-C and Axiom M6 for fibrations in pro-C), the composite mapping

\[
f': X \xrightarrow{f} Y \xrightarrow{\simeq} Y'
\]

is a trivial fibration. Reindex \( f'([2], Section 2.1, see also Appendix in [1], and [3]) to obtain a level map \( f'': X'' \to Y'' \) in some \( C^K \), where \( K \) is a cofinite, strongly directed set. Extend this new indexing to obtain a sequence

\[
X'' \xrightarrow{f''} Y'' \xrightarrow{g''} Z''
\]

in \( C^K \). Note that \( f'' \) is a trivial fibration in pro-C and \( g'' \) is a levelwise trivial cofibration in \( C^K \). By Proposition (3.2.24) in [2] (Quillen's [4] Axiom M2 for \( C^K \)), \( f \) factors as

\[
X'' \xrightarrow{f} W \xrightarrow{p} Y''
\]

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For the second claim, consider a commutative solid-arrow diagram

A filler \( \psi \) exists by Proposition (3.3.26) in [2]. Then \( \psi h_1 \) extends \( \bar{\psi} \) and covers \( \varphi \). Use \( H \) to deform \( \psi h_1 \) into the required filler \( \psi' \) as follows: \( \psi' \) is the “1-end” of the filler \( K \) in the diagram.

This completes the proof.

REFERENCES


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