Proximity in Coalition Building

Julien Reynaud∗Fabien Lange†Łukasz Gątarek‡Christian Thimann§

Submitted: 21.12.2011, Accepted: 11.05.2012

Abstract

Voting power methodology offers insights to understand coalition building in collective decision making. This paper proposes a new measure of voting power inspired from Banzhaf (1965) accounting for the proximity between voters by capturing how often they appear in winning coalitions together. Using this proximity index, we introduce a notion of relative linkages among coalition participants as determinant of coalition building. We propose an application to the governance structure of the International Monetary Fund, with linkages being represented by bilateral volumes of trade between voters. The results are able to explain several important features of the functioning of this particular voting body, and may be useful for other applications in international politics.

Keywords: voting power index, coalition building, International Monetary Fund, linkage, proximity

JEL Classification: C71, F33.
1 Introduction

Collective decision making is an everyday phenomenon. In some situations formal voting procedures exist, in other situations decisions are made by consensus. Majority thresholds are most often simple majorities, but higher thresholds can exist for far-reaching decisions. Sometimes, the votes of individuals are weighted, in other contexts one person carries one vote. Despite many differences in setup, most of these situations have one element in common, namely that a single person, group, party or constituency alone does not hold a majority; hence, coalitions are necessary to reach the required majority and take decisions.

Political economy theory, through cooperative game analysis, offers insights to understand coalition building. A first insight is that the notional size of a constituency (individual, country or group) can differ from its effective weight in the decision process. Theory therefore distinguishes the former, also called voting share, and the latter, also called voting power. Voting power can be larger or smaller than the notional voting share, and the difference depends on how important a single constituency is in the overall context of coalition building. This insight into voting power is well-known in the context of domestic politics. Suppose for example that there are three parties, Red, Rose and Green, in a Parliament of 100 seats, with the first two having 49 seats, and the Green party having 2 seats. If a majority of 51% is needed, each party needs a coalition with one of the other parties to win the vote. All the parties are therefore equally critical in terms of their capacity to build a majority coalition, having the same voting power of 33.3%. For the larger parties, voting power is lower than their notional weights and the contrary holds true for the smallest party. A second insight from coalition theory is that voting power depends also on the structure of the decision-making body, i.e. the number of constituencies, the distribution of votes and the majority threshold. Again, in the example of a national Parliament, suppose the Rose and Green parties form a new party. This new party would have a majority of 51% and therefore a voting power of 100% as no other party would be needed for a majority. Hence, the new party’s voting power exceeds that of its constituting members, and the voting power of Red would be zero.

The usefulness of formalised voting power indices has been broadly recognized. The most widely used indices so far were developed by Shapley and Shubik (1954) and Banzhaf (1965). Following these seminal works, an important number of scholars have developed a battery of indices to account for stability of the coalitions by taking into account different measures of the size of voters and coalitions (e.g. Johnston, 1978; Deegan and Packel, 1978; and Colomer and Martinez, 1995). Others have focused on defining voting power taking into account ex ante or a priori connections between voters as a determinant of coalition building (e.g. between others Myerson, 1977; Shenoy, 1982; Owen, 1982; Edelman, 1997; Calvo and Lasaga, 1997; Bilbao, 2000; and Perlinger, 2000). None of these indices, however, account for the proximity between voters within the game structure. Indeed, it might well be the case that some voters are not able to frequently build winning coalition together. For example,
a voter $i$ might coalesce more often with voter $j$ than with voter $k$; and this may not depend only on the relative importance of voter $j$ compared to the one of $k$ but also on the number of winning coalitions voter $i$ and $j$ share together compared to the ones voter $i$ shares with voter $k$. In this paper, we develop this idea, building on the Banzhaf (1965) index, proposing a new voting power index accounting for proximity as the frequency voters are able to form coalitions together should be taken into account while computing voting power.

Furthermore, we introduce linkages between voters in the proximity index as a relative determinant of coalition building. Although the measure of linkages in itself is an \textit{a priori} one, i.e. we do not model linkages \textit{per se}, our contribution is to introduce linkages at the coalition level while it has been until now introduced at the individual level of agents. Indeed, we argue that coalition building does not depend only on the linkages between two voters but rather between all coalition participants, and this is taken into account via our proximity index.

Finally, we present an application to international politics on the governance structure of the International Monetary Fund (the Fund or the IMF). The Fund has become over time the most prominent institution for global governance. As such, its representation and governance structures are increasingly called into question, and countries’ quotas are therefore central in the discussions. Consequently, we apply our proximity index to the constituencies represented at the Fund’s Executive Board, its decision making body. The results of our computations highlight that proximity between voters matters since they do not build coalitions equiprobably with other voters. Indeed, some voters appear more often together in winning coalitions than others.

For example, we found that Japan’s voting power rely mainly on the fact that it is an important coalescer for the USA but not for other constituencies. In this exercise, we also argue that bilateral volumes of trade is a good indicator reflecting the importance of country relationships in building coalitions. We therefore also compute our proximity index introducing this notion of linkages. Our results are able to explain several important features of IMF Executive Board functioning. For example we are able to differentiate between Brazil and Iran, the latter getting less voting power than the former, although they have similar voting shares.

Our work heavily draws on Gatarek \textit{et al.} (2007). However the subsequent references in the text to \textit{voting power index} have been restrained. The reason being that the index provided is not \textit{perse} a voting power index. The underlying reason is actually dependent on the definition of power. In our understanding, ability of an agent to participate and build coalition is directly linked to its voting power, but the term voting power has been rather focusing on pivotal players in the literature. We therefore decided to refer to a measure of power that is the \textit{proximity index} we present in this paper. Introducing this new measure we have developed a full section on the axiomatic properties of the proximity index. Finally, because our index focuses on proximity in coalition building, in the empirical part of the paper we have focused on identifying coalition members. In particular, we have exposed the novelty of our
The remainder of this paper is organized as follows. In section 2, we introduce voting power methodology, our proximity index and develop its properties. We also introduce linkages between voters in our proximity set-up. In section 3, we provide an application of our proximity index to the IMF and present also the results of the computation accounting for linkages among constituencies in the IMF, the latter being proxied by the bilateral volumes of trade. Section 4 concludes.

2 An index of proximity

2.1 Voting power methodology

Voting power methodology is a valuable tool for policy analysis because it captures numerically the distribution of power in collective decision-making processes. Its usefulness is twofold: first, it reveals the distribution of effective power between voters, which can be quite different from the notional distribution of voting shares and which is more relevant in understanding the effective decision-making. In this respect, by comparing notional shares with effective powers, voting power indices allow to evaluate the fairness of voting rules and provide positive as well as normative tools in understanding the practices and outcomes by voting bodies. Second, voting power analysis can quantify the impact of changes in voting procedures on the voting power of individual or groups of voters. This allows assessing the implications of procedural or institutional changes on the overall voting body. Voting power methodology was particularly prominent in international politics in assessing the consequences on the EU’s and ECB’s internal voting rules as a result of the enlargement rounds (see among others Kirman et al., 1995; Lane and Maeland, 1995; Bindseil and Hantke, 1997; König and Bräuninger, 1997; Laruelle and Widgrén, 1997; Holler and Widgrén, 1997; Steunenberg et al., 1999; Baldwin et al., 2000 and Leech, 2001).

Formally a voting body can be represented as a set $N$, which contains the $n$ voters in the voting body, with $N = \{1, 2, ..., n\}$ representing the set of voters. Let the set $W$ include all winning subsets of $N$. Three reasonable restrictions are made (Straffin, 1978): first, elements of $W$ must contain one voter, i.e. the empty set of members cannot ensure acceptance. Second, the entire set of voters $N$ ensure acceptance. Third, if $S$ and $T$ are subsets of $N$, with $S \in W$ and $S \subset T$ then $T \in W$. This implies that if $S$ can ensure acceptance and $T$ contains all the members of $S$, then $T$ can also ensure acceptance. Hence the total number of possible subsets is $2^n$. If these three conditions are satisfied, a simple game in characteristic function form is then defined as a pair $(N,v)$ s.t. $S : v(S) = 1$ if $S$ is winning and $v(S) = 0$ otherwise.

In many voting bodies, voters may have weights, represented by their voting shares. The voting weights for $N$ with a specified vote threshold $q$ are denoted as $[q; w_1, w_2, ..., w_n]$. Therefore, $v(S) = 1$ iff $[S \in W \Leftrightarrow \sum_{i \in S} w_i \geq q]$, otherwise $v(S) = 0$. A voter $i$ is said to be critical when she has a negative swing in a coalition.
Proximity in Coalition Building

$S$ if $v(S) = 1$, but $v(S \setminus i) = 0$. In order to avoid heavy notations, we will omit braces to write a singleton coalition, writing e.g. $S \setminus i$ for the coalition $S$ deprived of the player $i$. In other words, if voter $i$ withdraws her support, she can turn a winning coalition into a losing one.

The most widely used index is the so-called normalized Banzhaf index (1965) that calculates the share of a critical voter’s negative swings in the overall number of negative swings. Hence, it measures how important a given voter is among the group of critical voters that can all bring down winning coalitions. For any player $i \in N$, the normalized Banzhaf index, denoted $\Omega_i^B$, is then defined as follows:

$$\Omega_i^B := \frac{\sum_{S \subseteq N} [v(S) - v(S \setminus i)]}{\sum_{j \in N} \sum_{S \subseteq N} [v(S) - v(S \setminus j)]}$$

Following Banzhaf, scholars developed indices that, at least indirectly, try to account for the stability of coalitions. There are three main proposals in the literature that we present briefly to put our work in context: Johnston (1978) for example, refined the Banzhaf calculation method to account for the overall number of critical voters in the winning coalitions. Specifically, he aimed at capturing the fact that in a coalition with only one critical voter, the latter is more powerful than in a coalition where there are more critical voters, because the threat of one to withdraw reduces the power of the others. Deegan and Packel (1978) proposed an index to capture another element associated with the costs of maintaining a coalition arguing that the larger a coalition in terms of the number of members, the more vulnerable it is because it is generally more subject to the withdrawal of members. Finally, the index developed by Colomer and Martinez (1995) captures the fact that coalitions are more stable with small critical voters than with larger ones, for any given number of voters. Overall, these indices put different weights to the critical voter’s worth accounting for the size of coalitions or the characteristics of other voters sharing the coalition. In our spirit, the size of coalitions is indeed an important determinant of voting power, yet we go further in that direction at the agent level by accounting for bilateral proximity between voters. We develop hereafter our proximity index.

2.2 Definition of the index of proximity

Our approach aims at providing an index that captures the frequency voters may be able to form coalitions according to the characteristics of the game. Classical voting power indices are capturing individual measures of power and provide no information on the frequency voters are able to coalesce given the structure of the vote, i.e. their voting shares, the size of the voting body and the majority threshold. Therefore, we propose a new measure of voting power: the numerical proximity, that captures how often individual voters appear in a winning coalition together. Formally, our index is
Julien Reynaud et al.

constructed in the following way:

\[ \Omega^P_{ij} := \sum_{S \ni i,j} [v(S) - v(S \setminus i)], \text{ for all players } i, j \in N. \] (2)

Note that \( \Omega^P_{ii} = 0 \). Actually, \( \Omega^P_{ij} \) represents the times \( i \) is critical among coalitions containing player \( j \). We then have to sum these bilateral scores w.r.t. \( j \) to get the normalized version of the index, in the spirit of Banzhaf. Hausken and Mohr (2001) derives a matrix value in the same spirit from the Shapley value and apply it to the European Council of Ministers between 1981 and 1995. For any player \( i \in N \), the normalized proximity index is defined by:

\[ \Omega^P_i := \frac{\sum_{j \in N} \Omega^P_{ij}}{\sum_{k \in N} \sum_{j \in N} \Omega^P_{kj}}. \] (3)

The index provides therefore an indication of proximity between voters defined as the frequency they may build a winning coalition together, when one of them is critical. As a simple example, let us use the following simple game \([6; 4, 3, 2, 1]\). Computing (2) will lead to the following results:

\[
\Omega^P_{ij} = \begin{pmatrix}
A & B & C & D \\
A & 0 & 3 & 3 & 2 \\
B & 2 & 0 & 1 & 2 \\
C & 2 & 1 & 0 & 2 \\
D & 0 & 1 & 1 & 0 \\
\end{pmatrix}
\]

Interestingly, we can see from the computation of (2) that voter B builds more frequently winning coalitions with voter D than with voter C although \( w_C > w_D \). Moreover, voter A does not participate in coalitions when voter D is critical, and therefore we can conclude that voter D is closer, in terms of proximity, with voters B and C than with voter A. We then normalize (2) to obtain (3) and compare the results to other indices (see Table 1). Columns one and two give the name of players and their respective voting shares. Column three gives the number of swings for each voter. Column four presents our proximity index and the rest of the columns exposes the outcomes of computing Banzhaf indices. This little example provides a clear representation of the importance of taking into account proximity between players when estimating voting power. We develop more formal properties of (3) in the following sub-section.
2.3 Properties of the normalized proximity index

A first observation that one can make about this index is that \( \Omega_i^P \) can be written in terms of marginal contributions of player \( i \) in the game, as well as other well-known indices (Shapley-Shubik, Banzhaf, Deegan-Packel, Johnston, Colomer-Martinez, etc.). In other words, for all the cited indices, coalitions where \( i \) is critical are counted, and weighted with some coefficients:

\[
\Omega_i = \sum_{S \ni i} \alpha_S[v(S) - v(S \setminus i)],
\]

where the \( \alpha_S \)'s are some real coefficients. Indeed, this is the case since the double sum \( \sum_{k \in N} \sum_{j \in N} \Omega_{kj}^P \) is a constant that only depends on the game \( v \). In particular, whenever a power index \( \Omega \) can be written in terms of marginal contributions of the voters, \( \Omega \) satisfies the following property (Weber, 1988):

Player \( i \in N \) is said to be null for \( v \) if for any coalition \( S \), \( v(S) = v(S \setminus i) \), i.e., she is never critical.

**Nullity axiom:** for any game \((N,v)\) and any \( i \in N \) null for \( v \), \( \Omega_i = 0 \).

Note that according to the philosophy of the proximity index, the larger is the coalition \( S \) containing \( i \), the bigger is the coefficient \( \alpha_S^i \). In particular, \( \Omega_i^P \) does not depend on the marginal contribution of \( i \) when she is the only player of the coalition, i.e., \( \alpha_i^i = 0 \).

More accurately, the above coefficients \( \alpha_S^i \)'s of the proximity index are:

\[
\alpha_S^i = \frac{|S| - 1}{\sum_{k \in N} \sum_{j \in N} \Omega_{kj}^P}.
\]

Indeed, let \( S \subseteq N \) such that \( S \ni i \). Thus the difference \( [v(S) - v(S \setminus i)] \) appears once in the bilateral score \( \Omega_{ij}^P \), for every \( j \in S \setminus i \).

A consequence of this is that the proximity index satisfies the symmetry property.

<table>
<thead>
<tr>
<th>Voter</th>
<th>Weight</th>
<th>Swings</th>
<th>Proximity</th>
<th>Norm. Banzhaf</th>
<th>Abs. Banzhaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>5</td>
<td>0.400</td>
<td>0.417</td>
<td>0.625</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>3</td>
<td>0.250</td>
<td>0.250</td>
<td>0.375</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>3</td>
<td>0.250</td>
<td>0.250</td>
<td>0.375</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>1</td>
<td>0.100</td>
<td>0.833</td>
<td>0.125</td>
</tr>
</tbody>
</table>

that is to say, does not depend on the labelling of the players. Indeed, the \( p_S^i \)'s do not depend on \( i \).

Last but not least, it is noteworthy that the proximity index satisfies the monotonicity postulate, which is a very reasonable property that power indices should satisfy (Felsenthal and Machover, 1995).
The monotonicity postulate: Let \((N,v)\) be a weighted-voting game \([q;w_1,\ldots,w_n]\). A power index \(\Omega\) satisfies the monotonicity postulate if for all players \(i,j \in N\), \(w_i \leq w_j\) implies \(\Omega_i(v) \leq \Omega_j(v)\).

However, it appears that not all power indices satisfy this very natural property. In particular, the Deegan-Packel and the Colomer-Martinez do not satisfy it.

**Proposition 1** The proximity index satisfies the monotonicity postulate.

**Proof:** Let \(i\) and \(j\) be two players such that \(w_i \leq w_j\). Consider any coalition \(S\) in which \(i\) is decisive. If player \(j\) belongs to \(S\), she/he is also clearly critical in \(S\). Otherwise, \(j\) is critical in \((S \setminus i) \cup j\). Therefore, this describes an injection \(\iota\) between the set \(W_i\) of coalitions in which \(i\) is critical, and the set \(W_j\) of coalitions in which \(j\) is critical: \(\iota : S \mapsto \begin{cases} S & \text{if } S \ni j \\ (S \setminus i) \cup j & \text{else} \end{cases}\). Thus, \(|W_i| \leq |W_j|\). Moreover, \(\iota\) preserves the cardinality. Consequently, \(\sum_{S \in W_i} (|S| - 1) \leq \sum_{S \in W_j} (|S| - 1)\), and thus \(\Omega_i^P \leq \Omega_j^P\).

Table 2: Comparison of classical voting power indices and the proximity index. A strictly minimal winning coalition is a coalition containing only critical voters (see for example Lees and Taylor, 2006).

<table>
<thead>
<tr>
<th>Index</th>
<th>Admissible coalitions</th>
<th>Worth of voter (i) in (S)</th>
<th>Dimension of power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banzhaf</td>
<td>Min. winning</td>
<td>(1)</td>
<td>Normalized (individual)</td>
</tr>
<tr>
<td>Johnston</td>
<td>Min. winning</td>
<td>(1/p, p: \text{number of critical voters in } S)</td>
<td>Normalized (individual)</td>
</tr>
<tr>
<td>Deegan-Packel</td>
<td>Strictly</td>
<td>(1/q, q: \text{number of voters in } S)</td>
<td>Normalized (individual)</td>
</tr>
<tr>
<td>Colomer-Martinez</td>
<td>Min. winning</td>
<td>(w_i: \text{weight of voter } i \text{ in } S)</td>
<td>Normalized (individual)</td>
</tr>
<tr>
<td>Proximity (2)</td>
<td>Min. winning</td>
<td>Number of times (j) appears in coalition when (i) is critical</td>
<td>Bilateral between voters (i) and (j)</td>
</tr>
<tr>
<td>Proximity (3)</td>
<td>Min. winning</td>
<td>Number of voters sharing coalitions when (i) is critical</td>
<td>Normalized (individual)</td>
</tr>
</tbody>
</table>

2.4 Introducing linkages in the proximity index

Another path of research has developed indices introducing preferences or ideologies of voters in the building of coalitions, the so-called preference-based indices. Researchers start by laying out \(a \text{ priori}\) the political space that either pre-connects certain voters in a deterministic fashion or assigns \(ex \text{ ante}\) probabilities to different coalitions.
This then helps to exclude certain combinations or to the contrary makes 'outlier coalitions' more likely. In this strand of research, the approaches of Owen (1977 and 1982), Myerson (1977) and Bilbao (2000) consist for example of taking into account pre-existing coalitions’ structures between similar voters. Shenoy (1982) follows this line of research introducing ideology in a Banzhaf index using geometry to define political space of voters. Stenlund et al. (1985) provided an extension of the power index approach to take into account how actors behave in order to restrict the coalition possibilities. In a first step, they examine the power in the Swedish Riksdag considering more than 5000 decisions made during the period covering the shifts in government in 1976 and 1978. In the second step, they use the obtained relative frequencies of historical decisions to proxy the probabilities of the various parties to be a member of a (winning) coalition. Perliger (2000) extends Edelman’s (1997) model and put different weights on the allowable coalitions. The underlying idea is that the possibility of coalition building is given by aligning coalition on an ideological spectrum. Along the same lines, Calvo and Lasaga (1997) and Calvo, Lasaga and van den Nouweland (1999) use probabilistic graphs to define coalition building in the Spanish Parliament using probabilities defined by a survey. Finally, Aleskerov (2006) defines preferences to go together in a coalition as a linear order and thus is able to rank preferred coalitions.

Generally, researchers have modelled preferences as probabilities of forming coalition between two voters/parties. However, none of them deal with the fact that preferences or ideologies depend also on the composition of other voters sharing the coalitions and that some voters may appear together in coalitions more often than others, i.e. taking into account proximity between voters. Yet, the very notion of coalition implies the principle of commonly shared interests. Gupta (2003) argues that the purpose of these shared interests is to create a certain degree of linkages, which do not necessarily involve a formal commitment or a commonality of purposes among members of the coalition. In other words, linkages between voters may vary over time, across members and importantly across outcomes. One should thus model linkages at the coalition level rather than at the agent level. In this respect, our proximity index already provides the first step, by identifying voters sharing coalitions together. Consequently, we introduce, on top of the proximity concept, a measure of linkages between voters accounting for other voters in the coalition.

As we discussed above, modelling linkages between voters is subjective. Taking this as given, our proxy of linkage between voters is exogenous, i.e. the intensity of linkages between voters is not per se modelled. Nonetheless, what is modelled is the introduction of a measure of linkages that is relative to each coalition and each voter. More precisely, while the measure of linkages in itself may be defined by very different proxies, in our application we used bilateral volumes of trade as a proxy of coalition building determinants in International Financial Institution, our contribution is to introduce linkage at the coalition level accounting for proximity between voters. For example, two voters might have strong linkages to build coalitions together but might
not appear often in coalitions together. It might also be the case that some voters appear often in coalitions together but do not share strong linkages. Moreover, coalition building does not depend only on the linkages between two voters but rather on linkages shared among all coalition participants. In a proximity framework, this is modelled as the relative linkages shared by all members of the coalition with the critical voter. More formally, suppose that a \(|N\times N|\) matrix \(M\) is given, representing linkages between players. For example, \(N\) may be a set of countries, and for any \(i,j \in N\), \(M\) may represent the worth of trade between them: \(M_{ij}\) can be the worth of all goods imported by country \(i\) from country \(j\). We suppose that for all players \(i \in N\), \(M_{ii} = 0\).

On the same principle as the construction of the proximity index, we introduce a relative notion of linkages as follows:

\[
\Omega_{ij}^L := \sum_{S\ni i,j} \Omega_{ij}^S [v(S) - v(S \setminus i)], \text{ for all players } i,j \in N, \quad (4)
\]

where \(\Omega_{ij}^S := \frac{M_{ij}}{\sum_{k \in S} M_{kj}}\), is the bilateral normalized relative notion of linkages between voters.

For any player \(i \in N\), the normalized proximity index including linkages is then defined by:

\[
\Omega_i^L := \sum_{j \in N} \sum_{k \in N} \frac{\Omega_{ij}^L}{\Omega_{kj}^L}. \quad (5)
\]

We provide a formal example of this notion in the annex and in the following section where we present an application of our proximity index.

3 Application of the proximity index to the IMF Executive Board

IMF quotas are an important issue in IMF governance because they constitute the building blocks for many aspects of the IMF and its operations. A country’s quota directly translates into voting rights because the number of votes a country has in the Fund is based primarily on the size of its quota. In addition, a member’s quota fixes how much that country may be called upon to lend to other members through the Fund. Finally, it also determines how much a member can borrow from the Fund. These roles indisputably imply that proximity and linkages between voters are strategic elements when it comes to voting since the action of voting in the IMF includes the notion of an agreement to politically and/or financially support another member country.

The representation and governance structures of the Fund are increasingly called into question. Among members, pressure for change is growing. Emerging market
and developing countries consider that their "under-representation undermines the legitimacy" of the IMF (G24, 2005). In his report for the 2005 Annual Meetings, IMF Managing Director de Rato argued that "governance imbalances in the Fund now rival current account imbalances", adding that "neither imbalance is sustainable" (IMF, 2006). Similar concerns are expressed in academic circles, as captured by Ted Truman, who argues that "issues of governance are substantively crucial if the IMF is to regain the trust and respect of all of its member nations" (Truman, 2005). The purpose of this section is therefore to provide on top of a simulation exercise of our proximity index, a new view of the representation at the Fund given proximity between voters and given their relative linkages.

Few scholars have used voting power to study IMF’s governance issues. Between others, Leech (2002), Leech and Leech (2005), Alonso-Meijide and Bowles (2005) and Bini Smaghi (2006) found that the voting structure of the IMF EB gives disproportionate power to the United States at the expense of all other members. This means that for the United States, effective voting power exceeds their notional voting share, whereas for all other 23 constituencies the opposite result holds. Overall, these studies provide an interesting view of the relative voting power of constituencies but they do not inform us on the frequency constituencies are bound to build coalition together.

Figure 1: Computation of the proximity index for the IMF Executive Board
We start therefore our exercise by computing (2) using the voting structure of the IMF Executive Board. Note that all simulations are based on a simple majority threshold. Figure 1 plots for each constituency the difference of their proximity score to the mean of the proximity scores for all constituencies sharing a winning coalition with the critical voter. More precisely, critical constituencies are listed on the x axis while voters sharing coalitions with the latter are listed on the y axis. The difference to the mean is on the z axis in percentage points. Quite clearly, the domination of the American constituency is significant as the number of constituencies sharing winning coalitions with them are significantly higher than other critical voters. Moreover, proximity appears very obvious since some voters appear more often (than the average voter, in this representation) in coalitions with others. For example, Japan (JPN) share a more important number of coalitions with the American constituency than with the Rwandan and the Indian ones. Another good example is the case of the Swiss constituency that appears in rather large number of coalitions when the Dutch constituency is critical.

To put these results in perspective, we propose a ranking analysis of (2) in Table 3. For each critical constituency, we have ranked the other constituencies by decreasing order of appearance. The table should be read as follows taking the example of the USA being critical on the second column: Japan is the constituency with which the USA are sharing the most important number of winning coalitions, followed by Germany, Belgium, etc. It is therefore now possible to clearly show how proximity matters. It is also apparent from this ranking analysis that the USA are always ranked first in the proximity index, reinforcing thus their importance in the process of coalition building. However the original ranking (see first column of Table 4), i.e. the distribution of voting shares, is not respected for other constituencies. Quite noticeably, the case of Japan is interesting (JPN is in bold in Table 3). While the country’s constituency is ranked second with 6.02% of voting shares, it is never ranked second using our proximity index. At best, Japan is ranked third when Peru’s constituency is critical. At worst, it is four times ranked 20th. This shows that given its size, the distribution of voting shares, the number of voters and the majority threshold, Japan appears relatively less often in coalitions when other members are critical although it is ranked second in terms of voting share. In the case of Japan, we can argue that part of its voting power is therefore due to the fact that it appears more often than others in coalition when the USA are critical, since it does not appear to coalesce significantly with other players. Proximity appears therefore as an important determinant of coalition building in the sense that some voters are restricted to build coalitions together more or less often depending on their relative size.

We now introduce linkage in our proximity index. For this illustration, we choose to compute (3), i.e. the normalized index (see table 4), using bilateral volumes of trade as a proxy for linkages between voters (see annex for a discussion of this issue). The domination of the USA is increased compared to the normalized Banzhaf index. Indeed, the USA appear in a large number of winning coalitions including large players...
### Table 3: Ranking analysis of the proximity index computation

<table>
<thead>
<tr>
<th></th>
<th>USA</th>
<th>JPN</th>
<th>DEU</th>
<th>BEL</th>
<th>FRA</th>
<th>GBR</th>
<th>NLD</th>
<th>VEN</th>
<th>ITA</th>
<th>AUS</th>
<th>GHQ</th>
<th>CAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>JPN</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
<td>USA</td>
</tr>
<tr>
<td>2</td>
<td>DEU</td>
<td>PER</td>
<td>VEN</td>
<td>ITA</td>
<td>VEN</td>
<td>VEN</td>
<td>GE</td>
<td>NLD</td>
<td>CAN</td>
<td>2IDN</td>
<td>NLD</td>
<td>2PER</td>
</tr>
<tr>
<td>3</td>
<td>BEL</td>
<td>CHE</td>
<td>CHE</td>
<td>CHE</td>
<td>CHN</td>
<td>CHN</td>
<td>VEN</td>
<td>CHN</td>
<td>BEL</td>
<td>3CHE</td>
<td>CHE</td>
<td>3VEN</td>
</tr>
<tr>
<td>4</td>
<td>FRA</td>
<td>CAN</td>
<td>ITA</td>
<td>4ITA</td>
<td>KEN</td>
<td>GBR</td>
<td>4FRA</td>
<td>CHN</td>
<td>DEU</td>
<td>4RWA</td>
<td>PER</td>
<td>4DEU</td>
</tr>
<tr>
<td>5</td>
<td>GBR</td>
<td>ITA</td>
<td>FIN</td>
<td>FRA</td>
<td>5RUS</td>
<td>RUS</td>
<td>RUS</td>
<td>BEL</td>
<td>5RUS</td>
<td>5BEL</td>
<td>5RWA</td>
<td>5SAU</td>
</tr>
<tr>
<td>6</td>
<td>NLD</td>
<td>CHN</td>
<td>6RUS</td>
<td>6GBR</td>
<td>6GBR</td>
<td>6BEL</td>
<td>6BEL</td>
<td>6IND</td>
<td>6FIN</td>
<td>6DEU</td>
<td>6BRA</td>
<td>6FRA</td>
</tr>
<tr>
<td>7</td>
<td>VEN</td>
<td>RUS</td>
<td>7BEL</td>
<td>7DEU</td>
<td>7DEU</td>
<td>7DEU</td>
<td>7SAU</td>
<td>7FRA</td>
<td>7IND</td>
<td>7HRN</td>
<td>7GBR</td>
<td>7DEU</td>
</tr>
<tr>
<td>8</td>
<td>ITA</td>
<td>NLD</td>
<td>8FRA</td>
<td>8SAU</td>
<td>8CAN</td>
<td>8CAN</td>
<td>8EGY</td>
<td>8GBR</td>
<td>8AUS</td>
<td>8ITA</td>
<td>8KEN</td>
<td>8FRA</td>
</tr>
<tr>
<td>9</td>
<td>AUS</td>
<td>BEL</td>
<td>9GBR</td>
<td>9CHN</td>
<td>9IDN</td>
<td>9IDN</td>
<td>9KEN</td>
<td>9BRA</td>
<td>9JPN</td>
<td>9EGY</td>
<td>9FIN</td>
<td>9GBR</td>
</tr>
<tr>
<td>10</td>
<td>CAN</td>
<td>IND</td>
<td>10EGY</td>
<td>10IDN</td>
<td>10ITA</td>
<td>10ITA</td>
<td>10RUS</td>
<td>10IRN</td>
<td>10VHQ</td>
<td>10SAU</td>
<td>10BEL</td>
<td>10BRA</td>
</tr>
<tr>
<td>11</td>
<td>GHQ</td>
<td>11DEU</td>
<td>11AUS</td>
<td>11VEN</td>
<td>11RWA</td>
<td>11RWA</td>
<td>11JPN</td>
<td>11NLD</td>
<td>11VEN</td>
<td>11ITA</td>
<td>11CHN</td>
<td>11RWA</td>
</tr>
<tr>
<td>12</td>
<td>FIN</td>
<td>12FRA</td>
<td>12CAN</td>
<td>12FIN</td>
<td>12EGY</td>
<td>12EGY</td>
<td>12DEU</td>
<td>12ITA</td>
<td>12FRA</td>
<td>12RUS</td>
<td>12BRA</td>
<td>12NLD</td>
</tr>
<tr>
<td>13</td>
<td>EGY</td>
<td>13CHN</td>
<td>13FRA</td>
<td>13SAU</td>
<td>13BRA</td>
<td>13JPN</td>
<td>13FIN</td>
<td>13IRN</td>
<td>13RUS</td>
<td>14IDN</td>
<td>14SAU</td>
<td>14SAU</td>
</tr>
<tr>
<td>14</td>
<td>KEQ</td>
<td>15IDN</td>
<td>15IDN</td>
<td>15CHN</td>
<td>15RUS</td>
<td>15RUS</td>
<td>15BEL</td>
<td>15RWA</td>
<td>15SAU</td>
<td>15AUS</td>
<td>15SAU</td>
<td>15AUS</td>
</tr>
<tr>
<td>15</td>
<td>KEQ</td>
<td>15IT</td>
<td>15RUS</td>
<td>15BEL</td>
<td>15RWA</td>
<td>15RWA</td>
<td>15SAU</td>
<td>15FRA</td>
<td>15RUS</td>
<td>15IDN</td>
<td>15IDN</td>
<td>15IDN</td>
</tr>
<tr>
<td>16</td>
<td>SAU</td>
<td>16BRA</td>
<td>16JPN</td>
<td>16BRA</td>
<td>16NLD</td>
<td>16JPN</td>
<td>16IDN</td>
<td>16NLD</td>
<td>16JPN</td>
<td>16BRA</td>
<td>16NLD</td>
<td>16JPN</td>
</tr>
<tr>
<td>17</td>
<td>RUS</td>
<td>17BRA</td>
<td>17BRA</td>
<td>17BRA</td>
<td>17JPN</td>
<td>17RWA</td>
<td>17BRA</td>
<td>17FRA</td>
<td>17EGY</td>
<td>17SAU</td>
<td>17CHN</td>
<td>17RWA</td>
</tr>
<tr>
<td>18</td>
<td>BRA</td>
<td>18VEN</td>
<td>18IRN</td>
<td>18EGY</td>
<td>18KEN</td>
<td>18KEN</td>
<td>18IDN</td>
<td>18IDN</td>
<td>18IRN</td>
<td>18SAU</td>
<td>18SAU</td>
<td>18JPN</td>
</tr>
<tr>
<td>19</td>
<td>IRN</td>
<td>19KEQ</td>
<td>19IRN</td>
<td>19SAU</td>
<td>19IRN</td>
<td>19IRN</td>
<td>19IRN</td>
<td>19IRN</td>
<td>19IRN</td>
<td>19IRN</td>
<td>19IRN</td>
<td>19IRN</td>
</tr>
<tr>
<td>20</td>
<td>IRN</td>
<td>20FRA</td>
<td>20BRA</td>
<td>20IRN</td>
<td>20IRN</td>
<td>20IRN</td>
<td>20IRN</td>
<td>20IRN</td>
<td>20IRN</td>
<td>20IRN</td>
<td>20IRN</td>
<td>20IRN</td>
</tr>
<tr>
<td>21</td>
<td>ITA</td>
<td>21BRA</td>
<td>21CHN</td>
<td>21IDN</td>
<td>21RWA</td>
<td>21RWA</td>
<td>21RWA</td>
<td>21RWA</td>
<td>21RWA</td>
<td>21RWA</td>
<td>21RWA</td>
<td>21RWA</td>
</tr>
<tr>
<td>22</td>
<td>PER</td>
<td>22BRA</td>
<td>22IDN</td>
<td>22CAN</td>
<td>22BRA</td>
<td>22BRA</td>
<td>22BRA</td>
<td>22BRA</td>
<td>22BRA</td>
<td>22BRA</td>
<td>22BRA</td>
<td>22BRA</td>
</tr>
<tr>
<td>23</td>
<td>RWA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
<td>23BRA</td>
</tr>
</tbody>
</table>

with which they share relatively important linkages. Interestingly, some constituencies are emerging, in the sense that they gain in ranking compared with their voting share ranking: the Canadian, the Venezuelan and the Brazilian ones. Introducing linkage permits therefore to identify the fact that these constituencies are more likely to...

---

123 J. Reynaud et al.  
CEJEME 3: 111-132 (2011)
Table 4: Voting shares, Banzhaf and proximity with linkage indices and ranking analysis in the IMF Executive Board

<table>
<thead>
<tr>
<th>Rank</th>
<th>Voting share</th>
<th>Country</th>
<th>Norm. Banzhaf</th>
<th>Norm. proximity with linkage</th>
<th>Ranking of proximity with linkage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.80</td>
<td>USA</td>
<td>20.93</td>
<td>25.56</td>
<td>USA</td>
</tr>
<tr>
<td>2</td>
<td>6.02</td>
<td>JPN</td>
<td>5.73</td>
<td>5.86</td>
<td>CAN</td>
</tr>
<tr>
<td>3</td>
<td>5.88</td>
<td>DEU</td>
<td>5.60</td>
<td>4.78</td>
<td>JPN</td>
</tr>
<tr>
<td>4</td>
<td>5.15</td>
<td>BEL</td>
<td>4.90</td>
<td>4.22</td>
<td>VEN</td>
</tr>
<tr>
<td>5</td>
<td>4.86</td>
<td>FRA</td>
<td>4.62</td>
<td>3.64</td>
<td>DEU</td>
</tr>
<tr>
<td>6</td>
<td>4.86</td>
<td>GBR</td>
<td>4.62</td>
<td>4.00</td>
<td>BEL</td>
</tr>
<tr>
<td>7</td>
<td>4.76</td>
<td>NLD</td>
<td>4.53</td>
<td>3.75</td>
<td>GBR</td>
</tr>
<tr>
<td>8</td>
<td>4.45</td>
<td>VEN</td>
<td>4.23</td>
<td>5.23</td>
<td>NLD</td>
</tr>
<tr>
<td>9</td>
<td>4.11</td>
<td>ITA</td>
<td>3.90</td>
<td>3.58</td>
<td>FRA</td>
</tr>
<tr>
<td>10</td>
<td>3.85</td>
<td>AUS</td>
<td>3.66</td>
<td>2.95</td>
<td>ITA</td>
</tr>
<tr>
<td>11</td>
<td>3.66</td>
<td>CHN</td>
<td>3.47</td>
<td>2.97</td>
<td>FIN</td>
</tr>
<tr>
<td>12</td>
<td>3.64</td>
<td>CAN</td>
<td>3.45</td>
<td>5.87</td>
<td>CHN</td>
</tr>
<tr>
<td>13</td>
<td>3.44</td>
<td>FIN</td>
<td>3.26</td>
<td>3.10</td>
<td>AUS</td>
</tr>
<tr>
<td>14</td>
<td>3.20</td>
<td>EGY</td>
<td>3.04</td>
<td>2.56</td>
<td>BRA</td>
</tr>
<tr>
<td>15</td>
<td>3.17</td>
<td>SAU</td>
<td>3.01</td>
<td>2.41</td>
<td>IDN</td>
</tr>
<tr>
<td>16</td>
<td>3.12</td>
<td>IDN</td>
<td>2.96</td>
<td>2.57</td>
<td>EGY</td>
</tr>
<tr>
<td>17</td>
<td>2.94</td>
<td>KEN</td>
<td>2.79</td>
<td>2.49</td>
<td>KEN</td>
</tr>
<tr>
<td>18</td>
<td>2.79</td>
<td>CHE</td>
<td>2.64</td>
<td>2.38</td>
<td>SAU</td>
</tr>
<tr>
<td>19</td>
<td>2.70</td>
<td>RUS</td>
<td>2.56</td>
<td>2.16</td>
<td>CHE</td>
</tr>
<tr>
<td>20</td>
<td>2.42</td>
<td>BRA</td>
<td>2.29</td>
<td>2.67</td>
<td>RUS</td>
</tr>
<tr>
<td>21</td>
<td>2.42</td>
<td>IRN</td>
<td>2.29</td>
<td>1.95</td>
<td>IRN</td>
</tr>
<tr>
<td>22</td>
<td>2.35</td>
<td>IND</td>
<td>2.23</td>
<td>1.92</td>
<td>IND</td>
</tr>
<tr>
<td>23</td>
<td>1.96</td>
<td>PER</td>
<td>1.85</td>
<td>1.82</td>
<td>PER</td>
</tr>
<tr>
<td>24</td>
<td>1.39</td>
<td>RWA</td>
<td>1.31</td>
<td>1.44</td>
<td>RWA</td>
</tr>
</tbody>
</table>

enter coalition building with the USA than others because they share relatively larger linkages with the latter as shown in the annex. Not surprisingly, since the USA are the most important members in the IMF EB, introducing linkages in the proximity index permits to capture the fact that countries are closely related to the USA, i.e. that trade much with them, gain from the fact that the USA is the largest member in the voting body. Indeed, since Venezuela, Canada trade much with the USA, both have a stronger will to share a coalition with the USA and the inverse is true since the USA trade much with these countries. Another interesting case is the one of the Great Britain (GBR) and France (FRA). Indeed, these two countries have exactly the same voting share of 4.86% in the IMF inducing that they have the same voting power. Still, the historical political position of the GBR is oriented towards a quasi-unconditional support to the USA. Our results explain GBR’s strategy since their power is larger than the one of France, respectively 4.0% and 3.65% in normalized terms as shown in Table[1]. The same applies also for Iran and Brazil. Indeed, both have the same voting share of 2.42% and the same Banzhaf index of 2.3%. However, Brazil is much
closer to the USA, politically speaking which is translated also in larger trade volumes between the two countries. As a result, the voting power of Brazil is 2.67% whereas Iran gets 1.96%. Introducing linkages in the proximity index appears therefore as useful tool since it permits to capture other determinants of coalition building. Quite clearly, while the proximity index shows the importance of proximity in coalition building, linkage permits to introduce another dimension in coalition building which reflect some interesting features of international politics in the particular case of the International Monetary Fund.

4 Conclusions

We have presented a new framework to capture the numerical proximity of voters in coalition building. While standard voting power indices, such as the Banzhaf index, capture the relative importance of critical voters, our index provides bilateral representation of power as the frequency voters may build coalition together according to the structure of the game. Indeed, given the distribution of votes, the numbers of voters and the majority threshold, some voters may appear more often in coalition together than others, given their characteristics.

We provide a formal definition of our index of proximity as well as a normalized form, in the spirit of Banzhaf. We present also our proximity index introducing linkages as determinant of coalition building. Although preference-based indices have been quite criticized for using a priori measure of linkages, we do not overcome this criticism but provide a different way to model linkage at the level of coalition rather than at the agent level, introducing linkage as a relative measure between all coalition participants.

We present a simple example of our proximity index and apply our index to the countries and constituencies represented at the IMF’s Executive Board. We compute our proximity index and find interesting patterns in coalition building within the institution. We provide a ranking analysis of the frequency voters are building coalitions together. While former studies found that the USA dominate decision making since they are important critical voters, our index permits to show that this is also the case due the fact that they always appear first in the ranking analysis when the other constituencies are critical. For example, our results exhibit that Japan is an important voter thanks to the fact that it coalesces more often with the USA than with other constituencies. In other words, to coalesce more often with frequent critical voters increases your voting power.

In addition, we provide an application of our proximity index introducing linkage between constituencies. For this application in the context of international economic cooperation, we use the relative volumes of bilateral trade as an indicator reflecting the importance of constituencies’ relationships. The results are able to explain several important features of the IMF functioning such as the relative larger voter powers of the Canadian, the Venezuelan and the Brazilian constituencies.
Looking ahead, we would like to apply our index of proximity to other voting bodies in international politics. On the practical side, better measuring and conceptualizing linkages among coalition participants constitutes a challenging but appealing research agenda.

References


Proximity in Coalition Building


Appendix - Determinants of coalition building in the international financial institutions

In the special case of international politics, for which the Bretton Woods institutions are good examples, decisions are involving actions (financial or not) on a member or a group of members. The IMF decides on financial assistance to members, on organising principles of the international monetary system (rules of cooperation, sharing of financial contributions among members, composition of the special drawing rights, etc.) and on members’ compliance with these principles (for example in the surveillance framework that can entail decisions on whether countries comply with their obligation to report data to the Fund and maintain policies that are conducive to overall international stability). Moreover, the IMF promotes international monetary cooperation more broadly including through the provision of analysis of international linkages and spillovers, it gives policy advice to members and it provides technical assistance to help countries build and maintain strong economies. The World Bank decides on loans and grants for project in member countries, and it lays the rules for international development assistance.

Therefore, coalition building is likely to be influenced a priori by economic bilateral relations between members. Of course, we cannot ignore the general policy content of IMF and World Bank interventions, and the choice of the proxy to account for economic bilateral relations between members should therefore capture this element.

In this respect, we believe that bilateral trade between members is a reasonable first candidate as a proxy for a measure of proximity between members of the Executive board of the IMF and the World Bank. For one thing, one should choose a proxy that captures not only economic, but also political relations between countries. Indeed, the literature on the determinants of IMF lending make it clear that voting in the IMF involves strong bargaining when it comes to financially support another member. Thacker (1999), Oatley and Yackee (2004), Oatley (2002) and Barro and Lee (2005), for example, found evidence that access to Fund resources is skewed towards countries that are aligned with the US. To estimate this alignment, Bird and Rowlands (2001) used bilateral trade of the borrowing countries with the US and France.

As suggested by the geographic literature economic (see Anderson and van Wincoop, 2004 for a survey), bilateral trade is a superior proxy for bilateral relations between countries because it captures the border effect associated by at least 6 determinants: (1) the distance barrier (Srivastava and Green, 1986; and Krugman, 1991), (2) the language barrier (Eaton and Kortum, 2002; and Hummels, 2001), (3) the currency barrier (Rose and Wincoop, 2001; Rose, 2004, Estevadeordal, Frantz and Taylor, 2003; and Lopez-Cordova and Meissner, 2003), (4) the informational barrier (Portes and Rey, 2002; Rauch and Trindade, 2002; Head and Ries, 1998), (5) the contracting costs and insecurity barrier (Evans, 2001; Anderson and Marcouiller, 2002) and (6) the non-tariff policy barrier (Haarigan, 1993; Head and Mayer, 2000; and Chen, 2002).

Finally, Rose (2004) has shown using a gravity model that bilateral trade may also...
Figure 2: Repartition of export shares to EU members for the US and Germany; Repartition of US imports share by EU members.
capture historical (i.e. colonial) and geographic dimensions. The top panel of Figure 2 below illustrates this for two interesting countries, the US and Germany. Indeed, as we can see, bilateral relations can be illustrated by a sort of 'snail' if we rank members by decreasing intensity of bilateral trade. We note for example that Germany’s trade is spread more equally across trading partners (with France taking the highest share at about 17%), whereas US external trade is more heavily concentrated (with Canada taking the highest share at 30%). In political terms, we can interpret, these distributions loosely speaking as exports of country X to country Y capturing 'how much country X is willing to coalesce with country Y' and imports of country Y to country X capturing 'how much country X is willing to coalesce with country Y'. Comparing for example the top panel of Figure 2, part 1 for the US and the bottom one which graph levels of US exports shares into others Executive Board constituencies, we are able to distinguish between the linkages of the US and the linkages of others members with the US. For example, we notice that the US are closer to a number of countries, such as Malaysia and India to pick two examples, but that only few countries are closer to the US, which would include Canada or Venezuela.