PROBLEM PRESENTED AT THE WORKSHOP IN KRYNICA 2004

This is a problem by Michael Kubesa, Technical University Ostrava, presented by Dalibor Froncek.

Let $K_{2n}$ be a complete graph and $T$ a tree, both with $2n$ vertices. A $T$-factorization of $K_{2n}$ is a collection of edge disjoint spanning subgraphs (i.e., factors) $T_1, T_2, \ldots, T_n$ of $K_{2n}$, all isomorphic to $T$. Every edge of $K_{2n}$ then appears in exactly one copy of $T$.

M. Kubesa asked the following question: Suppose that there exists a $T$-factorization of $K_{2n}$. Is it then true that the vertex set of $T$ can be decomposed into two subsets, $X$ and $Y$, such that

1. $|X| = |Y| = n$,
2. $\sum_{x \in X} \deg(x) = \sum_{y \in Y} \deg(y)$.

Notice that the sets $X, Y$ in general are not the partite sets of the bipartition of $T$. 