EXPONENTIAL TEMPERATURE EFFECT ON FREQUENCIES OF A RECTANGULAR PLATE OF NON-LINEAR VARYING THICKNESS: A QUINTIC SPLINE TECHNIQUE

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The differential equation governing the transverse motion of an elastic rectangular plate of non-linear thickness variation with thermal gradient has been analyzed on the basis of classical plate theory. Following Levy’s approach, i.e. the two parallel edges are simply supported, the fourth-order differential equation governing the motion of such plates of non-linear varying thickness in one direction with exponentially temperature distribution has been solved by using the quintic splines interpolation technique for two different combinations of clamped and simply supported boundary conditions at the other two edges. An algorithm for computing the solution of this differential equation is presented for the case of equal intervals. The effect of thermal gradient together with taper constants on the natural frequencies of vibration is illustrated for the first three modes of vibration.

Key words: exponentially temperature, non-linear, thickness variation, vibration, rectangular plate

1. Introduction

In this era of science and technology, plates of various shapes and variable thickness may be regarded as a first approximation to wings and blades and occur as panels in many forms of engineering structures. Thus knowledge of their natural frequencies is of considerable importance at the design stage in order to avoid resonances excited by internal or external forces. Therefore, their design requires an accurate determination of their natural frequencies and mode shapes.

With the advancement of technology, plates of variable thickness are being extensively used in civil, electronic, mechanical, aerospace and marine engineering applications. Nowadays, it becomes very necessary to study the vibration behavior of plates to avoid resonance excited by internal or external forces. Modern engineering structures are based on different types of design, which involve various types of anisotropic and non-homogeneous materials in the form of their structure components. Depending upon the requirement, durability and reliability, materials are being developed so that they can be used to give better strength and efficiency. In the recent past, there has been a phenomenal increase in the development of elastic materials due to high demand for lightweight, high strength, corrosion resistance and high-temperature performance requirements in modern technology. Plates of composite materials are widely used in many engineering structures and machines.

A number of researchers have worked on free vibration analysis of plates of different shapes and variable thickness. Rectangular plates of non-linear varying thickness are widely used in various structures; however, they have been poorly studied, unlike linearly varying thickness. Rectangular plates of non-linear varying thickness with thermal gradient find various applications in the construction of modern high speed aircraft. The vibration characteristics of such plates are of interest to the designer.
An extensive review of the work up to 1985 on linear vibration of isotropic/anisotropic plates of various geometries was given by Leissa (1969). The studies on vibration of rectangular plates with uniform/non-uniform thickness with various edge conditions after 1985 were carried out by a number of researchers and were reported by Leissa (1977, 1978, 1981, 1987).

Here, a quintic splines procedure is developed for obtaining the natural frequencies of a rectangular plate of nonlinear varying thickness with the thermal gradient effect. The consideration of the present type of thickness variation was taken earlier by Gupta et al. (2006) for a circular plate. The plate type structural components in aircraft and rockets have to operate under elevated temperatures which causes non-homogeneity in the plate material, i.e. elastic constants of the material become functions of the space variables. In an up-to-date survey of literature, authors have come across various models to account for non-homogeneity of plate materials proposed by researchers dealing with vibration.

Gupta et al. (2010a) studied the thermal gradient effect on vibration of a non-homogeneous orthotropic rectangular plate having bi-direction linearly thickness variation. Gupta et al. (2011a) did the vibration analysis of a visco-elastic orthotropic parallelogram plate with linearly thickness variation in both directions. Lal et al. (1997) studied the transverse vibrations of non-uniform orthotropic rectangular plates by Quintic splines method. Gupta and Kaur (2008) studied the effect of thermal gradient on free vibration of clamped visco-elastic rectangular plates with linearly thickness variation in both directions. Gupta and Khanna (2007) studied the vibration of a visco-elastic rectangular plate with linearly thickness variations in both directions. Gupta et al. (2007) observed the thermal effect on vibration of a non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Tomar and Gupta (1983, 1985) studied the effect of thermal gradient on frequencies of an orthotropic rectangular plate of variable thickness in one and two directions. Gupta et al. (2010c, 2011b) studied the thermal effect on vibration of a parallelogram plate of linearly varying thickness and bi-directional linearly varying thickness. Gupta et al. (2010b) did the vibration study of a visco-elastic parallelogram plate of linearly varying thickness.

As the thickness variation is not perfectly linear and the same for quadratic, therefore non-linear variation in thickness is very useful for scientists and engineers to study vibration of the plate and find modes of vibrations.

Since there is no work available on the non-linear thickness variation on thermally induced vibration of rectangular plates, in this paper, the thermal effect on vibration of a rectangular plate with non-linear varying thickness is studied. Here, vibration of a rectangular plate with non-linear varying thickness under a steady exponential temperature distribution is examined. The effect of temperature on the modulus of elasticity is assumed to vary exponentially along the $x$-axis. The non-linear thickness variation is taken as a combination of linear and parabolic variation factor. The differential equation of motion has been solved by the quintic spline interpolation technique. The two edges parallel to the $x$-axis ($y = 0$ and $y = b$) are assumed to be simply supported. Different sets of boundary conditions have been imposed at the other two edges. The frequency parameters for the first three modes of vibrations for C-S-C-S- and S-S-S-S- boundary conditions and for various values of taper constants, thermal constant and a fixed value of length-to-breadth ratio, are obtained. The results are presented in tabular form.

2. Analysis and equation of motion

Let us consider a rectangular plate which is subjected to an exponential temperature distribution along the length, i.e. in the $x$-direction

$$T = T_0 \frac{e^{-\alpha x}}{e - 1}$$

(2.1)
where \( T \) denotes the temperature excess above the reference temperature at any point at the distance \( X = x/a \) and \( T_0 \) denotes the temperature excess above the reference temperature at the end, i.e. \( x = a \) or \( X = 1 \).

The temperature dependence of the modulus of elasticity for most of engineering materials is given by Nowacki (1962)

\[
E(T) = E_0(1 - \gamma T) \tag{2.2}
\]

where \( E_0 \) is the value of Young’s modulus at the reference temperature, i.e. \( T = 0 \), and \( \gamma \) is the slope of the variation of \( E \) with \( T \).

Taking as the reference temperature, the temperature at the end of the plate, i.e. at \( X = 1 \), the modulus variation in view of (2.1) and (2.2) becomes

\[
E(X) = E_0 \left(1 - \alpha \frac{e^{-e^X}}{e-1}\right) \tag{2.3}
\]

where \( \alpha = \gamma T_0 \ (0 \leq \alpha < 1) \) is a constant known as the temperature constant.

The differential equation governing the free transverse motion of an elastic rectangular plate of the length \( a \), breadth \( b \), thickness \( h \) and density \( \rho \) is

\[
D \nabla^4 w + 2 \frac{\partial D}{\partial x} \frac{\partial}{\partial x} \nabla^2 w + 2 \frac{\partial D}{\partial y} \frac{\partial}{\partial y} \nabla^2 w + \nabla^2 \frac{\partial D}{\partial x} \nabla^2 w \\
+ (\nu - 1) \left( \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 D}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 w}{\partial x^2} \right) + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \tag{2.4}
\]

where \( w \) is the transverse displacement.

Assume now that the two opposite edges of the plate \( y=0 \) and \( y=b \) are simply supported. Further, let thickness vary non-linearly in the \( x \)-direction only. Thus, the thickness \( h \) and flexural rigidity \( D \) of the plate become a function of \( x \) only. For harmonic vibration, \( w \) can be expressed as

\[
w(x, y, t) = W_1(x) \sin \left(\frac{m\pi y}{b}\right) e^{ipt} \tag{2.5}
\]

where \( p \) is the circular frequency and \( m \) is a positive integer.

Substitution of equation (2.5) into (2.4) gives

\[
D W_{1,xxxx} + 2D_{xx} W_{1,xxx} + \left( \frac{2m^2\pi^2}{b^2} - D_{xx} \right) W_{1,xx} + \left( \frac{2m^2\pi^2}{b^2} D_{xx} \right) W_{1,x} \\
+ \left( \frac{m^4\pi^4}{b^4} - \frac{\nu m^2\pi^2}{b^2} D_{xx} \right) W_{1} = \rho h p^2 W_1 \tag{2.6}
\]

A comma followed by a suffix denotes partial differentiation with respect to that variable.

Thus equation (2.6) reduces to a form independent of \( y \) and on introducing the non-dimensional variables

\[
H = \frac{h}{a} \quad W = \frac{W_1}{a} \quad X = \frac{x}{a} \quad D_1 = \frac{D}{a^4} \tag{2.7}
\]

differential equation (2.6) reduces to

\[
D_1 W_{1,xxxx} + 2D_{1,xx} W_{1,xxx} + \left( D_{1,xx} - 2r^2 D_1 \right) W_{1,xx} - 2r^2 D_{1,x} W_{1,x} \\
+ r^2 \left( r^2 D_1 - \nu D_{1,x} \right) W = \rho H a^2 p^2 W \tag{2.8}
\]

where \( r^2 = \left( m\pi a/b \right)^2 \).
Since the thickness varies non-linearly in the \( x \)-direction only, therefore, one can assume

\[
H = H_0(1 + \beta_1 X + \beta_2 X^2)
\]  
(2.9)

where \( \beta_1 \) and \( \beta_2 \) are taper constants such that \( |\beta_1| \leq 1, |\beta_2| \leq 1 \) and \( \beta_1 + \beta_2 > -1 \), \( H_0 \) is the thickness at \( X = 0 \).

Considering equation (2.3) and (2.9) with the help of (2.7), the expression for rigidity \( D_1 \) comes out as

\[
D_1 = D_0\left(1 - \frac{e^{-e^X}}{e^X}\right)(1 + \beta_1 X + \beta_2 X^2)^3
\]  
(2.10)

where \( D_0 = E_0H_0^3/[12(1 - \nu^2)] \).

Using equations (2.8) to (2.10), one obtains the equation of motion as

\[
\left(1 - \frac{e^{-e^X}}{e^X}\right)(1 + \beta_1 X + \beta_2 X^2)^2W_{XXX} + 2\left[\frac{e^X}{e^X} - 1\right](1 + \beta_1 X + \beta_2 X^2)^2
\]

\[
+ 3\left[1 - \frac{e^{-e^X}}{e^X}\right](1 + \beta_1 X + \beta_2 X^2)(\beta_1 + 2\beta_2 X)W_{XXX}
\]

\[
+ \left[\frac{e^X}{e^X} - 1\right](1 + \beta_1 X + \beta_2 X^2)^2 + 6\left[\frac{e^X}{e^X} - 1\right]W_{XXX}
\]

\[
+ 6\left[1 - \frac{e^{-e^X}}{e^X}\right]W_{XXX}
\]

\[
+ 2\beta_2^2\left[1 - \frac{e^{-e^X}}{e^X}\right]W_{XXX}
\]

\[
- 2\beta_2^2\left[1 - \frac{e^{-e^X}}{e^X}\right]W_{XXX}
\]

\[
+ r^2\left[1 - \frac{e^{-e^X}}{e^X}\right]W_{XXX}
\]

\[
+ 6\left[1 - \frac{e^{-e^X}}{e^X}\right]W_{XXX}
\]

\[
W = \lambda^2 W
\]

where

\[
\lambda^2 = \frac{p^2a^2}{H_0^2 \rho} \frac{12(1 - \nu^2)}{E_0 H_0^2 / \rho}
\]  
(2.12)

is a frequency parameter.

3. Method of solution

Let \( f(X) \) be a function with continuous derivatives in the range \( (0, l) \). Choose \((n + 1)\) points \( X_0, X_1, X_2, \ldots, X_n \), in the range \( 0 < X < l \) such that \( 0 = X_0 < X_1 < X_2 < X_3 < \ldots < X_n = 1 \).

Let the approximating function \( W(X) \) for \( f(X) \) be a quintic spline with the following properties:

(a) \( W(X) \) is a quintic polynomial in each interval \( (X_k, X_{k+1}) \),
(b) \( W(X_k) = f(X_k), \ k = 0(1)n \),
(c) \( W'(X), W''(X), W'''(X) \) and \( W^iv(X) \) are continuous.
By definition, the quintic spline takes the form

\[
W(X) = a_0 + \sum_{i=1}^{4} a_i (X - X_i)^4 + \sum_{j=0}^{n-1} b_j (X - X_j)^5
\]

(3.1)

where

\[
(X - X_j) = \begin{cases} 
0 & \text{if } X < X_j \\
X - X_j & \text{if } X \geq X_j 
\end{cases}
\]

(3.2)

It is also assumed, for simplicity, that the knots \(X_i\) are equally spaced in \((0, l)\) with the spacing interval \(\Delta X\), so that

\[
\Delta X = \frac{l}{n} \quad \text{and} \quad X_i = i \Delta X \quad i = 0, 1, 2, \ldots, n
\]

(3.3)

The number of unknown constants in equation (3.1) is \((n+5)\). Satisfaction of differential equation (2.11) by collocation at the \((n + 1)\) knots in the interval \((0, l)\) together with the boundary conditions (to be explained in the next section) gives precisely the requisite number of equations for the determination of unknown constants.

Substitution of \(W(X)\) from equation (3.1) into equation (2.11), for satisfaction at the \(m\)-th knot, gives

\[
B_4 a_0 + [B_4(X_q - X_0) + B_3]a_1 + [B_4(X_q - X_0)^2 + 2B_3(X_q - X_0) + 2B_2]a_2 \\
+ [B_4(X_q - X_0)^3 + 3B_3(X_q - X_0)^2 + 6B_2(X_q - X_0) + 6B_1]a_3 \\
+ [B_4(X_q - X_0)^4 + 4B_3(X_q - X_0)^3 + 12B_2(X_q - X_0)^2 + 24B_1(X_q - X_0) + 24B_0]a_4 \\
+ \sum_{i=0}^{n-1} [B_4(X_q - X_i)^5 + 5B_3(X_q - X_i)^4 + 20B_2(X_q - X_i)^3 + 60B_1(X_q - X_i)^2 \\
+ 120B_0(X_q - X_i)]b_i = 0
\]

(3.4)

where

\[
B_0 = \left(1 - \frac{e^{-X_q}}{e-1}\right)(1 + \beta_1 X_q + \beta_2 X_q^2)^2 \\
B_1 = 2\left[\frac{e^{X_q}}{e-1}(1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 3\left(1 - \frac{e^{-X_q}}{e-1}\right)(1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q)\right] \\
B_2 = \left[\frac{e^{X_q}}{e-1}(1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 6\alpha\frac{e^{X_q}}{e-1}(1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q) \\
+ 6\left(1 - \frac{e^{-X_q}}{e-1}\right)(\beta_1 + 2\beta_2 X_q)^2 + 24\alpha\frac{e^{X_q}}{e-1}(1 + \beta_1 X_q + \beta_2 X_q^2)\beta_2 \\
- 2r^2\left(1 - \frac{e^{-X_q}}{e-1}\right)(1 + \beta_1 X_q + \beta_2 X_q^2)^2\right] \\
B_3 = -2r^2\left[\frac{e^{X_q}}{e-1}(1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 3\left(1 - \frac{e^{-X_q}}{e-1}\right)(1 + \beta_1 X_q + \beta_2 X_q^2)\beta_1 + 2\beta_2 X_q\right] \\
B_4 = r^2\left[r^2\left(1 - \frac{e^{-X_q}}{e-1}\right)(1 + \beta_1 X_q + \beta_2 X_q^2)^2 - \nu\left(\frac{e^{X_q}}{e-1}(1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 6\alpha\frac{e^{X_q}}{e-1}\right) \\
(1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q) + 6\left(1 - \frac{e^{-X_q}}{e-1}\right)(\beta_1 + 2\beta_2 X_q)^2 \\
+ 6\left(1 - \frac{e^{-X_q}}{e-1}\right)(1 + \beta_1 X_q + \beta_2 X_q^2)\beta_2\right] - \lambda^2
\]
Thus, one obtains a homogeneous set of equations in terms of the unknown constants $a_0, a_1, a_2, a_3, a_4, b_0, b_1, \ldots, b_{n-1}$, which, when written in matrix notation, takes the form

$$BC = 0 \quad (3.5)$$

where $B$ is an $(n + 1) \times (n + 5)$ matrix and $C$ is an $(n + 5) \times 1$ matrix.

### 4. Boundary conditions and frequency equations

The frequency equations for clamped (C) and simply supported (S) rectangular plates have been obtained by employing the appropriate boundary conditions.

#### 4.1. C-S-C-S-plates

For a rectangular plate clamped at both the edges $X = 0$ and $X = 1$ (and simply supported at the remaining two edges)

$$W \bigg|_{X=0,1} = \frac{\partial W}{\partial X} \bigg|_{X=0,1} = 0 \quad (4.1)$$

Applying boundary conditions (4.1), to deflection function (3.1), at the two edges $X = 0$ and $X = 1$, one obtains a set of four homogeneous equations in terms of the unknown constants, which can be written as

$$A_1 C = 0 \quad (4.2)$$

where $A_1$ is an $4 \times (n + 5)$ matrix and $C$ is an $(n + 5) \times 1$ matrix.

Equation (4.2) taken together with equation (3.5) gives a complete set of $(n + 5)$ equations for a C-S-C-S-plate. These can be written as

$$[B/A_1]C = 0 \quad (4.3)$$

For a non-trivial solution of equation (4.3), the characteristic determinant must vanish

$$|B/A_1| = 0 \quad (4.4)$$

This is the frequency equation for a C-S-C-S-plate.

#### 4.2. S-S-S-S-plates

For a rectangular plate simply supported at both the edges $X = 0$ and $X = 1$ (and simply supported at the remaining two edges), the following holds

$$W \bigg|_{X=0,1} = \frac{\partial^2 W}{\partial X^2} \bigg|_{X=0,1} = 0 \quad (4.5)$$

Employing boundary conditions (4.5) to deflection function (3.1) at the two edges $X = 0$ and $X = 1$, one gets the boundary equations for a S-S-S-S-plate as

$$A_2 C = 0 \quad (4.6)$$

where $A_2$ is an $4 \times (n + 5)$ matrix and $C$ is an $(n + 5) \times 1$ matrix.

Hence the frequency equation comes out for S-S-S-S-plate as

$$|B/A_2| = 0 \quad (4.7)$$
5. Results and discussion

Frequency equations (4.4) and (4.7) are transcendental equations in $\lambda^2$ from which infinitely many roots can be obtained. The frequency parameter $\lambda$ corresponding to the first three modes of vibration of C-S-C-S- and S-S-S-S-rectangular plates have been computed for $m = 1$ and various values of aspect ratio $(a/b)$, thermal constant $(\alpha)$ and taper constants $(\beta_1, \beta_2)$. The value of Poisson’s ratio $\nu$ has been taken as 0.3.

To choose the appropriate interpolation interval $\Delta X$, a computer program has been developed for the evaluation of the frequency parameter $\lambda$ and run for $n = 10(5)60$. The numerical values show consistent improvement with the increase of the number of knots. In the computation, the authors have fixed $n = 50$, since further increase in $n$ does not improve the results except for the fifth or sixth decimal places. These results are presented in Tables 1 to 3.

Table 1 shows the variation of the frequency parameter ($\lambda$) with the thermal constant $(\alpha)$ for different combinations of taper constants $(\beta_1, \beta_2)$ and the fixed aspect ratio $(a/b = 1.5)$ corresponding to the first three modes of vibration for C-S-C-S- and S-S-S-S-plates. The value of the frequency parameter decreases with the increase of the thermal constant for both boundary conditions considered here. Furthermore, it can be seen that the frequency parameter, for both boundary conditions, decreases gradually in the third mode of vibrations in comparison to the first two modes of vibration.

<table>
<thead>
<tr>
<th>$\beta_1$, $\beta_2$</th>
<th>$\alpha$</th>
<th>C-S-C-S-plate</th>
<th></th>
<th>S-S-S-S-plate</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First mode</td>
<td>Second mode</td>
<td>Third mode</td>
<td></td>
</tr>
<tr>
<td>$\beta_1 = -0.5$</td>
<td>0.0</td>
<td>29.3011</td>
<td>63.0984</td>
<td>111.0257</td>
<td>21.4941</td>
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<tr>
<td>$\beta_2 = -0.5$</td>
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<td>28.3908</td>
<td>61.1005</td>
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<td>20.4906</td>
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<tr>
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<td>0.2</td>
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<td>58.8310</td>
<td>102.9045</td>
<td>19.4041</td>
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<tr>
<td></td>
<td>0.3</td>
<td>26.4212</td>
<td>56.8112</td>
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<td>18.3100</td>
</tr>
<tr>
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<td>0.4</td>
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<td>54.7283</td>
<td>95.3142</td>
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<td></td>
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<td>52.4450</td>
<td>91.1528</td>
<td>15.8906</td>
</tr>
<tr>
<td>$\beta_1 = 0.5$</td>
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<td>36.0132</td>
<td>72.5490</td>
<td>127.4781</td>
<td>27.3761</td>
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<tr>
<td>$\beta_2 = 0.5$</td>
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<td>23.1530</td>
</tr>
<tr>
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<td>30.7983</td>
<td>61.7406</td>
<td>106.5164</td>
<td>22.0029</td>
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</table>

The results presented in Table 2 show a marked effect of variation of the taper constant $(\beta_1)$ on the frequency parameter for the taper constant $(\beta_2 = 0.5)$, two values of the thermal constant $(\alpha = 0.0, 0.4)$ and a fixed aspect ratio $(a/b = 1.5)$ corresponding to the first three modes of vibration. It is observed that the frequency parameter increases with the increase of the taper constant for both boundary conditions considered here.
Table 2. Values of the frequency parameter (λ) for different taper constants (β₁) with different combinations of the thermal constant (α) and a fixed aspect ratio (a/b = 1.5) for C-S-C-S- and S-S-S-S-plates for the first three modes of vibrations; β₂ = 0.5

<table>
<thead>
<tr>
<th>α</th>
<th>β₁</th>
<th>C-S-C-S-plate</th>
<th></th>
<th>S-S-S-S-plate</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First mode</td>
<td>Second mode</td>
<td>Third mode</td>
<td>First mode</td>
</tr>
<tr>
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<td>−0.5</td>
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<td>72.5490</td>
<td>127.4781</td>
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</tr>
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</tr>
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<td>63.8503</td>
<td>110.5314</td>
<td>23.1530</td>
</tr>
<tr>
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In Table 3, the effect of the taper constant (β₂) on the frequency parameter for the taper constant (β₁ = 0.5), two values of the thermal constant (α = 0.0, 0.4) and a fixed aspect ratio (a/b = 1.5) corresponding to the first three modes of vibration for C-S-C-S- and S-S-S-S-plates is shown. From this table, one can observe that the frequency parameter in the first three modes of vibration increases with the increase of the taper constant for C-S-C-S- and S-S-S-S-plates.

Table 3. Values of the frequency parameter (λ) for different taper constants (β₂) with different combinations of the thermal constant (α) and a fixed aspect ratio (a/b = 1.5) for C-S-C-S- and S-S-S-S-plates for the first three modes of vibrations; β₁ = 0.5

<table>
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<th>α</th>
<th>β₂</th>
<th>C-S-C-S-plate</th>
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<th>S-S-S-S-plate</th>
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<td>Second mode</td>
<td>Third mode</td>
<td>First mode</td>
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<td>0.0</td>
<td>43.1534</td>
<td>91.1293</td>
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<td>33.4221</td>
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<td>0.1</td>
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</table>
Moreover, it can be seen in Tables 2 and 3 that the frequency parameter, for both boundary conditions, increases gradually in the third mode of vibrations in comparison to the first two modes of vibration.

Also, one can observe from Tables 1 to 3, that the frequency parameter of the C-S-C-S-plate is higher than that of the S-S-S-S-plate.

References


2. Gupta A.K., Johri T., Vats R.P., 2010a, Study of thermal gradient effect on vibrations of a non-homogeneous orthotropic rectangular plate having bi-directional linearly thickness variations, Meccanica, 45, 3, 393-400.


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