Mathematical model for motion of weighted parts in curled flow

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Abstract. The article deals with the problem of providing high performance apparatuses for cleaning air from dust in various branches of industry in order to reduce hazardous emissions to the level conforming to sanitary-hygienic norms. The article describes new trends in the development of dust catching apparatuses based on the use of centrifugal-inertial forces, permitting to improve significantly the effectiveness of dust catching.

Key words: dust catching, air cleaning, pollution, centrifugal, cyclone.

PROBLEM STATEMENT

Ecological problems are well-known issues for those who investigate air surrounding in rooms with welding gases. In this process hazardous substances consist of gases and aerosols where some little fractions permeate to pulmonary tissue and blood. In most cases welding gas has parts of iron oxide, zinc, cadmium, manganese, fluorine, asbestos, nickel, chromium, copper and others. As the result of interaction of those parts eye mucous membranes will be traumatized, some allergic diseases, silicosis, swollen lungs, headaches, chest pains and cancer diseases will occur, kidneys will be injured.

Using of a system air ejector fan will provide a level of maximum allowable concentration in breathing in various industrial processes controlled by industrial safety and ecology legislation.

ANALYSIS OF RECENT DEVELOPMENTS

Electrostatic filters are widely used and spread in different spheres because of high level of the most hazardous parts catching of 1-0.01 micron and undersized. At the same time such disease-producing factors as microbes, viruses, bacterium, morbific fungus and wreckers will die at electrostatic field of filter. Therefore electrostatic filters are different from mechanical filters because of dangerous elements which can be source of pollution in unclean filter maintenance.

Electrostatic filters are used effectively for cleaning air from different parts of dust, oily mist and fine dust of 200-0.01 micron. The efficiency of dust cleaning is 94%.

Air filters with mechanical filters are used to clarify a polluted air from huge parts of different types of dust, oily mist, welding gas in welding galvanized steel, aluminium, stainless steel, galvanizer, dust of soldering and spot welding. Mechanical filters have high level of air cleaning measured by 200 – 0.1 micron.

Tool-making and laboratory measuring show the efficiency of cyclones cleaning is 81-85% in primary dustiness 5,3 g/m³, bag filter – 99%, installation coefficient of efficiency – 99,8% [1,2]. Dry cleaning of dust air in high effective apparatus of modern construction allows providing waste fall in atmosphere up to 20-40 mg/m³.

Nowadays, there is no apparatus to secure regulations of discharge harmful substances providing sanitary-and-hygienic conditions. The most appropriate apparatus can’t manage that task. Therefore we aimed at creating high effective apparatus for fine dust catching.

THE MAIN MATERIAL REVIEW

This chapter will describe task arrangement of motion two-phase flow in dust catching of new construction and numerical calculation will be made. Numerical analysis of dynamic processes can be realized if laws of motion processes are described in mathematical formula in differential equation as a rule.
Every equation describes specific law of conservation with physical determined variable quantity and balance between various factors influenced on this variable. Dependent variables in these differential equations are specific performances referred to unit of weight. Term of differential equation describes action on unit of volume [3].

Problem statement of air flow motion in equipment consists of:
- Description of general assumption, boundary and initial conditions;
- Mathematical model of air flow motion in equipment considering a turbulence model;
- Mathematical model of solid phase (dust parts) motion in apparatus;
- Choice of a method solving and its realization;
- Method of efficiency determination process flow cleaning of dust parts;

**ACCEPTED PROPOSAL, BOUNDARY AND INITIAL CONDITIONS**

For a proper description of separation process of little parts of dust in dust catcher of proposed construction assumptions were accepted to approximate the mathematical model to definite conditions of equipment functioning and prevent model complicating.

Developing two-phase flow in elaborated construction of dust catcher for calculation next assumptions were taken:
- Little parts of dust are solid and not interacted;
- A flow has an even field of rate on equipment entering;
- Stokes law is used for resistance movement of particles in a gaseous medium;
- tangential and axial velocities of little parts are similar to velocity of gas flow turnover, accordingly, radial velocities are different as a result of inertial forces.

We need to set initial and boundary conditions for a solution of the problem of air flow motion determined by the shape, design of the dust collector and conditions of work, that is, for each type of cyclone there is a set of initial and boundary conditions.

The initial conditions for this system are the characteristics of air pollution and characteristics of dust:
- normal atmospheric pressure \(p_0=101325\) Pa,
- normal air temperature \(m_0=293\) K,
- air density \(\rho_0=1.293\) kg/m³,
- middle diameter of small dust particles \(d_{10}=5\cdot10^{-6}\) m,
- maximal diameter of small dust particles \(d_{max}=50\cdot10^{-6}\) m,
- minimal diameter of small dust particles \(d_{min}=10^{-6}\) m,
- density of small dust particles \(\rho_e=2000-4000\) kg/m³.

Boundary conditions influence on the problem solution of flow motion and must be performed at each time of motion. Besides they are determined by the nature of air flow on the boundary surface. Boundary conditions are difficult to specify, because they depend on the shape of the dust collector, the characteristics of its operation, as well as the gas density. High-density gas "sticks" to the sides of the dust collector when a rarefied gas slips by boundary surface. However, despite the peculiarities of the process, there are certain laws as boundary conditions when driving in dusty air dust collector: the rate on a fixed solid boundary is zero, the rate of dust air flow at the entrance to the dust collector is stable and equal to \(18\) m/s [7,8,9].

**MODELLING MOTION OF AIR FLOWS IN ALIGNED-INERTIAL DUST COLLECTORS**

A motion of air flows in cyclones on the basis of set of equations of viscous substance is considered to be:

\[
\nabla \mathbf{v} = 0, \quad \frac{d\mathbf{v}}{dt} = -\nabla p + \nu \Delta \mathbf{v}.
\]

A flow is dimensional if speeds are parallel to space and hydrodynamic sizes have different values. Choose motion direction axe \(x\). Then:

\[
\nu_x = \nu_z = 0.
\] (1)

Write a set of equations of viscous substance, taking into consideration (1):

\[
\frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{v}}{dx^2} = 0,
\] (2)

\[
\frac{d\mathbf{v}}{dt} + \nu_x \frac{d\mathbf{v}}{dx} = -\frac{1}{\rho} \frac{d\rho}{dx} + \nu \left( \frac{d^2\mathbf{v}}{dx^2} + \frac{d^2\mathbf{v}}{dy^2} + \frac{d^2\mathbf{v}}{dz^2} \right),
\] (3)

\[
\frac{dp}{dy} = 0, \quad \frac{dp}{dz} = 0.
\] (4)

From (2) it turns out that \(\nu_x\) does not depend on \(x\), from (4) –\(p\) is independent of \(y\) and \(z\), i.e. :

\[
\nu_x = \nu_y(y,z,t),
\] (5)

\[
p = p(x,t).
\] (6)

Given (5), rewrite equation (3) as follows:

\[
\frac{d\mathbf{v}}{dt} - \nu \left( \frac{d^2\mathbf{v}}{dy^2} + \frac{d^2\mathbf{v}}{dz^2} \right) = \frac{1}{\rho} \frac{d\rho}{dx}.
\] (7)

Left side (7) does not depend on \(x\), therefore, can depend on the time:

\[
\frac{dp}{dx} = f(t), \quad p = f(t)x + f_1(t).
\] (8)

Thus, pressure in a dimensional motion is linear function \(x\). Functions \(f(t)\) and \(f_1(t)\) can be found if the pressure \(p\) will be given in two intersections:
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Then:

\( \frac{dp}{dx} - \frac{f_x(t) - f_0(t)}{2x^2} = \frac{\Delta p}{\Delta x} \). \hspace{1cm} (9)

For a given pressure drop rate is found from equation (7):

\( p \frac{dv_x}{dt} = \mu \left( \frac{d^2 v_x}{dy^2} + \frac{d^2 v_x}{dz^2} \right) \frac{\Delta p}{\Delta x} \). \hspace{1cm} (10)

Equation (10) is similar with the thermal conductivity equation. Non-homogeneous equation (10) can be reduced to homogeneous with following displacement:

\( v_x = v_{x_0} \left( \frac{1}{p} \int f(t) dt \right) \). \hspace{1cm} (11)

To solve equation (10) initial and boundary conditions must be given. Dimensional motions can be performed with fluid flow in cylindrical tubes (or beyond). Therefore, the boundary conditions are written on circuits \( l_k \) with cylinder plane \( x = \text{const} \):

\( v_x = v_{x_0} \left( \frac{1}{p} \int f(t) dt \right) \). \hspace{1cm} (11)

\( uk(t) \) is a speed of circuits points. The initial conditions are following:

\( v_x \bigg|_{t=0} = F(y, z) \). \hspace{1cm} (12)

The task is simplified if the flow is invariable. In this case, the pressure drop is constant and equation (10) reduces to Poisson equation:

\( \mu \left( \frac{d^2 v_x}{dy^2} + \frac{d^2 v_x}{dz^2} \right) = \frac{\Delta p}{\Delta x} \). \hspace{1cm} (13)

Initial conditions disappear and boundary conditions do not depend on time:

\( v_x \bigg|_{l_k} = u_k \). \hspace{1cm} (14)

In general a rate:

\( v_x \bigg|_{l_k} \) may depend on circuits points:

\( v_x \bigg|_{l_k} = v_x(t, M) \). \hspace{1cm} (15)

A free flow of liquid is a special case when:

\( \frac{dp}{dx} = 0 \), \hspace{1cm} p = \text{const}.

Instead (10) we have equation:

\( \frac{dv_x}{dt} - \mu \left( \frac{d^2 v_x}{dy^2} + \frac{d^2 v_x}{dz^2} \right) \). \hspace{1cm} (16)

If the motion is set, use Laplace equation to find a rate:

\( \frac{d^2 v_x}{dy^2} + \frac{d^2 v_x}{dz^2} = 0 \). \hspace{1cm} (15)

Corresponding to boundary conditions (14).

Tasks (15), (14) \( uk \) constant on circuits \( l_k \) are equal finding flow function \( \varphi \) in plane of flow incompressible liquid:

\( \frac{d^2 \varphi}{dy^2} + \frac{d^2 \varphi}{dz^2} = 0 \), \hspace{1cm} \varphi \bigg|_{l_k} = u_k \).

Hence, in particular, conformal transformation method can be used to solve the problem (15), (14). It is easy to show that the force \( f_k \) influencing on the circuit \( l_k \) in a viscous fluid is described with circulation \( G \) corresponding ideal fluid flow:

\( f_k = \oint \tau_{\omega} ds = \mu \oint \frac{dv}{\pi r} ds = \mu \oint \frac{dp}{\pi r} ds = \mu G \).

Fig. 1. Motion scheme

An air flow motion in the cyclone can be described by a motion flow between two infinitely long circular cylinders with a common axis of the radius \( R_1 \) and \( R_2 \), the absence of mass forces (Figure 1.). Use axis \( x \) along the axis of the cylinder. Assume that the inner cylinder rotates with angular velocity \( \omega_1 \), external cylinder - velocity \( \omega_2 \). It is convenient to solve the problem introducing cylindrical \( r, \theta, x \), coordinates and write in these coordinates the system of equations of viscous fluid. To do this, find expression: div \( v \), div \( v \), grad \( p \), \( \Delta v \) in this coordinate system. Naturally the velocity is directed tangent to the circle \( r = \text{const} \) and both with pressure depends on \( r \), that is \( v_r = v_{r_0}, v_\theta = v_\theta(r), v_x = v_x(r) \). A system of equations can be applied to this problem when the motion is set, takes a simple form and can immediately get the solution of the problem in:

\( v_0 = C_1 r + C_2 \), \hspace{1cm} p = p_1 + \int_0^r \frac{v_x(r)}{r} dr.

Steels \( C_1 \) and \( C_2 \) are determined from the boundary conditions. But to solve this problem we use other solution.

To find the dependence \( v = v(r) \), we write the law of conservation of momentum in a sphere:

\( R_1^2 \leq r^2 \leq R_2^2, r < R_2 \).
(fig.1). Let $M$ – moment of forces acting on the sphere. As a flow is flat, vector $M$ is recognized on axis $x$. Get the following equality $M = 0$. It seems that: $M = M_1 + M_\omega$, where: $M_1$ – moment of forces acting on the inner cylinder; $M_\omega$ – moment of viscous friction forces that are applied to the cylinder radius $r$. The magnitude of this vector:

$$M_1 = \int_0^{2\pi} r (\tau_{\theta r} d\theta) = r^2 \int_0^{2\pi} \tau_{\theta r} d\theta,$$

here: $\tau_{\theta r}$ – projection to axis $\theta$ (direction $v$) tension acting on a part with normal $r$. It depends only on $r$, so:

$$M_1 = \tau_{\theta r} 2\pi r^2.$$

Thus, the law of conservation gives angular equality:

$$\tau_{\theta r} 2\pi r^2 + M_1 = 0. \quad (16)$$

Let angle $\theta$ delayed from the axis $y$. Apparently:

$$\tau_{\theta r} |_{\theta = 0} = \tau_{\theta r} |_{y = z = 0}.$$

As: $\tau_{\theta r}$ doesn’t depend on $\theta$, the latter value is valid for all $\theta$.

$$\tau_{\theta r} = \tau_{\theta r} |_{y = z = 0} = \mu \left( \frac{\partial v_z}{\partial z} + \frac{\partial v_y}{\partial y} \right) |_{\theta = 0}. \quad (17)$$

We get:

$$v_y = -v \sin \theta = -\frac{z}{r}, \quad v_z = v \cos \theta = v \frac{y}{r},$$

$$\frac{\partial v_z}{\partial z} |_{\theta = 0} = \frac{\partial}{\partial z} \left( -v \frac{z}{r} \right) |_{\theta = 0} = -v,$$

$$\frac{\partial v_y}{\partial y} |_{\theta = 0} = v \left( \frac{d}{dr} \frac{v}{r} \right) \frac{y^2}{r} = v \frac{r}{r} + r \left( \frac{d}{dr} v \right). \quad (18)$$

Using this equality, from (18) we obtain:

$$\tau_{\theta r} = \mu r \left( \frac{d}{dr} \frac{v}{r} \right) \quad (19)$$

Substituting (19) in (16), we obtain the equation for finding $v$:

$$M_1 + 2\pi r^2 \mu \left( \frac{d}{dr} \frac{v}{r} \right) = 0. \quad (20)$$

The general solution of this equation is expressed by:

$$v = C_1 r + \frac{C_2}{r}, \quad (21)$$

where:

$$C_2 = \frac{M_1}{4\pi \mu}.$$

Steels $C_1$ and $C_2$ determined from the boundary conditions:

$$v |_{r = R_1} = \omega_1 R_1, \quad v |_{r = R_2} = \omega_2 R_2, \quad (22)$$

or more precisely:

$$C_1 R_1 + \frac{C_2}{R_1} = \omega_1 R_1, \quad C_1 R_2 + \frac{C_2}{R_2} = \omega_2 R_2. \quad (23)$$

Solving the system (8), we obtain:

$$\begin{align*}
\hat{N}_1 &= \frac{\omega_1 R_2^2 - \omega_2 R_1^2}{R_2^2 - R_1^2}, \quad C_2 = \frac{R_1^2 (\omega_1 - \omega_2)}{R_2^2 - R_1^2}. \quad (24)
\end{align*}$$

Thus, the velocity distribution between cylinders with a common axis is given by the formula:

$$v = \frac{\omega_1 R_2^2 - \omega_2 R_1^2}{R_2^2 - R_1^2} r + \frac{R_1^2 (\omega_1 - \omega_2)}{R_2^2 - R_1^2} \frac{1}{r}. \quad (25)$$

With formula (25) it is easy to calculate $\tau_{\theta r}$ and $M_1$:

$$\tau_{\theta r} = \mu r \left( \frac{d}{dr} \frac{v}{r} \right) = -2\mu \frac{C_2}{R_1^2}, \quad M_1 = \tau_{\theta r} 2\pi r^2 = -4\pi \mu C_2, \quad (26)$$

where: $C_2$ is (24).

Note also that, by measuring the experiment, we can determine viscosity. Partial flow cases:

a) Two cylinders rotate with the same angular velocity:

$$\omega_1 = \omega_2 = \omega,$$

from (1.26) get:

$$v = \omega r.$$

b) fluid fills infinite space outside the cylinder:

$$R_1 : R_2 = R, \quad \omega_1 = \omega_2 = \omega, \quad R = \infty, \quad \omega_2 = 0.$$

In this case:

$$v = \frac{R_1^2 \omega}{r}.$$

c) one of the cylinders is fixed: $\omega_1 = 0, \omega_2 = \omega$.

Then:

$$v = \frac{R_1^2}{R_2^2 - R_1^2} \omega r - \frac{R_1^2 R_2^2 \omega_2}{R_2^2 - R_1^2} r.$$

Next we consider the movement of the air flow in the moving coordinate system, which rotates around the axis of the cyclone with an angular velocity equal to the speed of the flow around a vertical axis. Then we will deal with the case where the speed is equal to the outer wall $w_2$ and internal (inertial separator) $w_1 [10,11,14]$.  

**MODELLING RESULTS**

We had constructed three-dimensional finite element model for dust collector (Fig. 1) to prepare calculation in solid modeling program to study the physical meaning processes that occur in our unit and to justify the choice of parameters of experimental studies. The finite element method (FEM) solved the system of Navier-Stokes equations for turbo-valence air flow. For this purpose we built geometric model with geometrical parameters of the apparatus, which have been identified.

1. The diameter of the cylindrical part of the cyclone $0.7$ m.
2. The height of the cylindrical part of cyclone $1.5$ m.
3. The diameter of the exhaust pipe $0.7$ m.
4. Jalousie separator diameter $0.75$ m.
5. Height cyclone $3.2$ m.
6. Size of cross section inlet $0.40 \times 0.10$, $70$m.

The investigation was conducted for a variety of directions and speeds of jalousie separator. We studied the motion of air flow, distribution of air flow values and static pressure value in the flat section of the apparatus.
Figure 2 shows the trajectory of the air flow in the proposed precipitators. The speed of air flow reflected color trajectory. Figure 3 shows the distribution of air velocity in the horizontal section of the dust collector.

Hydraulic resistance of the device without rotation jalousie separator was 651.2 Pa.

Hydraulic resistance of the device was 766.6 Pa in rotating separator towards jalousie movement of air flow in the inlet, with an angular velocity of 1 rad/s.

When rotating separator towards jalousie movement of air flow in the inlet, with an angular velocity of 3 rad/s - hydraulic resistance of the device was 701.52 Pa.

When rotating jalousie separator in the direction of air flow in the inlet of the angular velocity of 1 rad/s - hydraulic resistance machine was 1030.2 Pa.

When rotating jalousie separator and direction of air flow with an angular speed of 3 rad/s - hydraulic resistance of the device was 1017.75 Pa.

Figure 8 shows the dependence of the hydraulic resistance of the device on the value of the angular velocity jalousie separator. [20, 21]. Added considered angular velocity jalousie separator in the direction of air flow in the inlet of the machine.

**Fig. 2.** Model new construction unit  
**Fig. 3.** Characteristics of air flow in the machine with a fixed jalousie separator  
path of air flow  
Distribution of static pressure  
distribution of airflow

**Fig. 4.** Characteristics of air flow in the apparatus during rotation jalousie separator with an angular velocity of 1 rad / s  
Path of air flow  
distribution of static pressure  
distribution of airflow
Fig. 5. Characteristics of air flow in the apparatus during rotation jalousie separator with an angular speed of 3 rad/s forward movement of air flow in the inlet

Path of air flow  distribution of static pressure  distribution of airflow

Fig. 6. Characteristics of air flow in the apparatus during rotation jalousie separator with an angular velocity of 1 rad/s in the direction of air flow

path of air flow  distribution of static pressure  distribution of airflow
Fig. 7. Characteristics of air flow in the apparatus during rotation jalousie separator with an angular speed of 3 rad/s in the direction of air flow

path of air flow  
distribution of static pressure  
distribution of airflow

Fig. 8. Dependence of hydraulic resistance scrubber from the angular velocity of rotation jalousie separator.

Rotation jalousie separator leads to a symmetrical distribution of air flow in the machine. Between jalousie separator and the outer wall of the device adds a stream, whose main purpose is to counter the radial flow [22, 24, 25]. If we analyze the trajectory of the air flow in the cyclone (Fig. 3-7), we conclude that the rotation jalousie separator in the cyclone are more symmetrical velocity field and a more symmetrical distribution of static pressure in the flat section of the machine. This creates conditions for increasing the efficiency of the purification process [13, 15, 19, 23].

Taking into account the results of which are presented in Figure 8 shall decide whether to manufacture separator design with jalousie separator that spins forward movement of air flow in the inlet. The optimal value of the angular velocity of rotation will be determined during the experimental trials of the newly formed unit [26, 27].
CONCLUSIONS AND RECOMMENDATIONS
FOR FURTHER RESEARCHES

New constructions of equipment were created on the basis of the simulation design divided into 2 groups of prospective production implementation.

1. Production, using centrifugal, inertial dust collectors.
2. Production of preparation technical documentation for implementation.

Nowadays in the developed and presented in the paper model is developed and produced drawings production pilot plant for cleaning the air of dust in the smelting of metal.

REFERENCE

12. GOST 25631-83 (ST SEV 2355-81), GOST 25632-83 (ST SEV 3252-81). Wet dust collectors. Types and basic parameters. Specifications. Ukraine.