Abstract:
The article presents and describes the structure of a new piezoelectric gravimeter of automated aviation gravimetric system (AGS), which has a higher accuracy (1mGal) than other types of gravimeters known to date and automated signal processing. It also describes the operation principle of a piezoelectric gravimeter and presents its mathematical model. There have been determined conditions under which the need to use low-pass filter in automated AGS is prevented.

Keywords: piezoelectric gravimeter, aviation gravimetric system, gravitational acceleration, piezoelectric effect

1. Introduction

Information about gravitational field of the earth is used in gravimetry, seismology, aviation and space technology, geology, archeology etc. In classical physics, gravitational field is described only by potential and strength. Strength of gravitational field coincides with gravitational acceleration.

The most convenient way to determine features of earth’s gravitational field, that is gravitational acceleration $g_z$, is to use aviation gravimetric system (AGS) in which gravimeter is a sensitive element. By means of AGS gravimetric information in remote areas of the globe can be obtained much faster and at lower cost than by means of other gravimetric facilities or systems [1–7].

The effectiveness of AGS is mainly provided by a choice of sensitive element of the systems, i.e. a gravimeter. Today, the most famous AGS gravimeters are: quartz gravimeter GAL-S, developed by Air gravimetric Laboratory at the Institute of Physics of the Earth of the Russian Academy of Sciences; quartz gravimeter GI 1/1, developed by Ramensky Instrument Engineering Plant; string gravimeter “Graviton-M”, developed by State Scientific Production Enterprise “Aerogeofizika”, Bauman Moscow State Technical University and All-Union Scientific Research Institute of Geophysics; quartz gravimeter “Chekan-AM” developed by OJSC Concern “CSRI "Elektroprivor"; magnetic gravimeters MAG-1 and GT-IA, developed in CJSC “Gravimetric Technologies” and “Canadian Micro Gravity” respectively; gyroscopic gravimeters PIGA 16, PIGA 25, developed at Massachusetts Institute of Technology. The accuracy of these gravimeters is insufficient (3-10 mGal). The speed is slow as well. They are not automated. Results are processed on Earth after flight in a time-consuming process. Besides, these gravimeters measure vertical acceleration $\ddot{h}$ along with gravitational acceleration. High-accuracy measurement of $\ddot{h}$ is a complex scientific and technical challenge and requires the use of additional filters. Automated gyroscopic gravimeters $g_z$ have been developed and investigated in NTUU “KPI”. However, their accuracy is insufficient (2 mGal) [5, 8–13].

Therefore, improvement of accuracy and speed of gravitational acceleration measurement by creating a new gravimeter of automated AGS is a relevant scientific and technical challenge.

The objective of the article is to describe the structure and features of a new piezoelectric gravimeter (PG) of automated aviation gravimetric system which has a higher accuracy and speed that other gravimeters known today.

2. Single Channel Piezoelectric Gravimeter Description

A new piezoelectric gravimeter (PG) of automated aviation gravimetric system has been developed at Instrument Engineering Department of KPI under supervision of the honored master of sciences and engineering of Ukraine, Prof. Dr.-Ing. O.M. Bezvesilna [14]. This gravimeter has an accuracy of 1 mGal. The sensitive element of PG (Fig. 1) consists of a piezoelectric element (PE) which operates on the compression-stretching deformation, insulators at the ends of PE and inertial mass (IM) attached to the basis by a screw with purpose to increase reliability and durability of the structure. Due to cable PG is connected to an operational amplifier. The piezoelectric element is a multi-layer structure (piezoelectric packet) consisting of layers of crystalline lithium niobate.

Fig. 1. Structure of PG: 1 – PE; 2 – insulators; 3 – IM; 4 – basis; 5 – hermetic enclosure; 6 – screw; 7 – inlet cable
Let us consider PG oscillation system that includes the following elements: MI (m), stiffening element (k) and damping element (n). Air creating resistance to motion of IM is a damping element. Stiffening element is characterized by elastic properties of PE [5].

According to Newton’s second law, IM is under action of force which equals to:

\[ G = mg_z \]  

(1)

where G is gravity affecting IM; m is weight of IM.

\[ F_{ef} = -kx, \]  

(2)

where \( F_{ef} \) is PE elastic force; \( k \) is elasticity coefficient depending on PE properties.

IM damping force \( F_{rf} \):

\[ F_{rf} = -nx, \]  

(3)

where \( n \) is damping (friction) coefficient.

Giving the (1), (2) and (3) we get:

\[ mg_z = -nx - kx. \]  

(4)

The final equation of PG motion taking into account inertial force

\[ F_i = m \frac{d^2x}{dt^2} \]

would look like:

\[ m \frac{d^2x}{dt^2} + nx + kx = -mg_z. \]  

(5)

The equation of PG motion in terms of installation on Earth, would be

\[ g_z = -\frac{k}{m}x. \]  

(6)

In terms of installation of PG on an aircraft the component

\[ m \frac{d^2x}{dt^2} \]

should be considered.

Let’s consider the equation of automated AGS motion to determine anomalies of gravitational acceleration \( \Delta g \) [1, 7]:

\[ \Delta g = f_z + \]

\[ + \frac{v^2}{r} \left[ 1 - 2e \left( 1 - 2\cos^2\varphi \cdot \left( 1 - \frac{\sin^2 k}{2} \right) \right) + \right. \]

\[ + 2\omega_3 \cdot \sin k \cdot \cos \varphi \cdot -2h \cdot \frac{e}{r} \cdot \cos k \cdot \sin 2\varphi + \]

\[ + 2 \frac{\gamma_0}{r} + \omega_3 \cdot h \cdot \cos^2 \varphi - \hat{h} - \gamma_0. \]  

(7)

where \( f_z \) is an output signal; \( v \) is aircraft velocity; \( r \) is radius of aircraft location; \( e \) is ellipsoid compression; \( \varphi \) is latitude; \( k \) is aircraft course; \( \omega_3 \) is angular rotational rate of Earth; \( h \) is height of aircraft over ellipsoid; \( \hat{h} \) is vertical aircraft velocity; \( \gamma_0 \) is reference value of gravitational acceleration.

Let’s rewrite equation of AGS motion as

\[ \Delta g = f_z + E + A - \hat{h} - \gamma_0, \]  

(8)

where \( E \) is AGS output signal;

Eotvos correction;

\[ A = 2 \left( \frac{\gamma_0}{r} + \alpha_h \cos^2 \varphi \right) \] - elevation correction;

\[ \gamma_0 = \gamma_0(1 + 0.0052884 \sin^2 \varphi - 0.0000059 \sin^2 2\varphi) \] - reference value of gravitational acceleration;

\( \gamma_0e = 9,780,49 \) m/s\(^2\) - equatorial reference value of gravitational acceleration.

Output signal of AGS gravimeter after calculation and introduction of corrections and \( E \), \( A \) and \( \gamma_0 \) can be written as:

\[ T = f_z + g_z + \sigma_0, \]  

(9)
where \( \sigma_s \) is error due to impact of vertical aircraft acceleration.

Paper [1, 15, 17] presents analytical expressions of useful signal spectral densities \( G_m(\omega) \) and vertical aircraft acceleration \( G_a(\omega) \) and their characteristics (Fig. 3).

\[
\omega_a = \sqrt{\frac{s_p E_p}{h_p m}} = 0.1 \text{ rad/s,} \quad (10)
\]

where \( m \) is weight of inertial mass; \( s_p, E_p \) and \( h_p \) are area, material modulus of elasticity and height of piezoelement, respectively.

It is low-frequency filtration with a cutoff frequency of 0.1 rad/s through which \( g_z \) can be separated from \( g \), with an accuracy of 1 mGal. Besides, other components of perturbations with predominant frequency of more than 0.1 rad/s are also removed from PG output signal. These perturbations includes translational vibration acceleration with predominant frequency of more than 0.1 rad/s [16–19].

3. Double Channel Piezoelectric Gravimeter Description

As already noted sensitive element of a new PG [14] is a single-channel and constructed as a set of piezoplates. Frequency of sensitive element natural oscillations is 0.1 rad/s. Moreover, PG piezoplates serve both as a sensitive element and a low-pass filter, separating useful signal of gravitational acceleration from high-frequency interference (it is mostly vertical aircraft acceleration). However, the structure of a new PG does not provide for instrumental error compensation resulted from effects of changes in temperature, humidity and pressure of environment that are significant in extreme conditions, associated with gravimetric measurements on an aircraft.

Therefore, development of a new double-channel PG was proposed to compensate instrumental errors. For this sensitive element is made of two channels with one piezoelement in each. Piezoelements of both channels are identical and made in the form of piezoplates and inertial mass, which are fixed one over another. Piezoelement 3 of one channel is placed with piezoplates down, and piezoelement 4 of another channel is placed with piezoplates up. Outputs of piezoplates of both channels are connected to inputs of adder unit 5, the output of which is connected to an input of a gravimeter output signal calculation system 2 (Fig. 4).

![Fig. 3. Graph of useful signal spectral density (2) and vertical aircraft acceleration (1) [17]](image)

![Fig. 4. Block diagram of double-channel PG](image)
midity and ambient pressure. If these effects are projected on PG measuring axis O_z and given that piezoelement 3 of one channel works in compression and piezoelement 4 of another channel works in tension, we get:

\[ u_1 = k( mg_z + \dot{h} + \Delta t) \]

\[ u_2 = k( mg_z - m \dot{h} - \Delta t) \]

where \( u_1 \) is output electric signal of piezoplates of one channel; \( u_2 \) is output electric signal of piezoplates of another channel; \( m \) is weight of inertial mass in each channel; \( k \) is piezoelectric constant.

Output electric signals \( u_1 \) and \( u_2 \) of piezoplates of both channels are summed in adder unit 5:

\[ u_x = u_1 + u_2 = 2kmg_z \]

where \( u_x \) is output signal of adder unit 5.

Output signal \( u_x \) of adder unit 5 is supplied to gravimeter output signal processing system 2. In the final result we get an output signal \( T \) of PG output signal processing system 2. In the final result we get an output signal \( T \) of PG output signal processing system 2.

4. Conclusions

The paper considers a new piezoelectric gravimeter of automated AGS, which has higher accuracy (1 mGal) and speed (fully automated) than gravimeters known to date. It also describes the operation principle of a piezoelectric gravimeter and presents its mathematical model. It has been found that it is possible to set piezoelectric gravimeter natural frequency of rad/s and avoid the need to use low-pass filter in automated AGS due to selection of design parameters for a sensitive element of piezoelectric gravimeter. The paper analyzes the prospects for further improvement of accuracy of a new piezoelectric gravimeter by implementing the process of instrumental error compensation resulting from effects of changes in temperature, humidity and pressure of environment that are significant in extreme conditions, associated with gravimetric measurements on an aircraft.

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AUTHORS

Igor Korobiichuk* – Institute of Automatic Control and Robotics, Warsaw University of Technology, Boboli 8, 02-525 Warsaw, Poland.
E-mail: kiv_Igor@ist.ru.

Olena Bezvesilna – National Technical University of Ukraine “Kyiv Polytechnic Institute”, Avenue Peremogy, 37, Kyiv, Ukraine, 03056.
E-mail: bezvesilna@mail.ru

Andrii Tkachuk – Zhytomyr State Technological University, 10005 Zhytomyr; Ukraine.
E-mail: andrew_tkachuk@i.ua.

Tetyana Chilchenko – Zhytomyr State Technological University, 10005 Zhytomyr, Ukraine.
E-mail: xvulunka@mail.ru.

Michał Nowicki* – Industrial Research Institute for Automation and Measurements, Jerozolimskie 202, 02-486 Warsaw, Poland.
E-mail: mnowicki@mchtr.pw.edu.pl.

Roman Szewczyk – Institute of Metrology and Biomedical Engineering, Warsaw University of Technology, Boboli 8, 02-525 Warsaw, Poland.
E-mail: r.szewczyk@mchtr.pw.edu.pl.

*Corresponding author
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