APPLICATION OF NUMBER-SIZE (N-S) FRACTAL MODEL TO QUANTIFY OF THE VERTICAL DISTRIBUTIONS OF Cu AND Mo IN NOWCHUN PORPHYRY DEPOSIT (KERMAN, SE IRAN)

Determinaton of the vertical distribution of geochemical elemental concentrations is of fundamental importance in mineral exploration. In this paper, eight mineralized boreholes from the Nowchun Cu-Mo porphyry deposit, SE Iran, were used to identify of the vertical distribution directional properties of Cu and Mo values using number-size (N-S) fractal model. The vertical distributions of Cu and Mo in the mineralized boreholes show a positively skewed distribution in the former and a multimodal distribution in the latter types. Elemental threshold values for the mineralized boreholes were computed by fractal model and compared with the statistical methods based on the data obtained from chemical analysis of samples. Elemental distributions are not normal in these boreholes and their median equal to Cu and Mo thresholds. The results of N-S fractal analysis reveal that Cu and Mo values in mineralized boreholes are multifractals in nature. There are at least three geochemical populations for Cu and Mo in the boreholes and Cu and Mo thresholds have ranges between 0.07%-0.3% and 50-200 ppm, respectively. The results obtained by N-S fractal model were compared with geological observations in the boreholes. Major Cu and Mo enrichment correlated by monzonitic rocks and high amounts of observed Cu and Mo ores (Chalcopyrite and molybdenite) in the boreholes.

Keywords: Number-size (N-S) fractal model, Nowchun, Cu-Mo porphyry deposit, Borehole, Iran

ZASTOSOWANIE MODELU FRAKTALNEGO N-S (LICZBA-ROZMIAR) DO ILOŚCIOWEGO OKREŚLENIA PIONOWEGO ROZKŁADU Cu I Mo W ZŁOŻU PORFIROWYM (KERMAN, IRAN)

Określenie pionowego rozkładu stężenia danych pierwiastków chemicznych ma podstawowe znaczenie w trakcie prac poszukiwawczych. W artykule wykorzystano dane z ośmiu otworów w porfirytowym złożu Cu-Mo w Nowchum, w południowo-wschodnim Iranie, dla określenia pionowego rozkładu kierunkowych

Keywords: Number-size (N-S) fractal model, Nowchun, Cu-Mo porphyry deposit, Borehole, Iran
1. Introduction

Recognition of the geochemical elemental distributions in mineralized boreholes is significant and important to evaluate the quality and to interpret the quantity of mineral resources in the mine planning and extraction method selection. Many studies pointed out that, many elements, especially some trace elements, do not obey the normal distribution, but show a right skew or a power-law tail distribution (e.g. Ahrens, 1954a, b; 1966; Li et al., 2003). The distributions of geochemical elements in boreholes also exhibit power-law relationships and can be fitted by fractal models (Cheng et al., 1994; Sanderson et al., 1994; Monecke et al., 2001; Zuo et al., 2009).

Classical statistics methods for identifying thresholds and for separating different geochemical populations are existed (Hawkes & Webb, 1979; Reimann et al., 2005). If the frequency distribution of elemental concentrations is normal then mean + kσ (standard deviation, k ranges from 1 to 3) is equal to elemental threshold value. The statistical methods have been recommended that statistical parameters and graphics like as box-plot, cumulative probability plot, median + 2MAD (median absolute deviation) should be used in combination with spatial demonstration for separating anomalies and mineralization from the background (Reimann et al., 2005; Bai et al., 2010). Many studies depicted that the frequency distribution of elemental concentrations is not always normal but it is lognormal in many cases (Razumovsky, 1940; Li et al., 2003). The statistical methods have limited effectiveness in areas with complex geological settings. Additionally, these methods are based on quantities such as mean, percentile, and standard deviation could not recognize anomalies in regions with high-value of background or miss weak anomalies in regions with known mineral deposits, because the difference between the background and anomaly in such cases has been identified (Bai et al., 2010).

Fractal theory has been established and developed by Mandelbrot (1983), as an important branch of nonlinear mathematical sciences has been applied in different fields of geosciences since 1980s. Bolviken et al. (1992) and Cheng et al. (1994)'s studies show that geochemical dispersion patterns of different elements are fractals. Several fractal models have been developed and applied to geochemical exploration for separate anomalies from background, e.g., number-size (N-S) model (Mandelbrot, 1983), concentration-area (C-A) model and perimeter–area (P-A) model (Cheng et al., 1994) and concentration-distance (C-D) model (Li et al., 2003). Also, fractal methods have been applied to mineral resources studies for determining quantitative particulars of mineralization and mineral deposit characters (Turcotte, 1989, Agterberg, 1993; Sanderson et
al., 1994; Turcotte, 1996; 2002; Zuo et al., 2009; Wang et al., 2010; 2011; Afzal et al., 2010a, b; Afzal et al., 2011; Zuo 2011a, b; Zhao et al., 2011). Monecke et al. (2001) showed that base metal concentrations in boreholes from the Hellyer massive sulfide deposit located in Tasmania, Australia, obey N-S fractal model. Zuo et al. (2009) investigated the application of three fractal models including the characterization of vertical distribution of Cu concentration in Qulong copper deposit in Tibet, western China. In this work, the N-S fractal model is used to characterize the vertical distribution of the Cu and Mo to evaluate the mineralization continuity based on borehole datasets in Nowchun Cu-Mo porphyry deposit located in SE Iran is investigated. In this analysis the fractal model is briefly presented for demonstrating the data processing involved.

2. Number-size (N-S) fractal model

Mandelbrot (1983) defined a fractal distribution is that the number of objects \(N\) with a size greater than \(r\) scales with the a relationship between desired certain attributes (e.g., ore element) and their cumulative numbers of samples with those attributes. Agterberg (1995) proposed a grade-size multifractal model for giant and supergiant deposits. Monecke et al. (2005) applied N-S fractal model that describes element enrichments taking place by metasomatic processes causing the formation of hydrothermal ore deposits in Waterloo massive sulfide deposit, Australia. It is revealed that element enrichment can result in a fractal distribution. The N-S fractal model is used to describe the distribution of elements without pre-treatment and evaluation of data (Deng et al., 2010). The model is expressed by the following equation:

\[
N(\geq \rho) \propto \rho^{-\beta}
\]  

(1)

Based on Zuo et al. (2009), it also can be rewritten as

\[
\log[N(\geq \rho)] = -\beta \log (\rho)
\]

(2)

where \(\rho\) denotes the element concentration; \(N(\geq \rho)\) is the cumulative sample number with concentration values equal or greater than the concentration value (\(\rho\)) and \(\beta\) is the scaling exponent or fractal dimension of the concentration distribution. Frequency distributions of elements displayed in log-log plots show the logarithm of the cumulative number of samples exceeding a certain element concentration plotted against the logarithm of the element concentration (Monecke et al., 2005). Straight lines in the log-log plots have the slopes \(\beta\) within different concentration intervals (Deng et al., 2010).

3. Geological setting of Nowchun deposit

The study area, Nowchun Cu-Mo porphyry deposit, is located about 65 Km south of Rafsanjan city and 4 Km SW of Sarcheshmeh copper mine as named the biggest Iranian copper mine, SE Iran (Fig. 1). This deposit is situated in SE part of Urumieh-Dokhtar Cenozoic magmatic belt which is extended 1700 Km and 150 Km wide from NW to SE Iran, as shown in Fig. 1 (Alavi, 1994; Shahabpour, 1994; Alavi, 2004; Dargahi et al., 2010; Afzal et al., 2010a). This belt has been interpreted to be a subduction related Andean-type magmatic arc that has been active from the
Fig. 1. Location of Nowchun Cu-Mo porphyry deposit in Urumieh-Dokhtar magmatic belt modified based on Alavi (1994) and geological map of the deposit
Late Jurassic to the present. The rock types of this belt are composed of voluminous tholeiitic, calcalkaline, and K-rich alkaline intrusive and extrusive rocks with associated pyroclastic and volcanoclastic successions, along the active margin of the Iranian plates (Berberian & King, 1981; Dargahi et al., 2010). The famous Iranian porphyry deposits, such as Sarcheshmeh, Sungun, Meiduk, Kahang and Darehzar occur in this belt (Shahabpour, 1994; Atapour & Aftabi, 2007; Boomeri et al., 2009; Afzal et al., 2010; 2011).

There is an Eocene volcano-sedimentary complex includes granite, diorite – porphyry, tuff and rhyodacite rocks based on Yugoslavian geologists’ exploration in 1972 (Fig. 1). Eocene andesitic units surround the complex and alteration zones. Porphyry Dioritic units expand in the southern part of the deposit, adjusted of the granite. Most of the Oligocene –Miocene rhyodacitic units are located in NE part of the area. There are two major structural systems with trends of E-W and NE-SW faults (BEOGRAD-Yugoslavia, 1972).

Alteration zones consist of potassic, phyllic, argillic and propylitic in this deposit. Argillic zone is several parts in this area but phyllic is extended in most parts of the deposit. The Cu mineral occurrences are not very abundant and consist mostly of malachite and azurite within quartz veins and veinlets contain chalcopyrite in several parts of the area. Sulfide minerals include pyrite, chalcopyrite, molybdenite, galena, sphalerite, tetrahedrite, pyrrhotite, magnetite, hematite, marcasite, chalcocite, bornite and covellite, malachite and azurite. Pyrite, chalcopyrite and molybdenite exist are numerous in this deposit.

4. Statistics analysis of geochemical data

From 8 drillcores in the deposit, 1948 lithogeochemical samples have been collected at 2 m intervals. 1948 samples were collected from 8 boreholes were drilled. The vertical distribution of rock types in each borehole show that there are monzonitic and granodioritic rocks (Fig. 2). Samples were collected from mineralized parts in boreholes by spacing of 2 m. These samples were analyzed by ICP-MS for determination of Cu, Mo and related elements same as Pb, Zn and Fe. Detection limits for Cu and Mo are 0.01% and 0.1 ppm, respectively. Statistical results of Cu and Mo in these boreholes are listed in Tables 1 and 2. Cu means range from 0.14% to 0.3% and Mo mean ranges from 197.84 ppm to 412.1 ppm. A high value of Cu and Mo means exist in NOC_23. The variations between maximum and minimum of Cu and Mo concentrations in these boreholes are wide. The maximum values of Cu and Mo are 2.46% and 4417 ppm with NOC_16 and NOC_23, respectively. The distribution of Cu and Mo values along the borehole are illustrated in Fig. 2.

Cu and Mo distributions are not normal based on their histograms (Figs. 3 and 4) and the distributions of Cu and Mo values in the mineralized boreholes are positively skewed. It can be assumed that elemental medians are equal to mineralization threshold values for Cu and Mo in these boreholes. Tukey (1977) illustrated median can to distinguish systematic nonadditivity from discrepant observations such as anomalies or mineralized zones. Median is better than mean and mode for determination of elemental thresholds in mineral exploration (Hawkes and Webb, 1962; Levinson, 1974). The median value is generally less sensitive to outliers and thus is a more appropriate estimate of average background (Garrett, 1993; Harris et al., 1997).
Fig. 2. Vertical distribution of Cu and Mo within rock types in drilled boreholes of Nowchun deposit
The Cu statistical properties of 8 mineralized boreholes data obtained from Nowchun Cu-Mo porphyry deposit

<table>
<thead>
<tr>
<th>Borehole Name</th>
<th>Number of samples</th>
<th>Cu Max (%)</th>
<th>Cu Min (%)</th>
<th>Cu mean (%)</th>
<th>Cu median (%)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOC_6</td>
<td>169</td>
<td>1.89</td>
<td>0.07</td>
<td>0.29</td>
<td>0.25</td>
<td>0.19</td>
</tr>
<tr>
<td>NOC_9</td>
<td>226</td>
<td>1.45</td>
<td>0.01</td>
<td>0.29</td>
<td>0.26</td>
<td>0.17</td>
</tr>
<tr>
<td>NOC_10</td>
<td>216</td>
<td>0.93</td>
<td>0.07</td>
<td>0.25</td>
<td>0.21</td>
<td>0.14</td>
</tr>
<tr>
<td>NOC_13</td>
<td>250</td>
<td>1.06</td>
<td>0.02</td>
<td>0.14</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>NOC_14</td>
<td>282</td>
<td>1.04</td>
<td>0.05</td>
<td>0.25</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td>NOC_15</td>
<td>305</td>
<td>1.19</td>
<td>0.06</td>
<td>0.24</td>
<td>0.20</td>
<td>0.14</td>
</tr>
<tr>
<td>NOC_16</td>
<td>213</td>
<td>2.46</td>
<td>0.05</td>
<td>0.22</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>NOC_23</td>
<td>287</td>
<td>2.11</td>
<td>0.06</td>
<td>0.30</td>
<td>0.25</td>
<td>0.19</td>
</tr>
</tbody>
</table>

The Mo statistical properties of 8 mineralized boreholes data obtained from Nowchun Cu-Mo porphyry deposit

<table>
<thead>
<tr>
<th>Borehole Name</th>
<th>Number of samples</th>
<th>Mo Max (ppm)</th>
<th>Mo Min (ppm)</th>
<th>Mo mean (ppm)</th>
<th>Mo median (ppm)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOC_6</td>
<td>169</td>
<td>3250</td>
<td>25</td>
<td>291.53</td>
<td>225</td>
<td>310</td>
</tr>
<tr>
<td>NOC_9</td>
<td>226</td>
<td>1600</td>
<td>3</td>
<td>197.84</td>
<td>145</td>
<td>188.78</td>
</tr>
<tr>
<td>NOC_10</td>
<td>216</td>
<td>1300</td>
<td>20</td>
<td>248.22</td>
<td>190</td>
<td>209.74</td>
</tr>
<tr>
<td>NOC_13</td>
<td>250</td>
<td>2500</td>
<td>5</td>
<td>255.95</td>
<td>114</td>
<td>202.42</td>
</tr>
<tr>
<td>NOC_14</td>
<td>282</td>
<td>1218</td>
<td>19</td>
<td>217.81</td>
<td>173</td>
<td>167.99</td>
</tr>
<tr>
<td>NOC_15</td>
<td>305</td>
<td>3197</td>
<td>26</td>
<td>286.49</td>
<td>211</td>
<td>292.32</td>
</tr>
<tr>
<td>NOC_16</td>
<td>213</td>
<td>1900</td>
<td>13</td>
<td>254.65</td>
<td>190</td>
<td>262.32</td>
</tr>
<tr>
<td>NOC_23</td>
<td>287</td>
<td>44717</td>
<td>8</td>
<td>412.10</td>
<td>322</td>
<td>369.54</td>
</tr>
</tbody>
</table>

The median value is then determined by counting half way through the data values from the minimum to the maximum or vice versa, thereby dividing the univariate data set into two equal parts (Carranza, 2009). The statistics results show that Cu median are between 0.09% and 0.26% and Mo median has a range between 114 ppm and 322 ppm. High values of Cu and Mo medians are presented in NOC_9 and NOC_23 (Tables 1 and 2). Based on statistics method, there is one mineralization zone for Cu and Mo in each borehole.

### 5. Implementation of N-S fractal model

Elemental threshold values are obtained from the log-log plots of Cu and Mo (Figs 5 and 6). The log–log plots of cumulative number versus Cu and Mo values, respectively, show the variation of elemental concentration satisfying a multifractal model. The threshold values for Cu ranges from 0.07% to 0.3% (Table 3), but in most of boreholes the range is between 0.16% and 0.19%. Additionally, threshold values of Mo are between 50 and 200 ppm in the boreholes (Table 4). Last populations in log-log plots reveal high grade mineralization of Cu and Mo in these boreholes. High grade threshold values for Cu are between 0.32% and 0.69% (Table 3). Threshold vales for Mo high grade mineralization are between 316 ppm and 1000 ppm (Table 4).
Fig. 3. The histograms of Cu values from 8 mineralized boreholes
Fig. 4. The histograms of Mo values from 8 mineralized boreholes
There are three populations of Cu enrichment in NOC_6, NOC_9, NOC_13, NOC_14, NOC_15 and NOC_16 that major Cu enrichment threshold is between 0.32% and 0.69%, as listed in table 3. NOC_10 and NOC_23 have four Cu enrichment populations that major Cu enrichment start in 0.57% and 0.63% in NOC_10 and NOC_23, respectively (Fig. 4). Three populations for Mo enrichment and mineralization exist in NOC_6, NOC_9, NOC_10, NOC_15 and NOC_23. Major stage of Mo enrichment begins in range of 501 and 1000 ppm in these boreholes. In the other hand, four population of Mo enrichment and two major Mo mineralization stages occurred in NOC_13, NOC_14 and NOC_16, as shown in table 4. Log-log plots of Mo in these boreholes illustrate a well known multifractal nature, as depicted in Fig. 5. Major mineralization stages start in 316 ppm, 316 ppm and 764 ppm in NOC_13, NOC_14 and NOC_16, respectively. Last populations of Mo enrichment in these boreholes reveal high grade mineralization of Mo which is started in 794 ppm, 794 ppm and 1259 ppm in NOC_13, NOC_14 and NOC_16, respectively.

TABLE 3

<table>
<thead>
<tr>
<th>Borehole No.</th>
<th>Threshold Cu%</th>
<th>Threshold of Cu% major mineralization</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOC_6</td>
<td>0.30</td>
<td>0.57</td>
</tr>
<tr>
<td>NOC_9</td>
<td>0.18</td>
<td>0.32</td>
</tr>
<tr>
<td>NOC_10</td>
<td>0.16</td>
<td>0.57</td>
</tr>
<tr>
<td>NOC_13</td>
<td>0.07</td>
<td>0.35</td>
</tr>
<tr>
<td>NOC_14</td>
<td>0.18</td>
<td>0.69</td>
</tr>
<tr>
<td>NOC_15</td>
<td>0.16</td>
<td>0.63</td>
</tr>
<tr>
<td>NOC_16</td>
<td>0.16</td>
<td>0.45</td>
</tr>
<tr>
<td>NOC_23</td>
<td>0.19</td>
<td>0.63</td>
</tr>
</tbody>
</table>

TABLE 4

<table>
<thead>
<tr>
<th>Borehole No.</th>
<th>Threshold Mo (*ppm)</th>
<th>Threshold of Mo (ppm) major mineralization</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOC_6</td>
<td>200</td>
<td>891</td>
</tr>
<tr>
<td>NOC_9</td>
<td>100</td>
<td>501</td>
</tr>
<tr>
<td>NOC_10</td>
<td>200</td>
<td>794</td>
</tr>
<tr>
<td>NOC_13</td>
<td>200</td>
<td>316</td>
</tr>
<tr>
<td>NOC_14</td>
<td>50</td>
<td>794</td>
</tr>
<tr>
<td>NOC_15</td>
<td>177</td>
<td>1000</td>
</tr>
<tr>
<td>NOC_16</td>
<td>200</td>
<td>794</td>
</tr>
<tr>
<td>NOC_23</td>
<td>200</td>
<td>1000</td>
</tr>
</tbody>
</table>

6. Correlated results of fractal model with geological particulars

Subsurface data collected from borehole include lithology and mineralogy used for validation results of fractal model. Major ore minerals are chalcopyrite and molybdenite in this
Fig. 5. Log-log plots of cumulative numbers versus Cu grade in 8
Fig. 6. Log-log plots of cumulative numbers versus Mo grade in 8 mineralized boreholes
deposit. High grade Cu mineralization in boreholes correlated with chalcopyrite accumulation, as depicted in Fig. 7. Also, major stage of Mo enrichment is proper positive relationship with high amounts of molybdenite in the boreholes. Major Cu enrichment and high grade mineralization obtained from fractal model in NOC_6 associate with chalcopyrite especially in depths from 80 to 120 m, but in pyritic zones Cu values are lower than high grade threshold values (Fig. 7). Chalcopyrite is observed in more parts of these boreholes, but major stage of Cu enrichment, derived via the fractal model, has correlated with high amounts of chalcopyrite association. Major stage of Mo enrichment correlated with molybdenite accumulation in these boreholes (Fig. 7). In high grade Mo enrichment resulted from fractal method, especially higher than 1000 ppm, molybdenite abundances are illustrated.

Correlation between lithological particulars of boreholes and fractal model results show that there are good correlation between Cu and Mo enrichment with monzonitic rocks (Fig. 2). Only in NOC_9 Cu high grade mineralization occurred in granodiorite.

Fig. 7. Correlation between Cu and Mo vertical distribution and ore minerals in drilled boreholes of Nowchun deposit
Fig. 7 continue. Correlation between Cu and Mo vertical distribution and ore minerals in drilled boreholes of Nowchun deposit
7. Conclusions

Results from this study show the potential usage of the fractal model in for geochemical populations’ separation in drilled boreholes as a useful tool for geochemical and mineral exploration. The mineralization potential in depth can be recognized by characterizing the vertical distribution of geochemical concentration values of the mineralized element in borehole datasets.

The obtained results demonstrate that the vertical distribution of Cu and Mo values in mineralized boreholes has a positive skewed distribution and exhibits a multifractal model. Statistics method based on elemental frequency distribution shows Cu and Mo distributions are not normal. In this case, median equal to elemental threshold values which show there is only two geochemical populations in the boreholes, but results obtained from fractal model reveal there are at least three enrichment stages for Cu and Mo in the mineralized boreholes. Elemental medians were proximate to first threshold values derived by fractal model. The results obtained by fractal model illustrate a multifractal model, especially in NOC_10 and NOC_23 for Cu and NOC_13, NOC_14 and NOC_16 for Mo.

Cu concentrations in the boreholes may be a result of the three steps of enrichment, i.e., mineralization that has a threshold range between 0.07% and 0.30%. Mo variation in the mineralized boreholes has threshold values between 50 ppm and 200 ppm. Also, data analysis shows that major Cu and Mo enrichment are started at least in 0.32% and 316 ppm in these boreholes.

Results from fractal model are correlated well with monzonitic rocks and ore minerals. Major stages of Cu and Mo enrichments associate within chalcopyrite and molybdenite accumulation, respectively. It shows that monzonitic rock is major rock types for hosting mineralization. Moreover, Cu and Mo high mineralization derived via the fractal model has good correlation with mineralographical observations.

Although it may be easier to study geochemical characteristics of elements in boreholes with the fractal model but multifractal nature of log-log curves could be of important help to geoscientists for interpreting the stages which an element is enriched.

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